Learning Summary

- Given a task, use
  - data/experience
  - bias/background knowledge
  - measure of improvement or error

  to improve performance on the task.

Learning Summary

- Given a task, use
  - data/experience
  - bias/background knowledge
  - measure of improvement or error

  to improve performance on the task.
- Representations for:
  - Data (e.g., discrete values, indicator functions)
  - Models (e.g., decision trees, linear functions, linear separators, neural networks)

Learning Summary

- Given a task, use
  - data/experience
  - bias/background knowledge
  - measure of improvement or error

  to improve performance on the task.
- Representations for:
  - Data (e.g., discrete values, indicator functions)
  - Models (e.g., decision trees, linear functions, linear separators, neural networks)
- A way to handle overfitting (e.g., trade-off model complexity and fit-to-data, cross validation).

Learning Summary

- Given a task, use
  - data/experience
  - bias/background knowledge
  - measure of improvement or error

  to improve performance on the task.
- Representations for:
  - Data (e.g., discrete values, indicator functions)
  - Models (e.g., decision trees, linear functions, linear separators, neural networks)
- A way to handle overfitting (e.g., trade-off model complexity and fit-to-data, cross validation).
- Search algorithm (usually local, myopic search) to find the best model that fits the data given the bias.

Learning Objectives - Reinforcement Learning

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning

Learning Objectives - Reinforcement Learning

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning
- Implement basic state-based reinforcement learning algorithms: Q-learning and SARSA

Learning Objectives - Reinforcement Learning

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning
- Implement basic state-based reinforcement learning algorithms: Q-learning and SARSA
- Explain the explore-exploit dilemma and solutions

Learning Objectives - Reinforcement Learning

At the end of the class you should be able to:

- Explain the relationship between decision-theoretic planning (MDPs) and reinforcement learning
- Implement basic state-based reinforcement learning algorithms: Q-learning and SARSA
- Explain the explore-exploit dilemma and solutions
- Explain the difference between on-policy and off-policy reinforcement learning

Reinforcement Learning

What should an agent do given:

- Prior knowledge
- Observations
- Goal

Reinforcement Learning

What should an agent do given:

- Prior knowledge   possible states of the world
  possible actions
- Observations
- Goal

Reinforcement Learning

What should an agent do given:

- Prior knowledge — possible states of the world; possible actions
- Observations — current state of world; immediate reward / punishment
- Goal

Reinforcement Learning

What should an agent do given:

- Prior knowledge — possible states of the world; possible actions
- Observations — current state of world; immediate reward / punishment
- Goal — act to maximize accumulated (discounted) reward

Reinforcement Learning

What should an agent do given:

- Prior knowledge possible states of the world
  possible actions
- Observations current state of world
  immediate reward / punishment
- Goal act to maximize accumulated (discounted) reward
- Like decision-theoretic planning, except model of dynamics and model of reward not given.

Reinforcement Learning Examples

- Game -

Reinforcement Learning Examples

- Game - reward winning, punish losing

Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog -

Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog - reward obedience, punish destructive behavior

Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog - reward obedience, punish destructive behavior
- Robot -

Reinforcement Learning Examples

- Game - reward winning, punish losing
- Dog - reward obedience, punish destructive behavior
- Robot - reward task completion, punish dangerous behavior

Experiences

- We assume there is a sequence of experiences:

$$state, action, reward, state, action, reward, \ldots.$$

Experiences

- We assume there is a sequence of experiences:

$$state, action, reward, state, action, reward, \ldots.$$

- The sequence of experiences up to the time the agent has to choose its action is it history
- The agent has to choose its action as a function of its history.

Experiences

- We assume there is a sequence of experiences:

$$state, action, reward, state, action, reward, \ldots.$$

- The sequence of experiences up to the time the agent has to choose its action is it history
- The agent has to choose its action as a function of its history.
- At any time it must decide whether to

Experiences

- We assume there is a sequence of experiences:

$$state, action, reward, state, action, reward, \ldots.$$

- The sequence of experiences up to the time the agent has to choose its action is it history
- The agent has to choose its action as a function of its history.
- At any time it must decide whether to
  - explore to gain more knowledge
  - exploit knowledge it has already discovered

Why is reinforcement learning hard?

- What actions are responsible for a reward may have occurred a long time before the reward was received.
  - The dog is expected to determine that eating the shoe at the start of the day is what was resposible for it being scolded at the end of the day.

Why is reinforcement learning hard?

- What actions are responsible for a reward may have occurred a long time before the reward was received.
  - The dog is expected to determine that eating the shoe at the start of the day is what was resposible for it being scolded at the end of the day.
- The long-term effect of an action depend on what the agent will do in the future.
  - It might be okay for a robot to create a mess as long as it cleans up after itself.

Why is reinforcement learning hard?

- What actions are responsible for a reward may have occurred a long time before the reward was received.
  - The dog is expected to determine that eating the shoe at the start of the day is what was resposible for it being scolded at the end of the day.
- The long-term effect of an action depend on what the agent will do in the future.
  - It might be okay for a robot to create a mess as long as it cleans up after itself.
- The explore-exploit dilemma: at each time should the agent be greedy or inquisitive?

Reinforcement learning: main approaches

- search through a space of policies (controllers)

Reinforcement learning: main approaches

- search through a space of policies (controllers)
- learn a model consisting of state transition function $P(s'|a, s)$ and reward function $R(s, a)$; solve this an an MDP.

Reinforcement learning: main approaches

- search through a space of policies (controllers)
- learn a model consisting of state transition function $P(s'|a, s)$ and reward function $R(s, a)$; solve this an an MDP.
- learn $Q^*(s, a)$, use this to guide action.

Recall: Asynchronous VI for MDPs, storing $Q[s, a]$

(If we knew the model:)

Initialize $Q[S, A]$ arbitrarily
Repeat forever:

- Select state s, action a
- $Q[s, a] := R(s, a) + \gamma \sum_{s'} P(s'|s, a) \left(\max_{a'} Q[s', a'] \right)$

Asynchronous VI for Deterministic RL

initialize $Q[S, A]$ arbitrarily
observe current state s
repeat forever:
    select and carry out an action a
    observe reward r and state s'
        What do we know now?

Asynchronous VI for Deterministic RL

initialize $Q[S, A]$ arbitrarily
observe current state s
repeat forever:
    select and carry out an action a
    observe reward r and state s'
    $Q[s, a] := r + \gamma \max_{a'} Q[s', a']$
    $s := s'$

Computing Averages: Temporal Differences

- Suppose we have a sequence of values:

  $$v_1, v_2, v_3, \ldots$$

  and want a running estimate of the average of the first k values:

  $$A_k = \frac{v_1 + \cdots + v_k}{k}$$

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$A_k = \frac{v_1 + \cdots + v_{k-1} + v_k}{k}$$

$$=$$

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$A_k = \frac{v_1 + \cdots + v_{k-1} + v_k}{k}$$
$$= \frac{(k-1)A_{k-1} + v_k}{k}$$

Let $\alpha_k = \frac{1}{k}$, then

$$A_k =$$

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$
\begin{aligned}
A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\
&= \frac{(k-1)A_{k-1} + v_k}{k}
\end{aligned}
$$

  Let $\alpha_k = \frac{1}{k}$, then

$$
\begin{aligned}
A_k &= (1 - \alpha_k)A_{k-1} + \alpha_k v_k \\
&= A_{k-1} + \alpha_k(v_k - A_{k-1})
\end{aligned}
$$

  "TD formula"

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$\begin{aligned} A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\ &= \frac{(k-1)A_{k-1} + v_k}{k} \end{aligned}$$

  Let $\alpha_k = \frac{1}{k}$, then

$$\begin{aligned} A_k &= (1-\alpha_k)A_{k-1} + \alpha_k v_k \\ &= A_{k-1} + \alpha_k(v_k - A_{k-1}) \end{aligned}$$

  "TD formula"

- Often we use this update with α fixed.

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$
\begin{aligned}
A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\
&= \frac{(k-1)A_{k-1} + v_k}{k}
\end{aligned}
$$

  Let $\alpha_k = \frac{1}{k}$, then

$$
\begin{aligned}
A_k &= (1 - \alpha_k)A_{k-1} + \alpha_k v_k \\
&= A_{k-1} + \alpha_k (v_k - A_{k-1})
\end{aligned}
$$

  "TD formula"

- Often we use this update with α fixed.
- We can guarantee convergence to average if

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$\begin{aligned} A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\ &= \frac{(k-1)A_{k-1} + v_k}{k} \end{aligned}$$

  Let $\alpha_k = \frac{1}{k}$, then

$$\begin{aligned} A_k &= (1 - \alpha_k)A_{k-1} + \alpha_k v_k \\ &= A_{k-1} + \alpha_k(v_k - A_{k-1}) \end{aligned}$$

  "TD formula"
- Often we use this update with α fixed.
- We can guarantee convergence to average if $\sum_{k=1}^{\infty} \alpha_k = \infty$ and $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$.

Temporal Differences (cont)

- Suppose we know A_{k-1} and a new value v_k arrives:

$$\begin{aligned} A_k &= \frac{v_1 + \cdots + v_{k-1} + v_k}{k} \\ &= \frac{(k-1)A_{k-1} + v_k}{k} \end{aligned}$$

  Let $\alpha_k = \frac{1}{k}$, then

$$\begin{aligned} A_k &= (1-\alpha_k)A_{k-1} + \alpha_k v_k \\ &= A_{k-1} + \alpha_k(v_k - A_{k-1}) \end{aligned}$$

  "TD formula"
- Often we use this update with α fixed.
- We can guarantee convergence to average if $\sum_{k=1}^{\infty} \alpha_k = \infty$ and $\sum_{k=1}^{\infty} \alpha_k^2 < \infty$.
- E.g., $\alpha_k = 10/(9+k)$ treats more recent experiences more, but converges to average.

Q-learning

- Idea: store $Q[State, Action]$; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).

Q-learning

- Idea: store $Q[State, Action]$; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).
- Suppose the agent has an experience $\langle s, a, r, s' \rangle$
- This provides one piece of data to update $Q[s, a]$.
- An experience $\langle s, a, r, s' \rangle$ provides a new estimate for the value of $Q^*(s, a)$:

which can be used in the TD formula giving:

Q-learning

- Idea: store $Q[State, Action]$; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).
- Suppose the agent has an experience $\langle s, a, r, s' \rangle$
- This provides one piece of data to update $Q[s, a]$.
- An experience $\langle s, a, r, s' \rangle$ provides a new estimate for the value of $Q^*(s, a)$:

$$r + \gamma \max_{a'} Q[s', a']$$

which can be used in the TD formula giving:

Q-learning

- Idea: store $Q[State, Action]$; update this as in asynchronous value iteration, but using experience (empirical probabilities and rewards).
- Suppose the agent has an experience $\langle s, a, r, s' \rangle$
- This provides one piece of data to update $Q[s, a]$.
- An experience $\langle s, a, r, s' \rangle$ provides a new estimate for the value of $Q^*(s, a)$:

$$r + \gamma \max_{a'} Q[s', a']$$

which can be used in the TD formula giving:

$$Q[s, a] := Q[s, a] + \alpha \left(r + \gamma \max_{a'} Q[s', a'] - Q[s, a] \right)$$

Q-learning

initialize $Q[S, A]$ arbitrarily
observe current state s
repeat forever:
    select and carry out an action a
    observe reward r and state s'
    $Q[s, a] := Q[s, a] + \alpha \left(r + \gamma \max_{a'} Q[s', a'] - Q[s, a]\right)$
    $s := s'$

Properties of Q-learning

- Q-learning converges to an optimal policy, no matter what the agent does, as long as it tries each action in each state enough.
- But what should the agent do?
  - exploit: when in state s,
  - explore:

Properties of Q-learning

- Q-learning converges to an optimal policy, no matter what the agent does, as long as it tries each action in each state enough.
- But what should the agent do?
  - exploit: when in state s, select an action that maximizes $Q[s, a]$
  - explore: select another action

Exploration Strategies

- The ϵ-greedy strategy: choose random action with probability ϵ & choose a best action with probability $1-\epsilon$.

Exploration Strategies

- The ϵ-greedy strategy: choose random action with probability ϵ & choose a best action with probability $1 - \epsilon$.
- Softmax action selection: in state s, choose a with probability

$$\frac{e^{Q[s,a]/\tau}}{\sum_a e^{Q[s,a]/\tau}}$$

where $\tau > 0$ is the *temperature*.

Exploration Strategies

- The ϵ-greedy strategy: choose random action with probability ϵ & choose a best action with probability $1 - \epsilon$.
- Softmax action selection: in state s, choose a with probability

$$\frac{e^{Q[s,a]/\tau}}{\sum_a e^{Q[s,a]/\tau}}$$

  where $\tau > 0$ is the *temperature*.
- "optimism in the face of uncertainty": initialize Q to values that encourage exploration.

Exploration Strategies

- The ϵ-greedy strategy: choose random action with probability ϵ & choose a best action with probability $1 - \epsilon$.
- Softmax action selection: in state s, choose a with probability

$$\frac{e^{Q[s,a]/\tau}}{\sum_a e^{Q[s,a]/\tau}}$$

  where $\tau > 0$ is the *temperature*.
- "optimism in the face of uncertainty": initialize Q to values that encourage exploration.
- "upper confidence bounds" - take into account average + variance

Exploration Strategies

- The ϵ-greedy strategy: choose random action with probability ϵ & choose a best action with probability $1 - \epsilon$.
- Softmax action selection: in state s, choose a with probability

$$\frac{e^{Q[s,a]/\tau}}{\sum_a e^{Q[s,a]/\tau}}$$

  where $\tau > 0$ is the *temperature*.
- "optimism in the face of uncertainty": initialize Q to values that encourage exploration.
- "upper confidence bounds" - take into account average + variance
- Maintain a stochastic policy (distribution over actions)

Problems with Q-learning

- It does one backup between each experience.
  - Is this appropriate for a robot interacting with the real world?

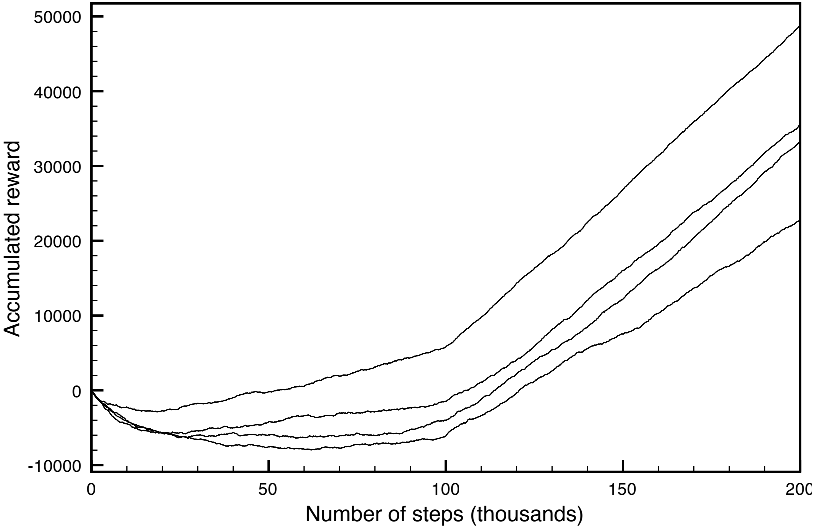
Problems with Q-learning

- It does one backup between each experience.
  - Is this appropriate for a robot interacting with the real world?
  - An agent can make better use of the data by

Problems with Q-learning

- It does one backup between each experience.
  - Is this appropriate for a robot interacting with the real world?
  - An agent can make better use of the data by
    — remember previous experiences and use these to update model (action replay)
    — building a model, and using MDP methods to determine optimal policy.
    — doing multi-step backups
- It learns separately for each state.

Evaluating Reinforcement Learning Algorithms



On-policy Learning

- Q-learning does off-policy learning: it learns the value of an optimal policy, no matter what it does.
- This could be bad if

On-policy Learning

- Q-learning does off-policy learning: it learns the value of an optimal policy, no matter what it does.
- This could be bad if the exploration policy is dangerous.
- On-policy learning learns the value of the policy being followed.

  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why?

On-policy Learning

- Q-learning does off-policy learning: it learns the value of an optimal policy, no matter what it does.
- This could be bad if the exploration policy is dangerous.
- On-policy learning learns the value of the policy being followed.

  e.g., act greedily 80% of the time and act randomly 20% of the time
- Why? If the agent is actually going to explore, it may be better to optimize the actual policy it is going to do.
- SARSA uses the experience $\langle s, a, r, s', a' \rangle$ to update $Q[s, a]$.

SARSA

initialize $Q[S, A]$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' using a policy based on Q
    $Q[s, a] :=$

SARSA

initialize $Q[S, A]$ arbitrarily
observe current state s
select action a
repeat forever:

    carry out action a
    observe reward r and state s'
    select action a' using a policy based on Q
    $Q[s, a] := Q[s, a] + \alpha \left(r + \gamma Q[s', a'] - Q[s, a]\right)$
    $s := s'$
    $a := a'$

Q-learning with Action Replay

initialize $Q[S, A]$ arbitrarily
$E = \{\}$
observe current state s
select action a
repeat forever:
  carry out action a
  observe reward r and state s'
  $E := E \cup \{\langle s, a, r, s' \rangle\}$
  $Q[s, a] :=$

Q-learning with Action Replay

initialize $Q[S, A]$ arbitrarily
$E = \{\}$
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    $E := E \cup \{\langle s, a, r, s' \rangle\}$
    $Q[s, a] := Q[s, a] + \alpha \left(r + \gamma \max_{a'} Q[s', a'] - Q[s, a]\right)$
    repeat for a while:
        select $\langle s_1, a_1, r_1, s'_1 \rangle \in E$
        $Q[s_1, a_1] :=$

Q-learning with Action Replay

initialize $Q[S,A]$ arbitrarily
$E = \{\}$
observe current state s
select action a
repeat forever:
  carry out action a
  observe reward r and state s'
  $E := E \cup \{\langle s, a, r, s' \rangle\}$
  $Q[s,a] := Q[s,a] + \alpha\,(r + \gamma \max_{a'} Q[s',a'] - Q[s,a])$
  repeat for a while:
    select $\langle s_1, a_1, r_1, s_1' \rangle \in E$
    $Q[s_1,a_1] := Q[s_1,a_1] + \alpha\,\left(r_1 + \gamma \max_{a_1'} Q[s_1',a_1'] - Q[s_1,a_1]\right.$
  $s := s'$
  $a := a'$

Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.

Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.
- Idea: learn the MDP and interleave acting and planning.

Model-based Reinforcement Learning

- Model-based reinforcement learning uses the experiences in a more effective manner.
- It is used when collecting experiences is expensive (e.g., in a robot or an online game); an agent can do lots of computation between each experience.
- Idea: learn the MDP and interleave acting and planning.
- After each experience, update probabilities and the reward, then do some steps of asynchronous value iteration.

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
    select and carry out action a
    observe reward r and state s'

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:

select and carry out action a
observe reward r and state s'
$T[s, a, s'] := T[s, a, s'] + 1$
$C[s, a] := C[s, a] + 1$
$R[s, a] := R[s, a] + (r - R[s, a])/C[s, a]$

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
    select and carry out action a
    observe reward r and state s'
    $T[s, a, s'] := T[s, a, s'] + 1$
    $C[s, a] := C[s, a] + 1$
    $R[s, a] := R[s, a] + (r - R[s, a])/C[s, a]$
    repeat for a while:
        select state s_1, action a_1
        $Q[s_1, a_1] :=$

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
  select and carry out action a
  observe reward r and state s'
  $T[s, a, s'] := T[s, a, s'] + 1$
  $C[s, a] := C[s, a] + 1$
  $R[s, a] := R[s, a] + (r - R[s, a])/C[s, a]$
  repeat for a while:
    select state s_1, action a_1

$$Q[s_1, a_1] := R[s_1, a_1] + \sum_{s_2} \frac{T[s_1, a_1, s_2]}{C[s_1, a_1]} \left(\gamma \max_{a_2} Q[s_2, a_2] \right)$$

  $s := s'$

Model-based learner

Data Structures: $Q[S, A]$, $T[S, A, S]$, $C[S, A]$, $R[S, A]$
Assign Q, R arbitrarily, $C = 0$, $T = 0$
observe current state s
repeat forever:
  select and carry out action a
  observe reward r and state s'
  $T[s, a, s'] := T[s, a, s'] + 1$
  $C[s, a] := C[s, a] + 1$
  $R[s, a] := R[s, a] + (r - R[s, a])/C[s, a]$
  repeat for a while:
    select state s_1, action a_1

$$Q[s_1, a_1] := R[s_1, a_1] + \sum_{s_2} \frac{T[s_1, a_1, s_2]}{C[s_1, a_1]} \left(\gamma \max_{a_2} Q[s_2, a_2] \right)$$

  $s := s'$

What goes wrong with this?

Reinforcement Learning with Features

- Usually we don't want to reason in terms of states, but in terms of features.
- In state-based methods, information about one state cannot be used by similar states.
- If there are too many parameters to learn, it takes too long.
- Idea: Express the value (Q) function as a function of the features. Most typical is a linear function of the features, or a neural network.

Reinforcement Learning

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x :=$$

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x := x - \eta \frac{df}{dx}$$

where η is the step size

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x := x - \eta \frac{df}{dx}$$

where η is the step size

To find a local minimum of real-valued function $f(x_1, \ldots, x_n)$:

- assign arbitrary values to $x_1, \ldots, x_n$
- repeat:

    for each x_i

$$x_i :=$$

Review: Gradient descent

To find a (local) minimum of a real-valued function $f(x)$:

- assign an arbitrary value to x
- repeat

$$x := x - \eta \frac{df}{dx}$$

where η is the step size

To find a local minimum of real-valued function $f(x_1, \ldots, x_n)$:

- assign arbitrary values to $x_1, \ldots, x_n$
- repeat:

  for each x_i

$$x_i := x_i - \eta \frac{\partial f}{\partial x_i}$$

Review: Linear Regression

- A linear function of variables $x_1, \ldots, x_n$ is of the form

$$f^{\overline{w}}(x_1, \ldots, x_n) = w_0 + w_1 x_1 + \cdots + w_n x_n$$

  $\overline{w} = \langle w_0, w_1, \ldots, w_n \rangle$ are weights. (Let $x_0 = 1$).
- Given a set E of examples.
  Example e has input $x_i = e_i$ for each i and observed value, o_e:

$$Error_E(\overline{w}) = \sum_{e \in E} (o_e - f^{\overline{w}}(e_1, \ldots, e_n))^2$$

- Minimizing the error using gradient descent, each example should update w_i using:

$$w_i :=$$

Review: Linear Regression

- A linear function of variables $x_1, \ldots, x_n$ is of the form

$$f^{\overline{w}}(x_1, \ldots, x_n) = w_0 + w_1 x_1 + \cdots + w_n x_n$$

$\overline{w} = \langle w_0, w_1, \ldots, w_n \rangle$ are weights. (Let $x_0 = 1$).

- Given a set E of examples.
Example e has input $x_i = e_i$ for each i and observed value, o_e:

$$Error_E(\overline{w}) = \sum_{e \in E} (o_e - f^{\overline{w}}(e_1, \ldots, e_n))^2$$

- Minimizing the error using gradient descent, each example should update w_i using:

$$w_i := w_i - \eta \frac{\partial Error_E(\overline{w})}{\partial w_i}$$

Review: Gradient Descent for Linear Regression

Given E: set of examples over n features
each example e has inputs $(e_1, \ldots, e_n)$ and output o_e:
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
repeat:
For each example e in E:
let $\delta = o_e - f^{\overline{w}}(e_1, \ldots, e_n)$
For each weight w_i:
$w_i := w_i + \eta \delta e_i$

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s, a) = w_0 + w_1 F_1(s, a) + \cdots + w_n F_n(s, a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value:
  - new "observed" value:

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s,a) = w_0 + w_1 F_1(s,a) + \cdots + w_n F_n(s,a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
  - old predicted value: $Q_{\overline{w}}(s,a)$
  - new "observed" value:

SARSA with linear function approximation

- One step backup provides the examples that can be used in a linear regression.
- Suppose $F_1, \ldots, F_n$ are the features of the state and the action.
- So $Q_{\overline{w}}(s,a) = w_0 + w_1 F_1(s,a) + \cdots + w_n F_n(s,a)$
- An experience $\langle s, a, r, s', a' \rangle$ provides the "example":
    - old predicted value: $Q_{\overline{w}}(s,a)$
    - new "observed" value: $r + \gamma Q_{\overline{w}}(s',a')$
- Treat $r + \gamma Q_{\overline{w}}(s',a')$ as a new training example for $Q(s,a)$ in linear regression (or other supervised learning algorithm).

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
    For $i = 0$ to n
        $w_i := w_i + \eta \delta F_i(s, a)$

SARSA with linear function approximation

Given γ:discount factor; η:step size
Assign weights $\overline{w} = \langle w_0, \ldots, w_n \rangle$ arbitrarily
observe current state s
select action a
repeat forever:
    carry out action a
    observe reward r and state s'
    select action a' (using a policy based on $Q_{\overline{w}}$)
    let $\delta = r + \gamma Q_{\overline{w}}(s', a') - Q_{\overline{w}}(s, a)$
    For $i = 0$ to n
        $w_i := w_i + \eta \delta F_i(s, a)$
    $s := s'$
    $a := a'$

Example Features

- $F_1(s, a) = 1$ if a goes from state s into a monster location and is 0 otherwise.
- $F_2(s, a) = 1$ if a goes into a wall, is 0 otherwise.
- $F_3(s, a) = 1$ if a goes toward a prize.
- $F_4(s, a) = 1$ if the agent is damaged in state s and action a takes it toward the repair station.
- $F_5(s, a) = 1$ if the agent is damaged and action a goes into a monster location.
- $F_6(s, a) = 1$ if the agent is damaged.
- $F_7(s, a) = 1$ if the agent is not damaged.
- $F_8(s, a) = 1$ if the agent is damaged and there is a prize in direction a.
- $F_9(s, a) = 1$ if the agent is not damaged and there is a prize in direction a.

Example Features

- $F_{10}(s, a)$ is the distance from the left wall if there is a prize at location P_0, and is 0 otherwise.
- $F_{11}(s, a)$ has the value $4 - x$, where x is the horizontal position of state s if there is a prize at location P_0; otherwise is 0.
- $F_{12}(s, a)$ to $F_{29}(s, a)$ are like F_{10} and F_{11} for different combinations of the prize location and the distance from each of the four walls.

  For the case where the prize is at location P_0, the y-distance could take into account the wall.

Problems and Variants of function approximation

- This algorithm tends to overfit to current experiences. "Catastrophic forgetting".
  Solution:

Problems and Variants of function approximation

- This algorithm tends to overfit to current experiences. "Catastrophic forgetting".
  Solution: remember old $\langle s, a, r, s' \rangle$ experiences and to carry out some steps of action replay

Problems and Variants of function approximation

- This algorithm tends to overfit to current experiences. "Catastrophic forgetting".
  Solution: remember old $\langle s, a, r, s' \rangle$ experiences and to carry out some steps of action replay
- Different function approximations, such as
  - a decision tree with a linear function at the leaves (regression tree)
  - a neural network

  could be used, but they requires a representation of the states and actions.

Problems and Variants of function approximation

- This algorithm tends to overfit to current experiences. "Catastrophic forgetting".
  Solution: remember old $\langle s, a, r, s' \rangle$ experiences and to carry out some steps of action replay
- Different function approximations, such as
  - a decision tree with a linear function at the leaves (regression tree)
  - a neural network

  could be used, but they requires a representation of the states and actions.
- Use the policy to do more than one-step lookahead (better estimate of $Q(s', a')$)

Evolutionary Algorithms

- Idea:
  - maintain a population of controllers
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers

Evolutionary Algorithms

- Idea:
  - maintain a population of controllers
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers
- If there are n states and m actions, there are policies.

Evolutionary Algorithms

- Idea:
  - maintain a population of controllers
  - evaluate each controller by running it in the environment
  - at each generation, the best controllers are combined to form a new population of controllers
- If there are n states and m actions, there are m^n policies.
- Experiences are used wastefully: only used to judge the whole controller. They don't learn after every step.
- Performance is very sensitive to representation of controller.