CPSC 421: Introduction to Theory of Computing
Assignment #2, due Monday September 26th by 12pm (noon), via GradeScope.com

[9] 1. Let R and S be regular expressions over some alphabet Σ. Formally prove or disprove each of the following statements. (Here we use "=" to mean that the two regular expressions describe the same language.)

[3] a. $(R^*S^*)^* = (R \cup S)^*$.

[3] b. $R^* \cup S^* = (R \cup S)^*$

[3] c. $(R \cup S)^* S = (R^* S)^*$.

[9] 2. Prove that the following languages over $\Sigma = \{0, 1\}$ are not regular. You may use the fact that the class of regular languages is closed under unions, concatenations and complements.

[3] a. $L = \{ 0^n 1^m : n \leq m \}$.

[3] b. $L = \{ w : |w| \text{ is a perfect square} \}$. (That is, $|w| = n^2$ for some integer $n \geq 0$.)

[3] c. $L = \{ w : w \text{ is not a palindrome} \}$. (A palindrome is a string that reads the same forward and backward.)

[15] 3. Consider the language $F = \left\{ a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k \right\}$.

[7] a. Show that F is not regular.

[7] b. Show that F satisfies the condition of the pumping lemma. In other words, give an integer $p \geq 1$ and demonstrate that for all strings $w \in F$ with $|w| \geq p$ we can find $x, y, z \in \{a, b, c\}^*$ such that

- $|y| > 0$,
- $|xy| \leq p$, and
- $xy^n z \in F$ for all $n \geq 0$.

[1] c. Explain why this does not contradict the pumping lemma.

[10] 4. Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, let

$$B = \{ uv : u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v| \}.$$

[5] a. Give a PDA that recognizes B.

[5] b. Give a CFG that generates B.

[20] 5. **OPTIONAL BONUS QUESTION**

Professor Dumas thinks he understands the pumping lemma. His interpretation is that "a finite automaton has a limited amount of complexity, so if it accepts a very long string, that string must have some repeated structure inside". To formalize his interpretation, he proposes the following conjecture.

Conjecture 1. Let L be regular language over Σ. Then there exists an integer $p \geq 1$ such that, for every string $w \in L$ with $|w| \geq p$,

$$\exists x, y, z \in \Sigma^* \text{ and } i \geq 2 \quad \text{such that} \quad w = xy^i z \text{ and } y \neq \epsilon. \tag{1}$$

Professor Dumas' conjecture is false. Let's try to understand why.

[10] a. Find a DFA $M = (Q, \Sigma, \delta, q_0, F)$ with with $|Q| \leq 3$ and $|\Sigma| \geq 3$ and a string w that is accepted by M, and has $|w| > |\Sigma|^2$, but does not satisfy (1).

[10] b. Can you find an even longer string w that is accepted by your M but does not satisfy (1)? You can use computer simulations if you like. If your solution involves any ideas found in the literature or online, be sure to give a citation.