Limitations and potential of lifted probabilistic inference

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Outline

1. Assumptions behind current lifted algorithms
2. Lifted VE vs Search
3. Unknown Number of Individuals
4. Identity and Existence Uncertainty
Assumptions behind current lifted algorithms

- Known population size
- Conditioning and querying on conjunctions of ground atoms
- Unique names assumption: different references to individuals denote different individuals. There is no identity uncertainty.
- No querying about equality
- No existence uncertainty
Modelling Assumptions

A priori, individuals are indistinguishable and so share the same probabilities (exchangeability)

- unique names, known ≠ individuals
- unique names, unknown ≠ individuals
- identity uncertainty
- identity uncertainty, existence/type uncertainty
What is Observed / Queried?

- conjunction of ground assignments for few individuals
- conjunction of ground assignments for all individuals
- arbitrary propositions of ground assignments
- quantified (first-order) query, without equality
- quantified (first-order) query, with equality
Example Observation

Suppose we observe exactly one person asking a question:

$$\exists x \text{ asks\_question}(x)$$
Example Observation

Suppose we observe exactly one person asking a question:

$$\exists x \text{ asks\_question}(x)$$

$$\land \forall y \; y \neq x \rightarrow \neg \text{asks\_question}(y)$$

[What is a reasonable language for observations? What is an agent physically able to observe?]
Jaeger [AIJ 2000] show that

- If the query/observation language includes first-order logic with equality

- then there are queries that are not polynomial in population size.
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- If the query/observation language includes first-order logic with equality
- then there are queries that are not polynomial in population size.

Are there weaker languages with equality that avoid this proof? Are there stronger results that are possible?
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Problems with Lifted VE

- Intermediate representations (parfactors, counting formulae...) are not closed under lifted operations.
  - We need to ground sometimes
  - We need better intermediate representations
  - We need alternatives to lifted VE.
Lifted Search

- Variable elimination is the dynamic programming variant of recursive conditioning (and related search methods).
- VE creates intermediate representations.
- Search just evaluates factors when fully instantiated.
- In search, everything is evaluated in a particular context — perhaps we don’t need complex intermediate representations that need to anticipate all eventualities.
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What if we don’t know the population size?

- The previous methods assumed that we know how many individuals there are.
- What if we don’t know the population size, but only have a distribution over population size?
If \( n \) is distributed according to a geometric distribution with \( \forall k \ q = P(n = k | n \geq k) \), the expected value of \( p^n \) is

\[
\frac{q}{1 - p(1 - q)}
\]

Proof:

\[
\mathbb{E}_n(p^n) = \sum_{n=0}^{\infty} q(1 - q)^n p^n
\]

\[
= q \sum_{n=0}^{\infty} (p(1 - q))^n
\]

\[
= \frac{q}{1 - p(1 - q)}
\]
If $n$ is distributed according to a Poisson distribution
$$f(n, \lambda) = \frac{\lambda^n e^{-\lambda}}{n!} \quad (\lambda \text{ is the expected number of individuals})$$
the expected value of $p^n$ is
$$e^{-\lambda(1-p)}.$$

Proof:
$$\mathcal{E}_n(p^n) = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!} p^n$$
$$= e^{-\lambda} e^{\lambda p} \sum_{n=0}^{\infty} \frac{(\lambda p)^n e^{-\lambda p}}{n!}$$
$$= e^{-\lambda(1-p)}$$
Counting for unknown population size

- **Open problem:** Is there an analog of counting formulae for unknown population?
- Can we do lifted inference after finding some evidence about the number of objects? (Is there a conjugate distribution for counting with an unknown population?)
What is the population size is infinite?

- If \( n = \infty \) then
  \[
  p^n = \begin{cases} 
  1 & \text{if } p = 1 \\ 
  0 & \text{if } p < 1 
  \end{cases}
  \]

- Is there a (finite) counting formula for \( n = \infty \)?
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Symbol Denotations

**Constants/Terms**

- \( a \)
- \( b \)
- \( c \)
- \( d \)
- \( f(a) \)
- \( e \)

**Individuals**

- \( \)
In logic, $x = y$ is true if $x$ and $y$ refer to the same individual. $a \neq b, b = c, b = f(a), d = e, d \neq b, \ldots$
Assumptions behind current lifted algorithms
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Symbol Partitioning

<table>
<thead>
<tr>
<th>Constants/Terms</th>
<th>Individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td></td>
</tr>
<tr>
<td>b</td>
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<tr>
<td>c</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td></td>
</tr>
<tr>
<td>e</td>
<td></td>
</tr>
<tr>
<td>f(a)</td>
<td></td>
</tr>
</tbody>
</table>
Equality

Equality can be axiomatized with:

- $x = x$
- $x = y \Rightarrow y = x$
- $x = y \land y = z \Rightarrow x = z$
- $y = z \Rightarrow f(x_1, \ldots, y, \ldots, x_n) = f(x_1, \ldots, z, \ldots, x_n)$
- $y = z \land p(x_1, \ldots, y, \ldots, x_n) \Rightarrow p(x_1, \ldots, z, \ldots, x_n)$

The most common theorem-proving method is **paramodulation**: map each equivalence class of equal terms to a canonical element.
Probability and Identity

- Have a probability distribution over partitions of the terms
- The number of partitions grows faster than any exponential (Bell number)
- The most common method is to use MCMC: one step is to move a term to a new or different partition.
- Can we do this in a analytic / lifted manner?
Existence Uncertainty

- What is the probability there is a plane in this area?
- What is the probability there is a large gold reserve in some region?
- What is the probability that there is a third bathroom given there are two bedrooms?
- What is the probability that there are (exactly) three bathrooms given there are two bedrooms?
Existence Uncertainty

Two approaches:

- **BLOG**: you have a distribution over the number of objects, then for each number you can reason about the correspondence.

- **NP-BLOG**: keep asking: is there one more? e.g., if you observe a radar blip, there are three hypotheses:
  - the blip was produced by plane you already hypothesized
  - the blip was produced by another plane
  - the blip wasn’t produced by a plane
Existence Example

- False alarm
- Plane
- Observe blip

- False alarm
- Plane
- Same plane
- Another plane

- False alarm
- Plane
- Same plane
- Another plane

- False alarm
- Plane
- Same plane
- Another plane

- False alarm
- Plane
- First plane
- Second plane
- Another plane

False alarm plane observe blip...

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Lifted Learning

For a Bayesian there is only inference.

Model: $\forall i \ P(e(i) = 1|\theta) = \theta$

Infer: $P(\theta|e(1)\ldots e(k)) \propto \theta^{e(i)=1}(1 - \theta)^{e(i)=0}P(\theta)$
Conclusion

- Big gap between what we know how to do and the potential of lifted inference.
- Limited knowledge of the limitations of lifted inference.
- Exact inference forms the foundations of approximate inference (e.g., Rao-Blackwellization, variational methods).
- How to do lifted inference for richer languages?
- Lifted planning, lifted MDPs, lifted POMDPs, lifted RL....
- What do we need for real applications?