CPSC 421/501 Intro to Theory of Computing (Term 1, 2012-13)
Assignment 2

Due: Wednesday, October 10th, in class.

For any question, if you write "I do not know the answer to this question", you will receive 20% of the marks for that question.

Question 1: Exercise 2.47 (3rd edition only).

Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, let $B = \{ \, uv \, : \, u \in \Sigma^*, \; v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v| \, \}$.

- **(a):** Give a PDA that recognizes B.
- **(b):** Give a CFG that generates B.

Question 2: For a string $w \in \Sigma^*$, let $\text{rev}(w)$ be the *reverse* of that string. (That is, if $w = w_1 w_2 ... w_k$, where each $w_i \in \Sigma$, then $\text{rev}(w) = w_k w_{k-1} ... w_2 w_1$.) For a language L over Σ, define

$$\text{rev}(L) \;=\; \{ \, \text{rev}(w) \, : \, w \in L \, \} \, .$$

Prove that if L is context-free, then so is $\text{rev}(L)$.

Question 3: Exercise 2.31 (2nd and 3rd edition)

Let L be the language over the alphabet $\{0, 1\}$ consisting of all palindromes that contain an equal number of 0s and 1s. (A *palindrome* is a string that equals its reverse.) Show that L is not a CFL.

Question 4: For the following questions, state whether the answer is yes or no, and *prove* your answer.

- **(a):** Are CFLs closed under intersection? (If L_1 and L_2 are CFLs, is $L_1 \cap L_2$ necessarily a CFL?)
- **(b):** Are CFLs closed under complement? (If L is a CFL over the alphabet Σ, is $\Sigma^* \setminus L$ necessarily a CFL?)

Question 5: Exercise 2.19 (2nd and 3rd edition)

Let G be the following CFG over $\Sigma = \{a, b\}$.

$$\begin{aligned} S \to & \quad aSb \mid bY \mid Ya \\ Y \to & \quad bY \mid aY \mid \epsilon \end{aligned}$$

Give a simple description for $L(G)$ in English. Use that description to give a CFG for the complement of $L(G)$, i.e. $\Sigma^* \setminus L(G)$,

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OPTIONAL BONUS QUESTION:

Question 6: Assignment 1 showed that, for any fixed k, the language L_k over $\Sigma = \{0, 1\}$ defined by

$$L_k \;=\; \{ \, xy \, : \, x, y \in \Sigma^* \text{ and } |x| = |y| = k \text{ and } x \neq y \, \}$$

is regular. We now consider a similar problem where k is not fixed. Define the language L over Σ by

$$L \;=\; \{ \, xy \, : \, x, y \in \Sigma^* \text{ and } |x| = |y| \text{ and } x \neq y \, \} \, .$$

Prove that L is a context-free language.