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# Long-term Stock Market Forecasting using Gaussian Processes

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5 **Abstract**

6 Forecasting stock market prices is an attractive topic to researchers from  
7 different fields. The accuracy of this forecasting is very critical for market  
8 dealers. The existing forecast models show valid results in short-term  
9 forecasting; however, the accuracy of these models degrades in long-term  
10 forecasting. In this project, the Gaussian processes are applied to forecast  
11 the stock market trend. We select three stocks from NASDAQ Stock Market  
12 to test the proposed model. The experiment results show worthy findings of  
13 the stocks behavior over different periods of time. This model could help  
14 investors to make the long-term investment or to validate their investment  
15 decisions.

16  
17 **1 Introduction**

18 Nowadays, most of the stock market traders relay on machine learning techniques to analyze  
19 and forecast stock prices and index changes. The accuracy of these techniques is still an  
20 issue due to several factors such as seasons, political situation and economic conditions that  
21 cause fluctuation of stock market movement (Ou & Wang, 2009). Although this movement  
22 does not follow exact seasonal cycles all the time, it is highly recommended not to ignore  
23 these cycles (Jeffrey & Kass, 2012). This project proposes a new application of long-term  
24 forecasting with the Gaussian processes (GP) model in (Chapados & Bengio, 2007) in stock  
25 market.

26 In general, there are two methodologies to predict stock prices: Fundamental Analysis and  
27 Technical Analysis. The Fundamental Analysis relies on the past performance of the  
28 company to make predictions. The Technical Analysis deals with past stock prices to  
29 understand its pattern change and predict the future prices. Although, most of machine  
30 learning application show more interest in Technical Analysis, hybrid approaches could  
31 combine both methodologies to make prediction (Ayodele, et al., 2012). In this paper,  
32 Technical Analysis will be used to perform long-term predictions in stock prices.

33  
34 **1.2 Motivation**

35 In stock market, investors need long-term forecasting techniques to choose the right time to  
36 buy/sell stocks to maximize their profits or to minimize their loss. The majority of existing  
37 stock market forecasting techniques require predictions over a single continuous time series.  
38 These techniques perform well in short-term (a day to weeks) time series prediction but the  
39 accuracy of these techniques degrades when long-term time series prediction is made. The  
40 motivation for this project comes from the presence of large amount of historical data in  
41 stock market and the ability to use of GP in long-term time series forecasting (Chapados &  
42 Bengio, 2007). The goal of this project is to help investors to choose the right stock to invest

43 in, based on long-term forecasting. Also, this project can assist investors to predict the right  
44 time to buy/sell in stock market to maximize the profit.

45 The rest of this paper is organized as follows. Section 2 sheds light on the related work and  
46 gives background about stock market and Gaussian processes GP .In Section 3, we present  
47 the methodology and the collected data. Section 4 gives a summary of the results obtained  
48 and the analysis of these results. Section 5 concludes with future directions of work.

49

## 50 **2 Background and related work**

51 Several forecasting models have considered Gaussian processes for time-series forecasting  
52 (Chapados & Bengio, 2007; Todd & Correa, 2007; Groot et al., 2011). In this section, we  
53 give an overview about some related studies. Also, brief introductions about stock market  
54 and Gaussian processes are covered.

55 Stock market trend prediction using Gaussian processes were tackled in (Todd & Correa,  
56 2007). This study shows that increasing the size of training data (a long time period) gives  
57 more accurate prediction. The drawback of this approach is the high computational time.  
58 Multiple-step time series forecasting using sparse Gaussian process was addressed in (Groot  
59 et al., 2011). This approach produced more accurate and faster predictions than standard GP  
60 approach. Chapados and Bengio (2007), introduced a Long-term forecasting approach using  
61 Gaussian processes. This approach used functional representation of time series to perform  
62 long-term forecasting. Commodity spread trading data was used as an application for this  
63 approach. In this project, the technique in (Chapados & Bengio, 2007) is applied to forecast  
64 long-term prices in stock market.

65

### 66 **2.1 Stock Market**

67 Stock markets are public markets for trading the companies' stocks (shares) at agreed prices.  
68 Investors (companies or individuals) are allowed to buy and sell stocks and these  
69 transactions are called trading. The stock prices depend on the demands and supplies; it goes  
70 high when there is high demand and falls down at low demand. In stock market, a quarter  
71 (Q) refers to one-fourth of a year. The four quarters are: January, February and March (Q1);  
72 April, May and June (Q2); July, August and September (Q3); and October, November and  
73 December (Q4). Investors use the past several quarters to forecast the future of the stocks  
74 (Wikipedia, 2013).

75 Stock markets are considered as one of the economic indicators of countries. The growth of  
76 stock prices attracts investors and increases the companies' values. In general, the growth in  
77 stock market reflects the strength and development of countries' economics so that countries  
78 watch and control the behavior of stock market (Preethi & Santhi, 2012). The size of global  
79 stock market was estimated at about \$54 Trillion in 2010 (anonymous, 2012).

80

### 81 **2.2 Gaussian Processes**

82 A Gaussian process (GP) is a popular technique in machine learning and is widely used in  
83 time series analysis (Mori & Ohmi, 2005). Rasmussen and Williams (2006) defined GP as “a  
84 collection of random variables, any finite number of which have a joint Gaussian  
85 distribution”. The GP is used to characterize probability distribution over functions by  
86 defining two functions: mean function  $m(x)$  and the covariance function mean function  
87  $k(x_1, x_2)$  (Rasmussen & Nickisch, 2006). To describe a real process  $f(x)$  as a GP, we write:

$$88 \quad f(x) \sim \mathcal{GP}(m(x), k(x_1, x_2)), \quad (1)$$

89 where,

90

$$m(x) = \mathbb{E}[f(x)],$$
$$k(x_1, x_2) = \mathbb{E}[(f(x_1) - m(x_1))(f(x_2) - m(x_2))].$$

91 In regression, given a data set  $D$  of  $N$  observations;  $D = \{(x_i, y_i) \mid i = 1, \dots, N\}$ , with  $x_i \in \mathbb{R}^D$

92 and  $y_i \in \mathbb{R}$ , the goal is to predict new  $y_*$  given  $x_*$  using  $f(x)$  such that:  $y_i = f(x_i) + \delta_i$  where  
 93  $\delta_i$  is a Gaussian noise with mean zero and variance  $\sigma^2$ . However, we assume that closing  
 94 prices in stock market are noise free because true prices are evaluated at closing time (Todd  
 95 & Correa, 2007). The prior distribution of the observed target  $y$  is given by

96 
$$y \sim \mathcal{N}(0, K(X, X)), \quad (2)$$

97 where,  $K(X, X)$  is the covariance matrix between all pairs of training points and  $X$  is  $(n \times m)$   
 98 matrix of input. In this project, (Gaussian) radial basis function kernel, or RBF kernel is  
 99 used:

100 
$$k(x_1, x_2) = \exp(-\sigma \|x_1 - x_2\|^2). \quad (3)$$

101 The predictive distribution of  $y_*$  can be computed by conditioning on the training data to get  
 102  $p(f(x_*)|x_*, D)$ . The joint distribution over  $y$  and predictions of  $x_*$  is given by:

103 
$$\begin{bmatrix} y \\ f(x_*) \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K(X, X) & K(X, x_*) \\ K(x_*, X) & K(x_*, x_*) \end{bmatrix}\right). \quad (4)$$

104 The conditional distribution of (2) allows us to get the predictive distribution of  $y_*$  with the  
 105 following mean and covariance (Ou & Wang, 2011):

106 
$$\bar{f}(x_*) = K(X, x_*)^T (K + \sigma_n^2 I)^{-1} y, \quad (5)$$

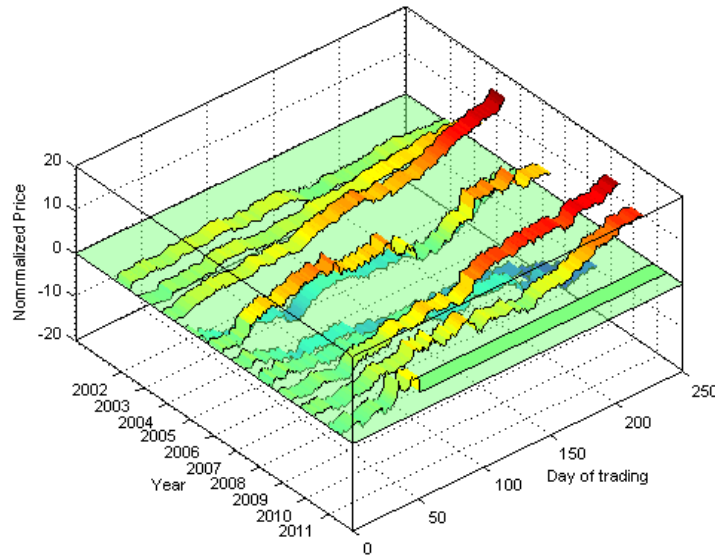
107 
$$V_f(x_*) = K(x_*, x_*) - K(X, x_*)^T (K + \sigma_n^2 I)^{-1} K(X, x_*) \quad (6)$$

108

109 **3 Methodology**

110 The main idea of this approach is to avoid representing the whole history as one time series.  
 111 Each time series is treated as an independent input variable in the regression model  
 112 (Chapados & Bengio, 2007). For trading year  $i$ , there are  $M_i$  trading days,  $i = 1, \dots, N$  and  
 113  $t = 1, \dots, M_i$ . The model problem is given  $M$  observations from  $i = 1, \dots, N - 1$  trading years  
 114 and partial trading days from  $N$ ,  $\{y_t^N\}$ ,  $t = 1, \dots, M_N$ , we want to predict the rest of trading  
 115 days in  $N$ ,  $\{y_\tau^N\}$ ,  $\tau = M_N + 1, \dots, M_N + H$ , where  $M_N + H$  is the last day of trading in  $N$ . Also,  
 116 it is given  $\{x_t^i\}$  for each series and our objective is to find  $P(\{y_\tau^N\}, M_N + 1, \dots, M_N +$   
 117  $H | \{x_t^i, y_t^i\}_{t=1, \dots, M_i}^{i=1, \dots, N})$ . See Figure 1.

118



119

120 Figure 1: Illustration of the regression variables (price history from 2002 to the first quarter  
 121 of 2011) of Starbucks stock. The objective of this model is to predict the "green strip" in  
 122 2011.

123 **3.1 Data description**

124 For this project, three random stocks were randomly selected from NASDAQ Stock Market,  
 125 namely Hewlett-Packard Company (HPQ), Yahoo Inc. (YHOO) and Starbucks Corporation  
 126 (SBUX). The daily changes of closing prices of these stocks were examined. The historical  
 127 data was downloaded from the yahoo finance section.

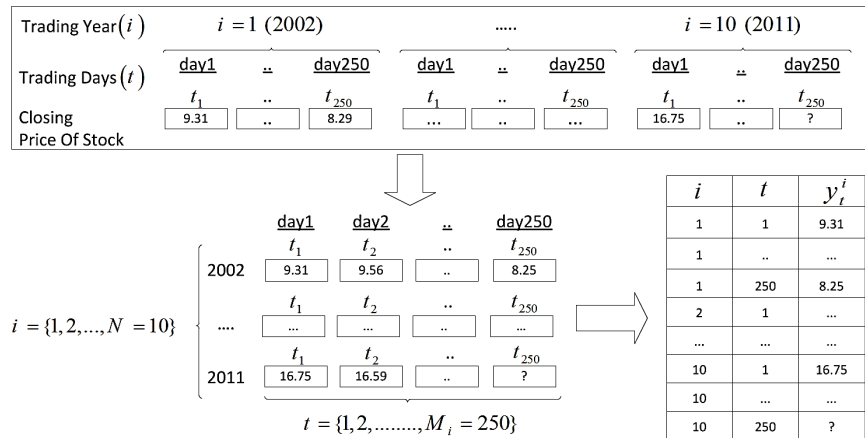
128 The sample period is from Jan 01 2002 to Dec. 31 2011 ( $N = 10$ ). We have about 250 days  
 129 of trading per year since no data is observed on weekends. However, some years have more  
 130 than 250 days of trading ( $M_i = 250$ ), we choose to ignore these days so that the whole  
 131 sample is of 2500 trading days.

132 We choose to use adjusted close prices because we aim to predict the trend of the stocks not  
 133 the prices. The adjusted close price is used to avoid the effect of dividends and splits  
 134 because when stock has a split, its price drop by half.

135 The adjusted close prices are standardized to zero mean and unit standard deviation. We also  
 136 normalize the prices in each year to avoid the variation from previous years by subtracting  
 137 the first day to start from zero.

138 As time-series model, we include a representation of the trading date as independent (input)  
 139 variables. The trading date is split into two parts: the trading year  $i$  (an integer, from 1 to 10)  
 140 and the days of trading  $t$  (an integer, from 1 to 250), Figure 1. These variables are  
 141 preprocessed before using them as input to the GP. They were standardized to zero mean and  
 142 unit standard deviation.

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144

145 Figure 2: Example of data processing to split trading date into two inputs: trading year ( $i$ )  
 146 and trading day ( $t$ )

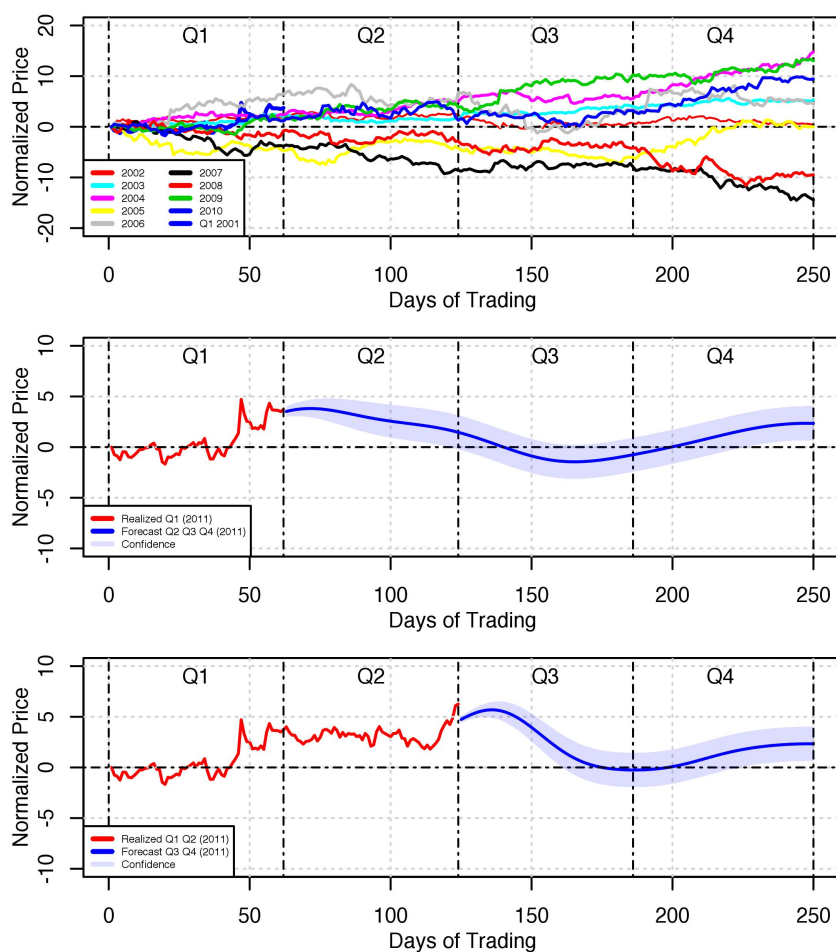
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148 **4 Evaluation**

149 To evaluate the performance of the proposed approach, "kernlab" R package is used. For  
 150 each stock, we applied two scenarios for long-term forecasting. The first scenario, given  
 151 complete observations from 9 years (2002 to 2010) and the first quarter (Q1) from 2011, we  
 152 want to predict the second, third and fourth quarters of 2011 (Q2, Q3 and Q4). The data is  
 153 divided into two sub-samples where the training data spans from Jan 01 2002 to the first  
 154 quarter of 2011 with 2312 trading days. The rest trading days of year 2011 of size 188 days  
 155 are reserved for test data.

156 The second scenario, given complete observations from 9 years (2002 to 2010) and the first  
 157 and second quarters (Q1 and Q2) from 2011, we want to predict the third and fourth quarters  
 158 of 2011 (Q3 and Q4). The data is divided into two sub-samples where the training data spans  
 159 from Jan 01 2002 to the second quarter of 2011 with 2374 trading days. The rest trading

160 days of year 2011 of size 126 days are reserved for test data. Figure 3 shows the training  
 161 data and the forecast results for Starbucks stock.  
 162

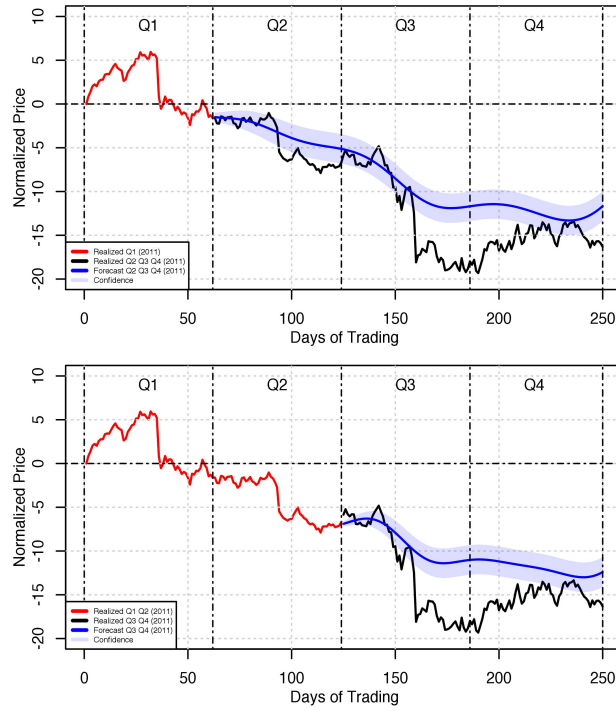


163  
 164 Figure 3: Top plot: Training set of Starbucks stock for the period from 2002 to the first quarter of  
 165 2011. Each line represent complete trading year. Middle plot: Shows the first scenario where  
 166 forecast made for the rest quarters of 2011 (Q2, Q3 and Q4). Bottom plot: shows the second where  
 167 training set is the period from 2002 to the second quarter of 2011. Forecast made for the third and  
 168 fourth quarters of 2011.

169  
 170 **4.1 Results and discussion**

171 The forecast results for the three stocks (HP, Yahoo and Starbucks) are shown in Figure 4, 5,  
 172 6. The “blue” lines show the forecast prices and the “black” lines show the actual prices. In  
 173 Figure 4, the results of scenario 1 (top part) shows drop in HP stock prices in Q2, Q3 and Q4  
 174 of 2011. Also, scenario 2 (bottom part) confirms this drop until the end of 2011. Based on  
 175 that, investors should not buy HP stock in 2011 and if they already did, it is highly  
 176 recommended to sell it to minimize their loss. Although, the model could not predict the  
 177 high drop in Q3, it keeps following the trend of the actual prices.

178 Figure 5 shows the forecast price of Yahoo stock. The results of scenario 1 (top part) show  
 179 slight decrease in Yahoo stock prices in Q2 and Q3 of 2011; however, the price shows some  
 180 improvement in Q4. The second scenario shows Yahoo stock prices reverse direction in Q4.  
 181 Investors can take the risk and buy in Q3 or wait until the beginning of Q4. The forecasting  
 182 model is able to track the trend of this stock most of the time.



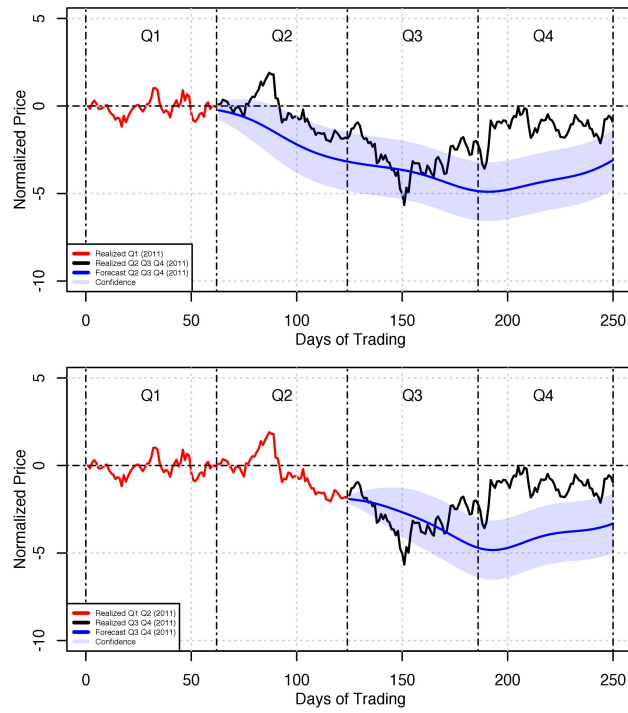
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Figure 4: Top part: Forecast result for HP stock from scenario 1. Bottom part: Forecast result for HP stock from scenario 2.

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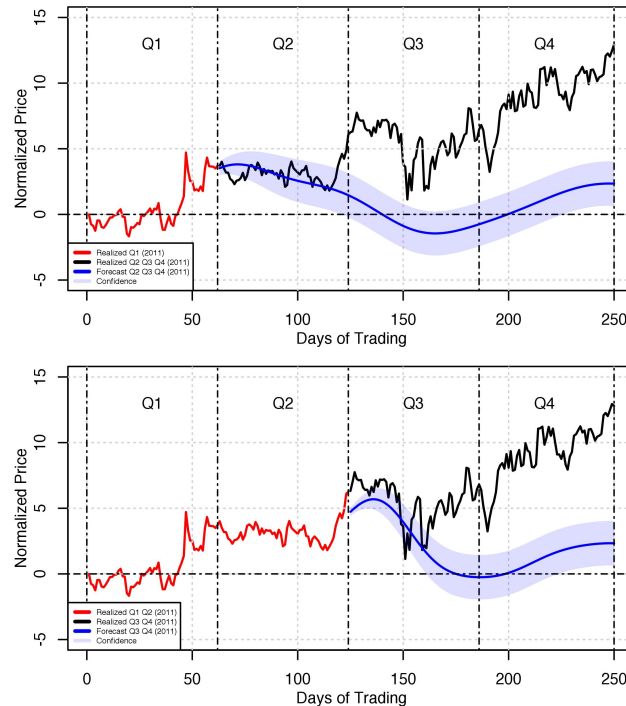
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Figure 5: Top part: Forecast result for Yahoo stock from scenario 1. Bottom part: Forecast result for Yahoo stock from scenario 2.

191

192 The forecasting result for Starbucks stock is shown in Figure 6. Although, the true model  
193 shows high fluctuation in 2011, our model keeps following the main trend of the stock.  
194 Scenario 1 shows falling in the price until the mid of Q3, however, scenario 2 updates the  
195 curve in Q3 to follow the increase at the end of Q2. Both scenarios agree that the mid of Q3  
196 is suitable to buy this stock. If investors own the stock before Q3, it is highly recommended  
197 to wait until the end of Q4.

198



199

200 Figure 6: Top part: Forecast result for Starbucks stock from scenario 1. Bottom part:  
201 Forecast result for Starbucks stock from scenario 2.

202

203 In general, this model is able to track the prices of the three stocks. As we know, stock price  
204 could be affected by several factors such as political situation and economic conditions,  
205 which may cause high fluctuations as shown in some areas of this experiment. As a long-  
206 term forecasting model, it is acceptable to not follow these fluctuations.  
207

## 208 5 Conclusion and future work

209 In this project, we applied Gaussian processes to perform long-term forecasting in stock  
210 market. This technique showed acceptable prediction to three stocks from NASDAQ Stock  
211 Market. The experiment showed highly acceptable time to buy and sell over different period  
212 of times. Due to the fast computation and the simplicity of this model, investors could use  
213 this model to do a long-term investment or to validate their investment decisions. More  
214 stocks could be tested on this model from other stock market.

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