CPSC 421/501 Intro to Theory of Computing (Term 1, 2013-14)
Assignment 1

Due: Friday, September 20th, in class.

Question 1: Give regular expressions for the following languages. You do not need to justify your answers.

For convenience, you can use the following notation:

- $(x)+$ (which matches one or or occurrences of the pattern x)
- $(x)?$ (which matches zero or one occurrences of the pattern x)
- Σ (which matches any single character in Σ)

The alphabet is always $\Sigma = \{0, 1\}$.

(a): The set of strings that begin with a 1 and end with a 0.

(b): The set of all strings that contain at least three 1s.

(c): The set of all strings that do not contain 110 as a substring.

(d): The set of *nonempty* strings that do not contain two consecutive occurrences of the same symbol.

Question 2: For each of the following languages, provide a DFA that accepts it. You should define each DFA by drawing it as a directed graph with accepting states marked by double concentric circles. You do not need to justify your answers.

Your DFAs should not have any undefined transitions. Any transitions that are intended for immediate rejection should be sent to infinite looping states (as in the solution to Exercise 1.4 (b) in the textbook).

Your solution does not need to give the smallest possible DFA, but marks may be deducted if your solution is unnecessarily complicated. For each of the questions there is a DFA with less than 10 states.

(a): The language $L_1 = \{\ x \in \{a, b, c\}^* \ : \ x$ has exactly one occurrence of the substring $ab\ \}$.

(b): The language $L_2 = \{\ x \in \{0, 1\}^* \ : \ x$ is nonempty and begins and ends with the same symbol $\}$.

(c): The language $L_3 = \{\ x \in \{0, 1\}^* \ : \ $the 3rd from last symbol in x is 1 $\}$.

Question 3: Exercise 1.14 (Sipser, 3rd edition).

(a): Show that if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

(b): Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

Question 4: For each claim below, state whether it is true or false, and prove your answer.

(a): Is it true that, for all languages A and B, we have $(A^* \cap B^*)^* = (A \cap B)^*$?

(b): Is it true that, for all languages A and B, we have $(A^* \cup B^*)^* = (A \cup B)^*$?

OPTIONAL BONUS QUESTIONS

Question 5: Exercise 1.69 (3rd edition only).

Let $\Sigma = \{0,1\}$. Let $WW_k = \{ \, ww \, : \, w \in \Sigma^* \text{ and } w \text{ is of length } k \, \}$.

(a): Show that, for each k, any DFA that recognizes WW_k must have at least 2^k states.

(b): Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k. (The number of states should at most a polynomial in k.)

Let

$$L_k \;=\; \{ \, xy \, : \, x, y \in \Sigma^* \text{ and } |x| = |y| = k \text{ and } x \neq y \, \} \, .$$

If you'd prefer to solve the problem for L_k instead of $\overline{WW_k}$, that is ok too.