CPSC 421/501 Intro to Theory of Computing (Term 1, 2013-14)
Assignment 3

Due: Wednesday Oct 9th, in class.

Question 1: [10 marks]

Give a context-free grammar that generates the language

$$L = \left\{ a^i b^j c^k \ : \ i = j \text{ or } j = k \text{ where } i, j, k \geq 0 \right\}.$$

Question 2: [20 marks]

Let G be the following CFG over $\Sigma = \{a, b\}$.

$$\begin{aligned} S &\to \quad aSb \mid bY \mid Ya \\ Y &\to \quad bY \mid aY \mid \epsilon \end{aligned}$$

- **(a):** Find a simple, English description of $L(G)$. You must formally prove that your description is correct.
- **(b):** Use your description from part (a) to give a CFG for the complement of $L(G)$, i.e. $\Sigma^* \setminus L(G)$.

Question 3: [20 marks]

- **(a):** Show that the class of decidable languages is closed under complement (i.e., if L is decidable then $\Sigma^* \setminus L$ is decidable).
- **(b):** Does the same argument as part (a) show that the class of Turing-recognizable languages is closed under complement? Why or why not?
- **(c):** Show that the class of Turing-recognizable languages is closed under concatenation (i.e., if L_1 and L_2 are Turing-recognizable then so is $L_1 \circ L_2 = \{ \, xy \, : \, x \in L_1, \ y \in L_2 \, \}$).

Question 4: [10 marks]

A **doubly-infinite multitape Turing machine** is like an ordinary Turing machine, except that it has $k \geq 1$ tapes, and the tapes are infinite to the left as well as to the right. Each tape has its own head for reading and writing. All tapes are initially blank, except for the first tape. A portion of the first tape contains the input string, and otherwise it is blank. The first tape's head is initially positioned on the first symbol of the input string. The transition function is

$$\delta \ : \ Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R, S\}^k ,$$

where L means "move left", R means "move right" and S means "stay still", as in Section 3.2 of the textbook.

Prove that every doubly-infinite multitape Turing machine has an equivalent single-tape Turing machine (whose tape is only infinite to the right). Your answer should have roughly the same level of detail as the proof of Theorem 3.13 in the text.

OPTIONAL BONUS QUESTION: [20 marks]

Question 5: Define the language L over $\Sigma = \{0, 1\}$ by

$$L \ = \ \{ \, x\#y \, : \, x, y \in \Sigma^* \text{ and } x \neq y \, \} .$$

Prove that L is context-free by either giving a CFG that describes L or a PDA that recognizes L.