CPSC 320: Intermediate Algorithm Design and Analysis
Assignment #2, due Monday, May 25th, 2015 at 2:15pm in Room x235, Box 2

One mark will be deducted if your solution uses multiple sheets of paper that are not stapled.

[10] 1. Let us consider how cycles, connectivity and searching works in directed graphs. (See Section 3.5 of the text.)

A path in a directed graph is much the same as in an undirected graph, except that all edges must be pointed in the same direction. So, a path is a sequence of vertices $v_1, \ldots, v_k$ such that $v_1, \ldots, v_k$ are distinct vertices, and (v_i, v_{i+1}) is a directed edge for every i.

A cycle in a directed graph is much the same as in an undirected graph, except that all edges must be pointed in the same direction. So, a cycle is a sequence of vertices $v_1, \ldots, v_k$ such that $v_1, \ldots, v_{k-1}$ are distinct vertices, $v_1 = v_k$, and (v_i, v_{i+1}) is a directed edge for every i.

A directed graph is called *strongly connected* if, for every ordered pair of vertices u and v, there is a directed path from u to v. (So there must be one from v to u as well.)

Breadth-first search (BFS) in a directed graph works much like BFS in an undirected graph, except that edges must be followed in the right direction: from their tail towards their head.

[5] (a) Prove or disprove: In a directed graph with $n \geq 2$ nodes, if BFS run from two different nodes reaches n nodes both times, then this directed graph has a directed cycle.

[5] (b) Prove or disprove: In a directed graph with $n \geq 2$ nodes, if BFS run from two different nodes reaches n nodes both times, then this directed graph is strongly connected.

[10] 2. Some of your friends from Texas are planning an expedition to a small town deep in Yukon during their Christmas holidays. They have researched all travel options and have drawn up a directed graph G whose nodes represent intermediate destinations and edges represent the roads between them.

During this investigation, they have learned that weather conditions may cause large travel delays in Winter (duh!). They found an excellent Web site that can accurately predict how fast they will be able to travel along the roads; however, the speed of travel depends on the time of the year. More precisely, the Web site answer queries of the following form: "Given two sites v and w that are connected by an edge of G, and given a proposed starting time from v, what is the predicted arrival time $a_{v,w}(t)$ at w?". The Web site guarantees that $a_{v,w}(t) \geq t$ (you can not go back in time), and that $t' > t \rightarrow a_{v,w}(t') \geq a_{v,w}(t)$ (you can not arrive earlier by leaving later). Other than that, the function $a_{v,w}(t)$ may be arbitrary.

[7] (a) Give an efficient greedy algorithm to determine the fastest way to travel from your friends' starting point to their intended destination. (Assume that the Web site's predictions are correct.)

[3] (b) Determine the worst-case number of queries to the Web site that your algorithm will make, as a function of the number of vertices and edges of the graph G.

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[18] 3. Alice wants to throw a party and is deciding whom to call. She has n people to choose from, and she has made up a list of which pairs of these people know each other. She wants to pick as many people as possible, subject to two constraints: at the party, each person should have at least five other people whom they know and five other people whom they don't know.

[6] a. Give a greedy algorithm that takes as input the list of n people and the list of pairs who know each other and outputs the best choice of party invitees. Hint: Try to identify people who should not be invited.

[6] b. Prove the correctness of your algorithm from part (a).

[6] c. Explain which data structures you will need to use to implement your algorithm from part (a) efficiently, and analyze its time complexity.

[1] 4. (Bonus) How long did it take you to complete this assignment (not including any time you spent revising your notes before starting?)