CPSC 421/501 Intro to Theory of Computing (Term 1, 2012-13)
Assignment 1

Due: Friday, September 28th, in class.

Question 1: Give regular expressions for the following languages. You do not need to justify your answers.

For convenience, you can use the notation $(x)+$ (which matches one or or occurrences of the pattern x) or $(x)?$ (which matches zero or one occurrences of the pattern x).

- **(a):** The set of strings over $\{a, b\}$ that do not contain two consecutive occurrences of the same letter.
- **(b):** The set of *nonempty* strings over $\{a, b\}$ that do not contain two consecutive occurrences of the same letter.
- **(c):** The set of strings over $\{a, b\}$ containing exactly three as.

Question 2: For each of the following languages, provide both a regular expression and a DFA that accepts it. You should define each DFA by drawing it as a directed graph with accepting states marked by double concentric circles. You do not need to justify your answers.

Your DFAs should not have any undefined transitions. Any transitions that are intended for immediate rejection should be sent to infinite looping states (as in the solution to Exercise 1.4 (b) in the textbook).

- **(a):** The language of strings over alphabet $\Sigma = \{0, 1, 2\}$ in which the first character of the string does not appear elsewhere in the string. (You can either assume ϵ is in the language or not, as you wish.)
- **(b):** The language of strings over alphabet $\Sigma = \{0, 1\}$ that contain 000 as a substring but not 111.

Question 3: For each claim below, state whether it is true or false, and prove your answer.

- **(a):** Is it true that, for all languages A and B, we have $(A^* \cap B^*)^* = (A \cap B)^*$?
- **(b):** Is it true that, for all languages A and B, we have $(A^* \cup B^*)^* = (A \cup B)^*$?

Question 4: Exercise 1.69 (3rd edition only).

Let $\Sigma = \{0, 1\}$. Let $WW_k = \{\, ww : w \in \Sigma^* \text{ and } w \text{ is of length } k \,\}$.

- **(a):** Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.
- **(b):** Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k. (The number of states should at most a polynomial in k.)

  Let
  $$L \;=\; \{\, xy : x, y \in \Sigma^* \text{ and } |x| = |y| = k \text{ and } x \neq y \,\}.$$
  If you'd prefer to solve the problem for L instead of $\overline{WW_k}$, that is fine with me.

Question 5: Exercise 1.71 (3rd edition only).

Let $\Sigma = \{0, 1\}$.

- **(a):** Let $A = \left\{\, 0^k u 0^k : k \geq 1 \text{ and } u \in \Sigma^* \,\right\}$. Show that A is regular.
- **(b):** Let $B = \left\{\, 0^k 1 u 0^k : k \geq 1 \text{ and } u \in \Sigma^* \,\right\}$. Show that B is not regular.