Tutorial 11

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1. **(NP Completeness of 4-coloring)** Graph coloring is discussed in the textbook (Kleinberg-Tardos Section 8.7, Erickson Section 30.10). Given a graph $G = (V, E)$, a k-coloring of G is a function f mapping the vertices V to the integers $\{1, ..., k\}$, such that $f(u) \neq f(v)$ for every edge $(u, v) \in E$.

   The **3-Coloring** decision problem is "Given a graph G, does G have a 3-coloring?" The **4-Coloring** decision problem is "Given a graph G, does G have a 4-coloring?"

   (a) Prove that the 4-Coloring problem is in NP.

   (b) The 3-Coloring problem is known to be NP complete. (This is proven in Kleinberg-Tardos Section 8.7 and Erickson Section 30.10.)

   Prove that the 4-Coloring problem is NP complete.

2. **(NP Completeness of Knapsack)** The **Subset Sum Problem** is defined as follows. (See Kleinberg-Tardos Sections 6.4 and 8.8 or Erickson Sections 3.3, 5.6.1 and 30.12.) The input is a list of numbers $a_1, ..., a_n$, and a target number A. The decision problem is "Does there exist a subset of $\{a_1, ..., a_n\}$ that adds up to exactly A?"

   The **Knapsack Problem** is defined as follows. The input is a list of item values $v_1, ..., v_n$, item weights $w_1, ..., w_n$, a target value V, and a maximum capacity W. The decision problem is "Does there exist a subset S of $\{1, ..., n\}$ for which $\sum_{i \in S} v_i \geq V$, and $\sum_{i \in S} w_i \leq W$?"

   (a) Prove that the Knapsack Problem is in NP.

   (b) The Subset Sum problem is proven to be NP-complete in Kleinberg-Tardos Section 8.8 and Erickson Section 30.12. Prove that the Knapsack Problem is NP-complete.