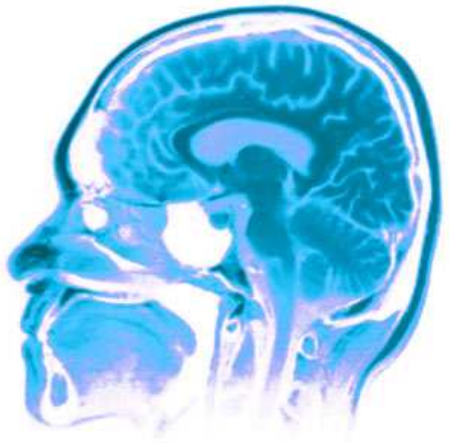
CPSC340



Decision trees & Random Forests

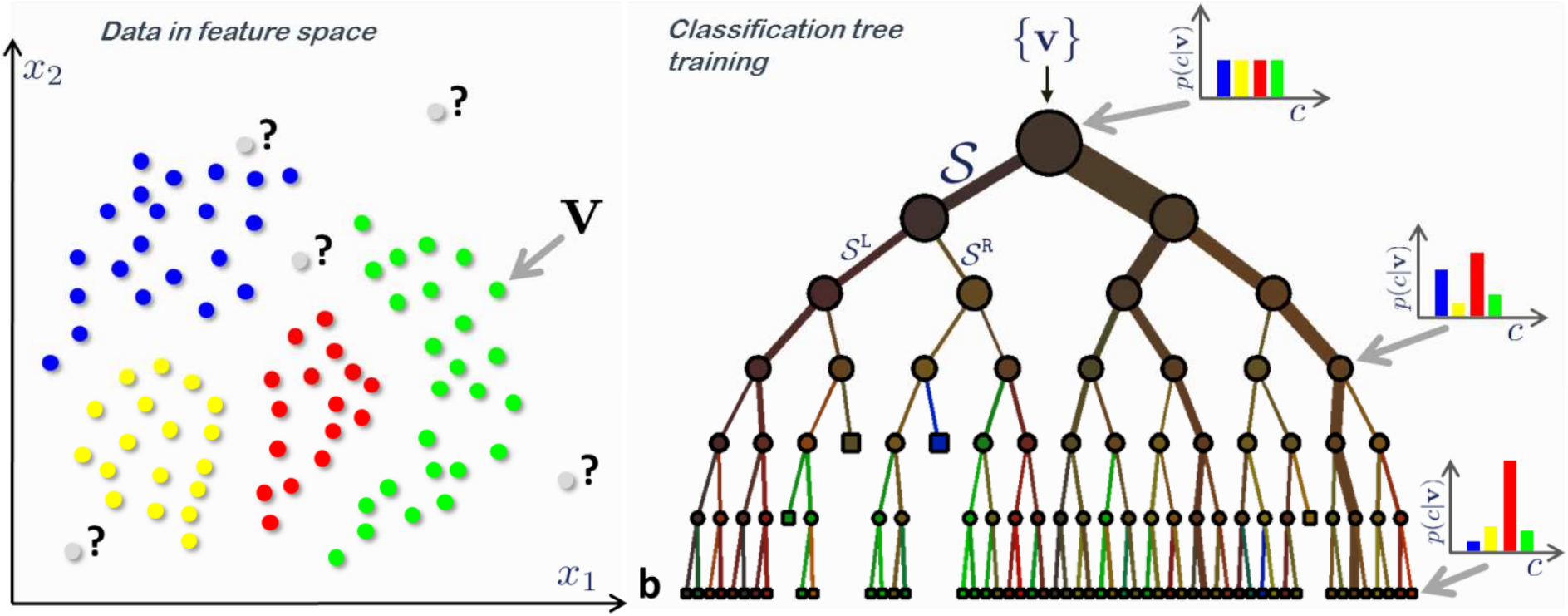
Nando de Freitas
November, 2012
University of British Columbia

Outline of the lecture

This lecture discusses classification trees and how to incorporate then into an ensemble (random forest). It discusses:

- Classification trees
- Random forests
- Object detection
- Kinect

Classification tree

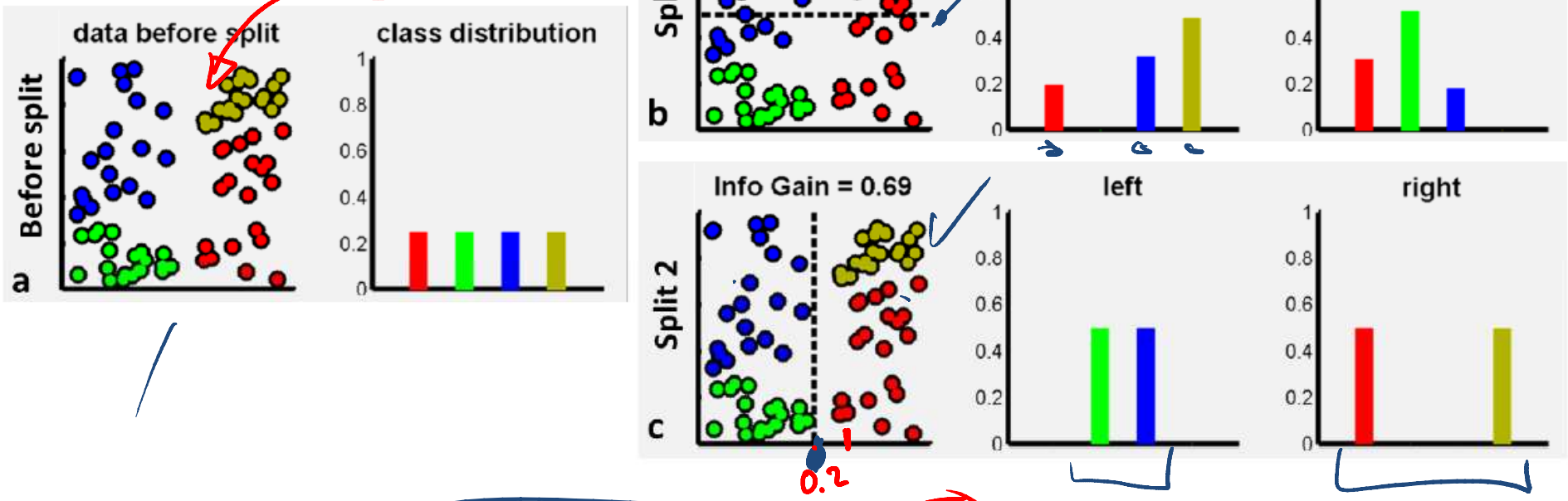


A generic data point is denoted by a vector $\mathbf{v} = (x_1, x_2, \cdots, x_d)$

$\mathcal{S}_j = \mathcal{S}_j^{\mathrm{L}} \cup \mathcal{S}_j^{\mathrm{R}}$

[Criminisi et al, 2011]

Use information gain to decide splits



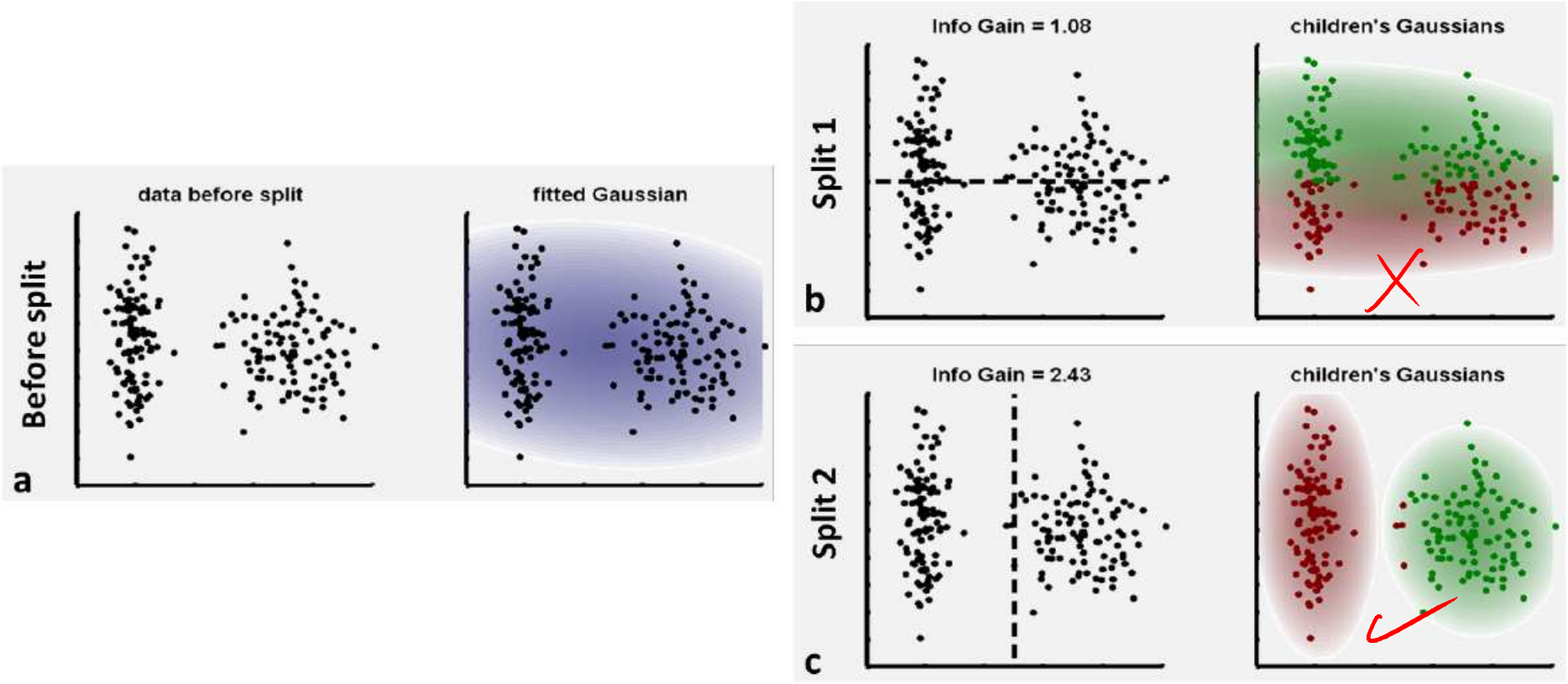
$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

node j

$I(2) > I(1)$

[Criminisi et al, 2011]

Advanced: Gaussian information gain to decide splits

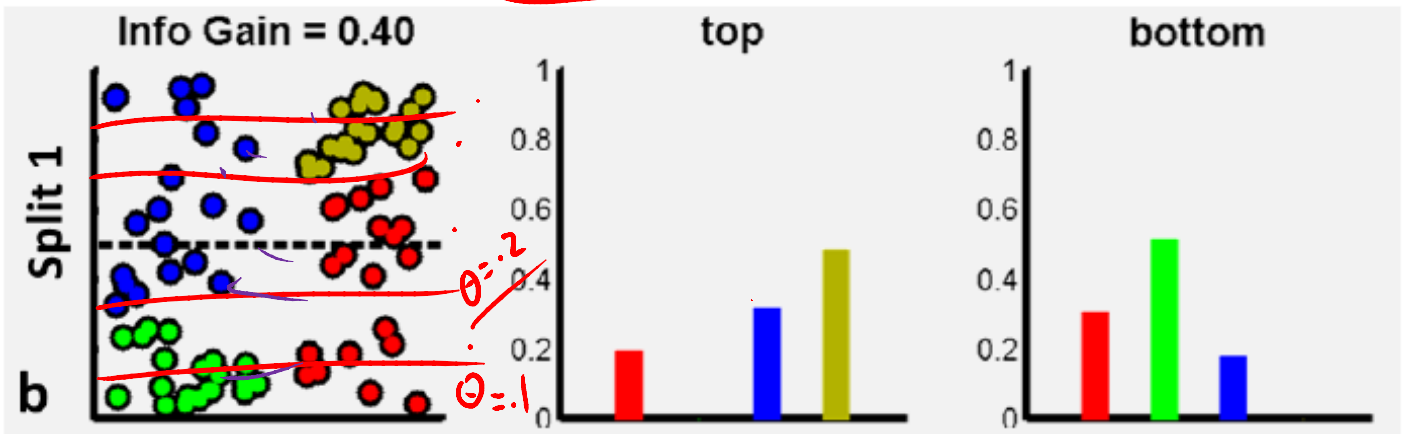


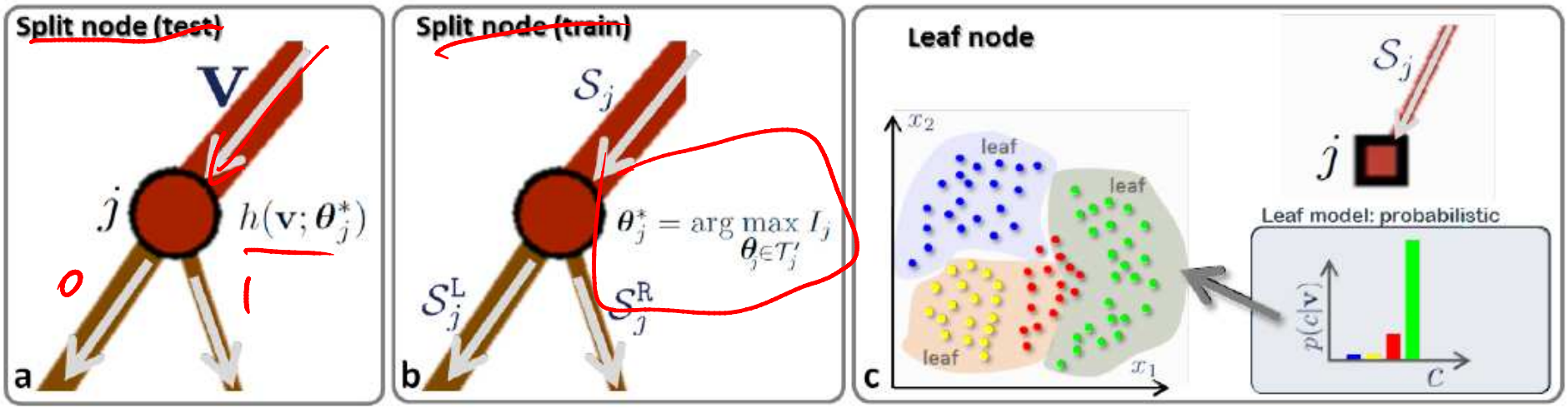
$$H(\mathcal{S}) = \frac{1}{2}\log\left((2\pi e)^d|\Lambda(\mathcal{S})|\right)$$

[Criminisi et al, 2011]

Each split node j is associated with a binary split function

$$h(\mathbf{v}, \boldsymbol{\theta}_j) \in \{0, 1\}$$

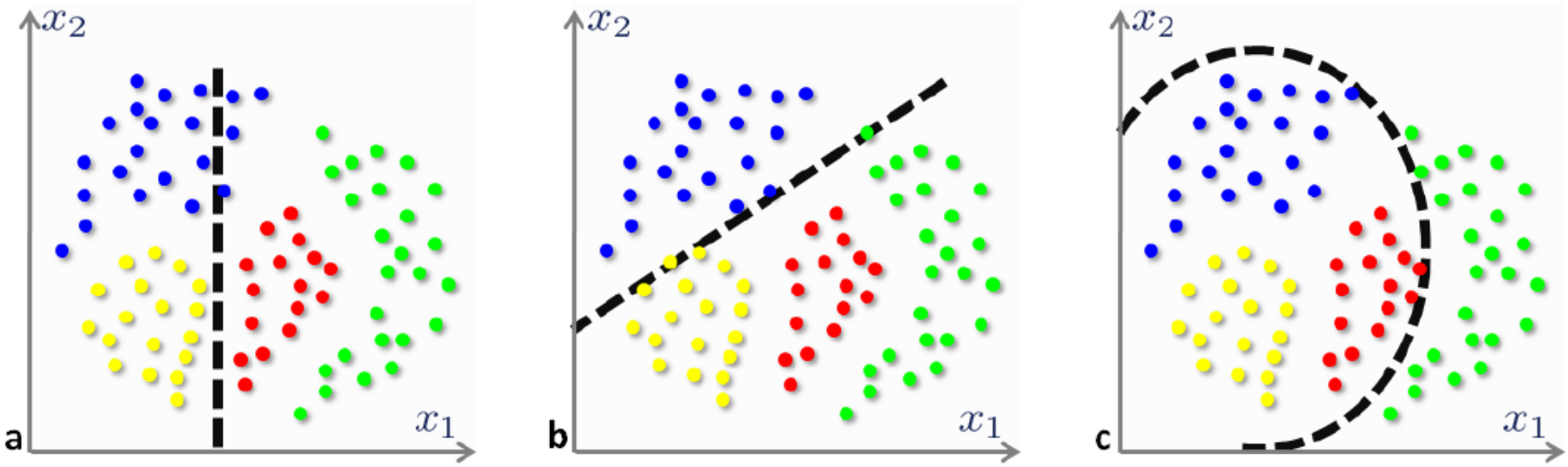




$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

[Criminisi et al, 2011]

Alternative node decisions



$$\mathbf{v} = (x_1 \; x_2) \in \mathbb{R}^2$$

[Criminisi et al, 2011]

Random Forests for classification or regression

1. For $b = 1$ to B:
   (a) Draw a bootstrap sample $\mathbf{Z}^*$ of size N from the training data.
   (b) Grow a random-forest tree T_b to the bootstrapped data, by recursively repeating the following steps for each terminal node of the tree, until the minimum node size n_{min} is reached.
      i. Select m variables at random from the p variables.
      ii. Pick the best variable/split-point among the m.
      iii. Split the node into two daughter nodes.
2. Output the ensemble of trees $\{T_b\}_1^B$.

[From the book of Hastie, Friedman and Tibshirani]

Randomization

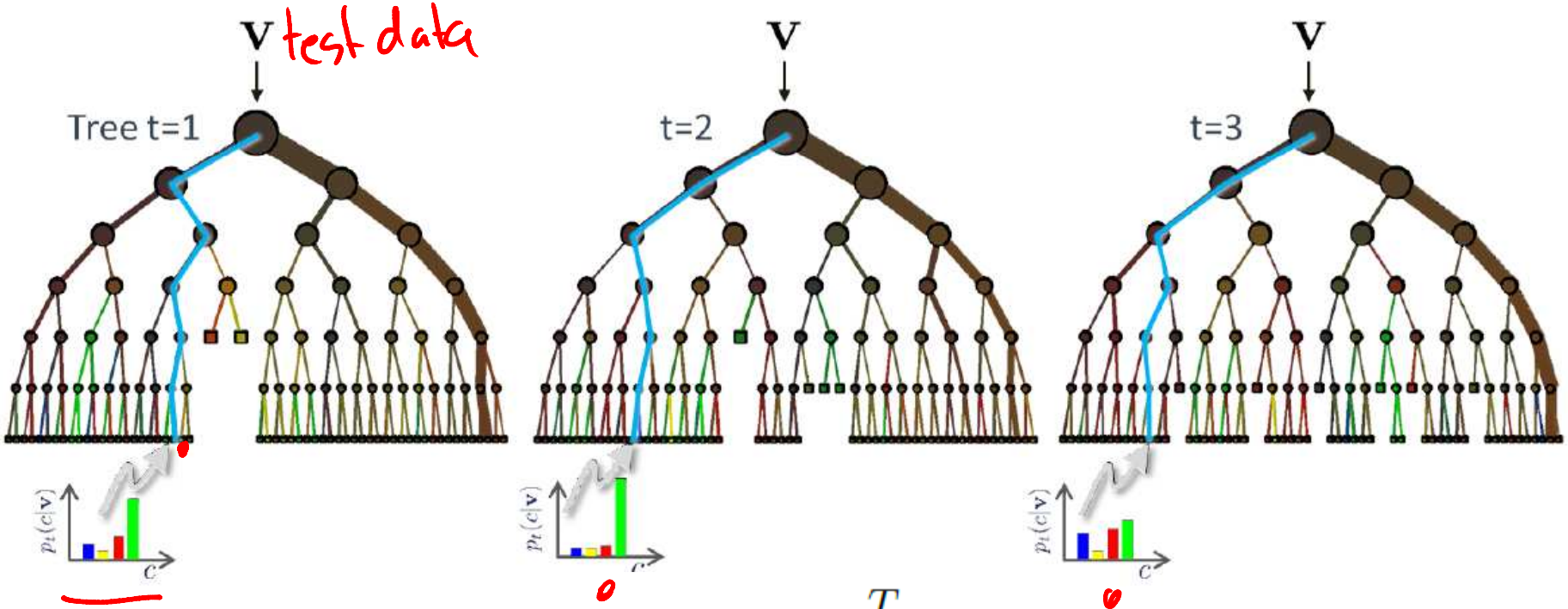
Randomized node optimization. If $\mathcal{T}$ is the entire set of all possible parameters $\boldsymbol{\theta}$ then when training the j^{th} node we only make available a small subset $\mathcal{T}_j \subset \mathcal{T}$ of such values.

$$\boldsymbol{\theta}_j^* = \arg\max_{\boldsymbol{\theta}_j \in \mathcal{T}_j} I_j.$$

[Criminisi et al, 2011]

Building a forest (ensemble)

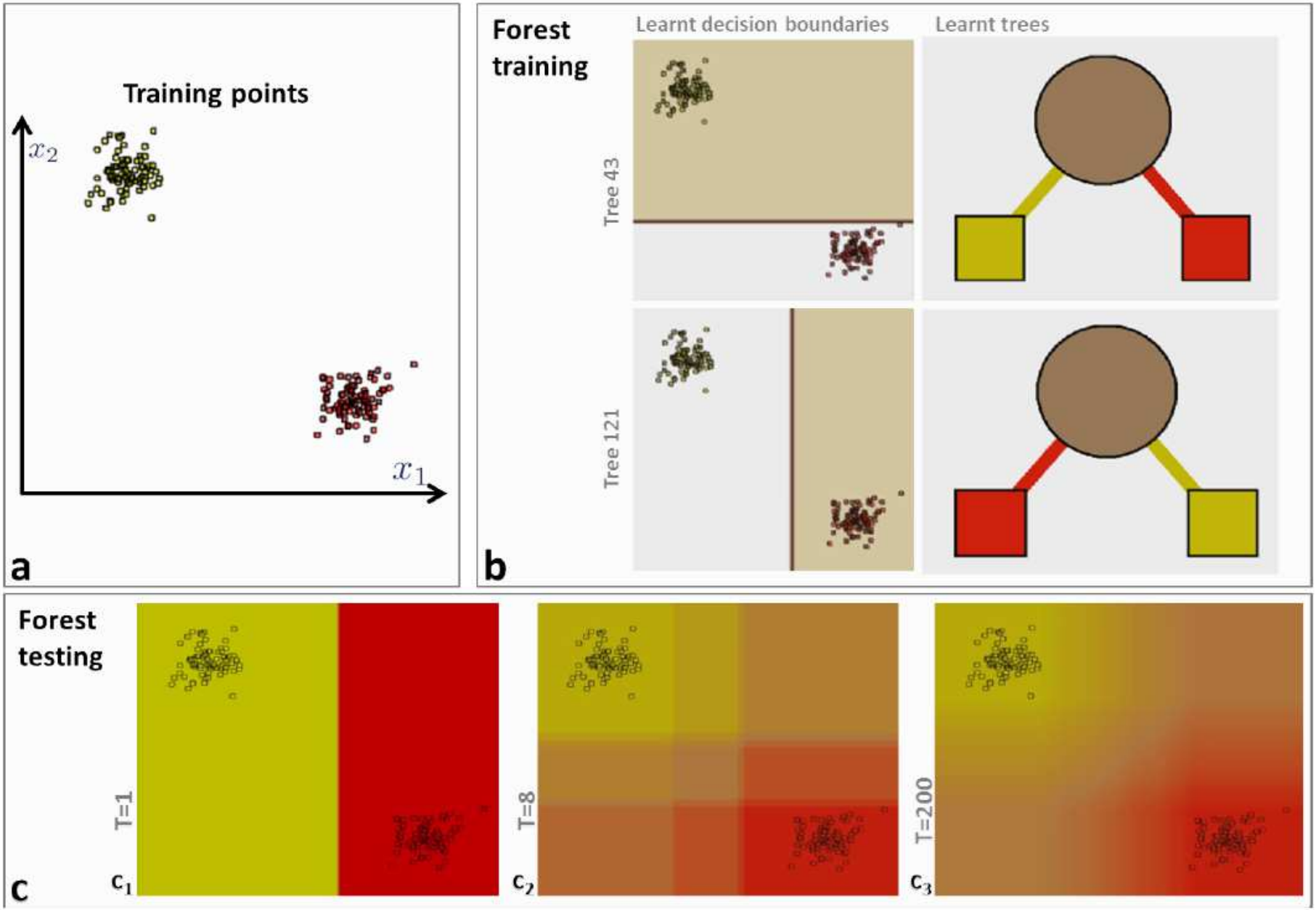
In a forest with T trees we have $t \in \{1, \cdots, T\}$. All trees are trained independently (and possibly in parallel). During testing, each test point $\mathbf{v}$ is simultaneously pushed through all trees (starting at the root) until it reaches the corresponding leaves.



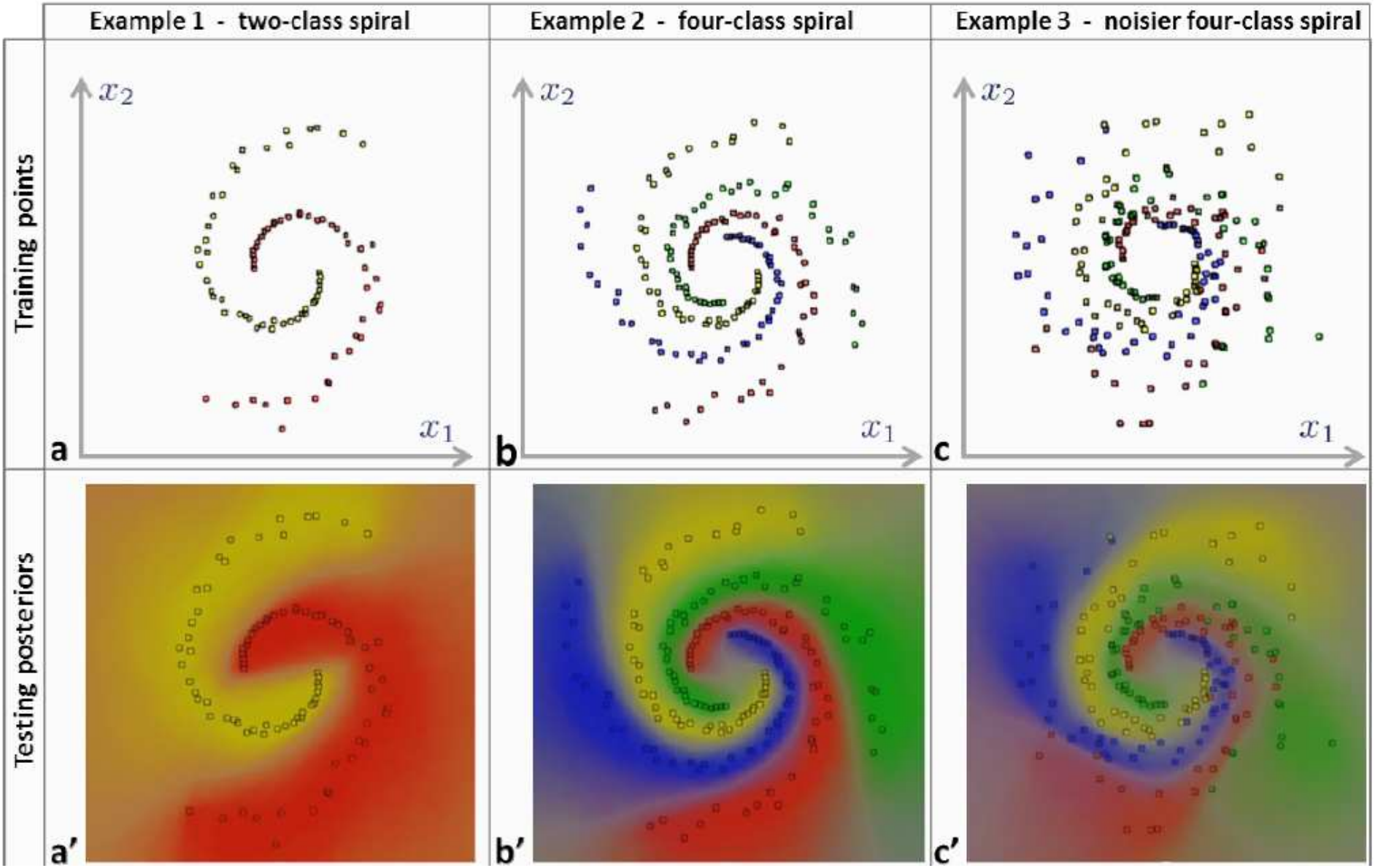
$$p(c|\mathbf{v}) = \frac{1}{T}\sum_{t=1}^{T} p_t(c|\mathbf{v})$$

[Criminisi et al, 2011]

Effect of forest size

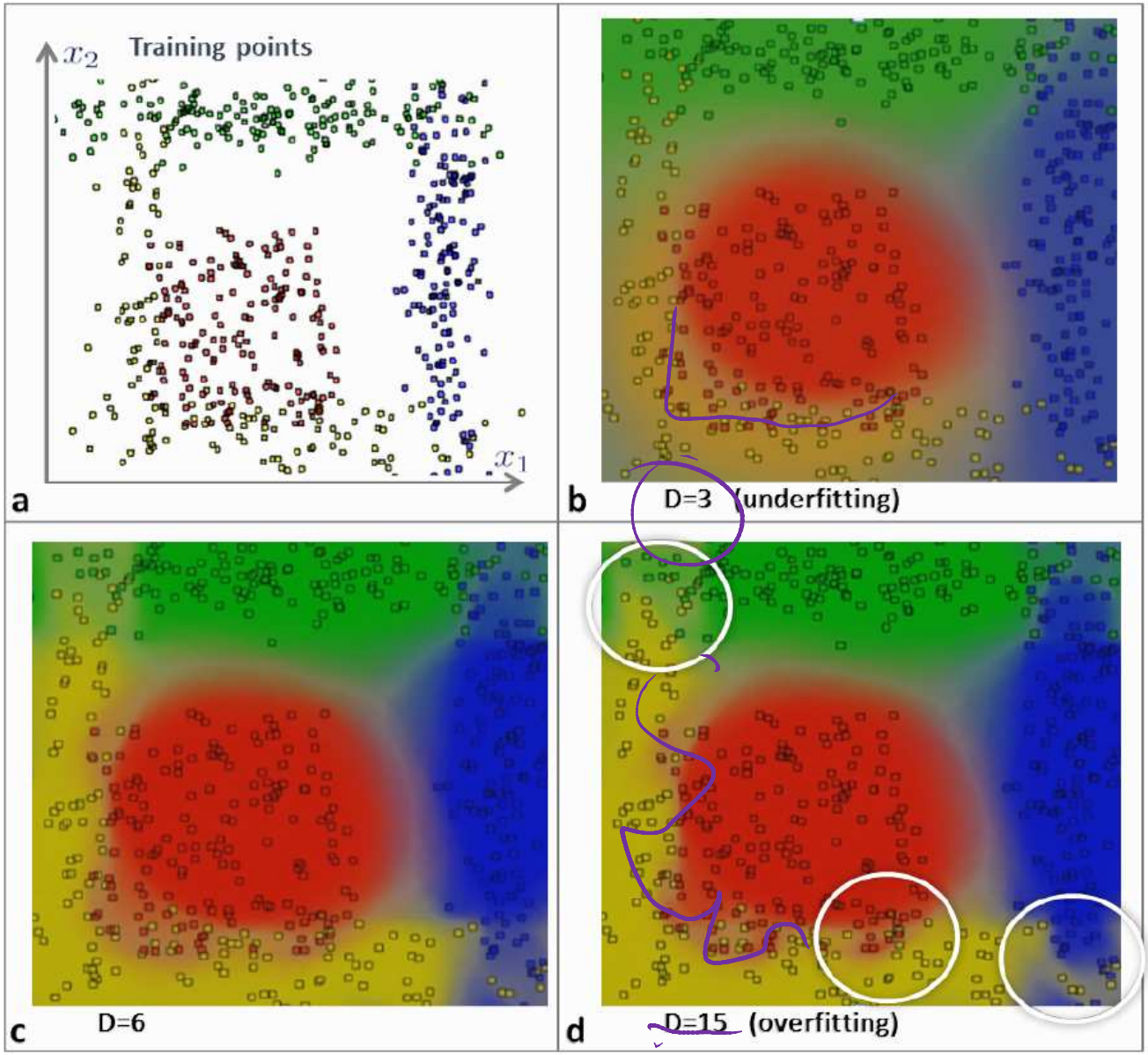


Effect of more classes and noise

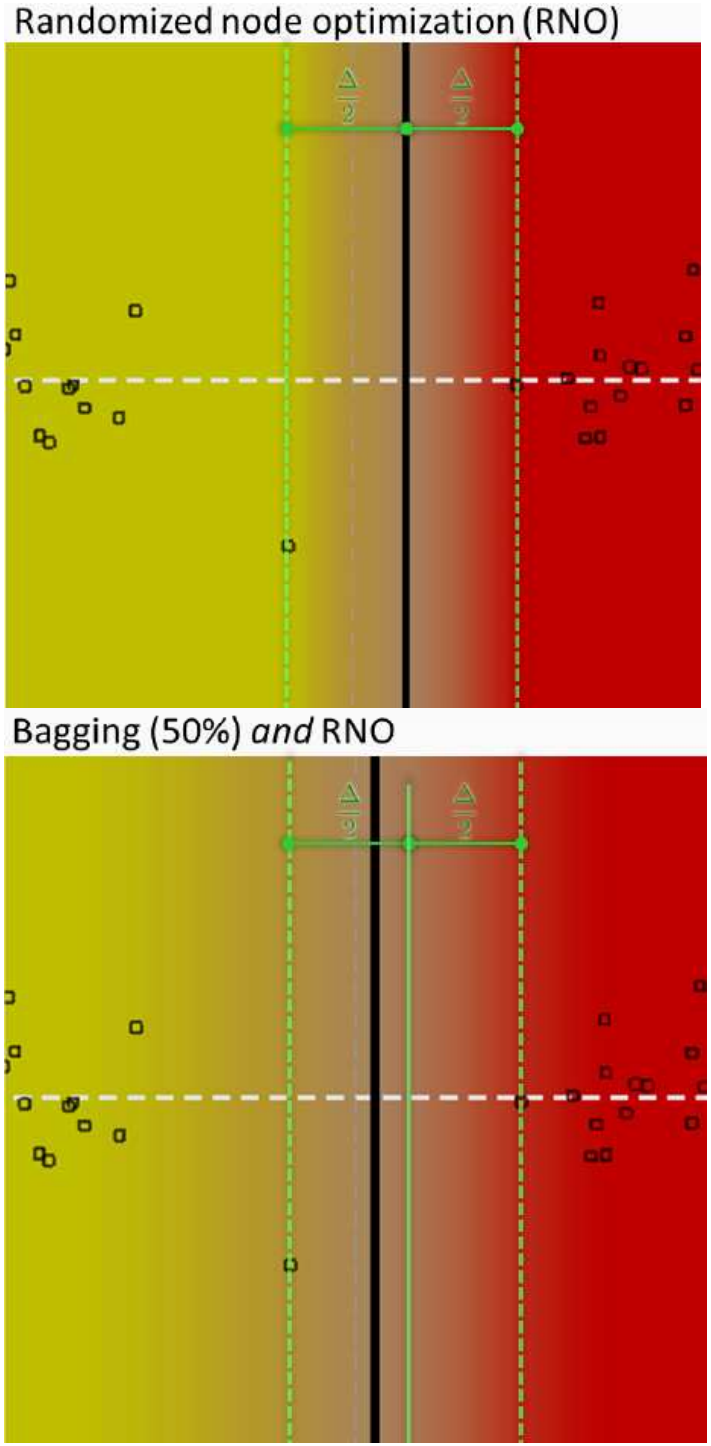


[Criminisi et al, 2011]

Effect of tree depth (D)



[Criminisi et al, 2011]



Effect of bagging

[Criminisi et al, 2011]

Application to face detection

Training Data

- 5000 faces
  - All frontal
- 300 million non faces
  - 9400 non-face images

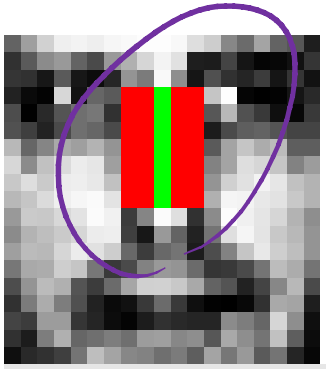
[Viola and Jones, 2001]

Object detection

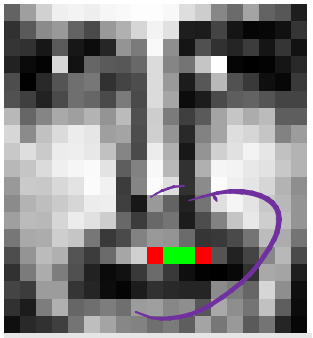
Idea: Extract simple features from all 24 by 24 pixel patches x_i. E.g., the value of a *two-rectangle feature is the difference between the sum* of the pixels within two rectangular regions. Then compare the level of activation (value of the feature *f*) with respect to a threshold (theta).

$$h_t(x_i) = \begin{cases} 1 & \text{if } f_t(x_i) > \theta_t \\ 0 & \text{otherwise} \end{cases}$$

$$h(x) = \begin{cases} 1 & \sum_{t=1}^{T} \alpha_t h_t(x) \geq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

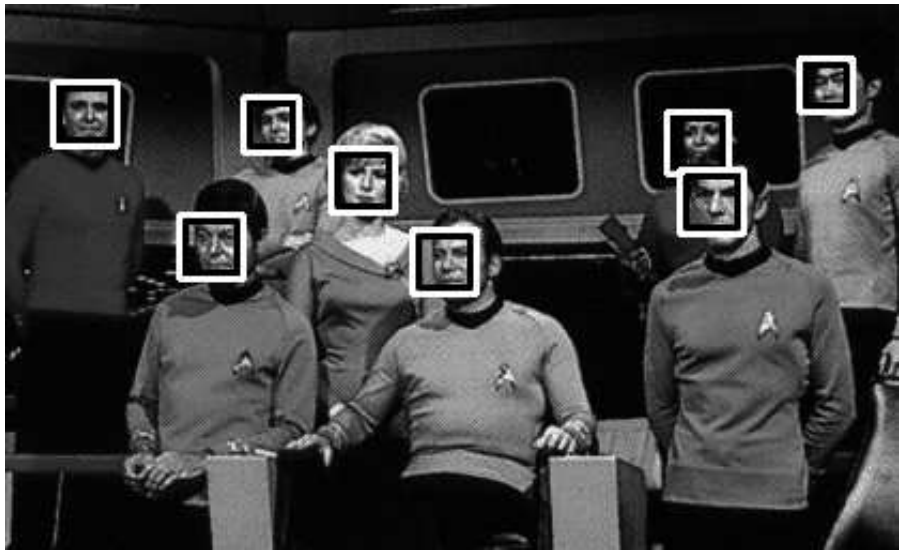


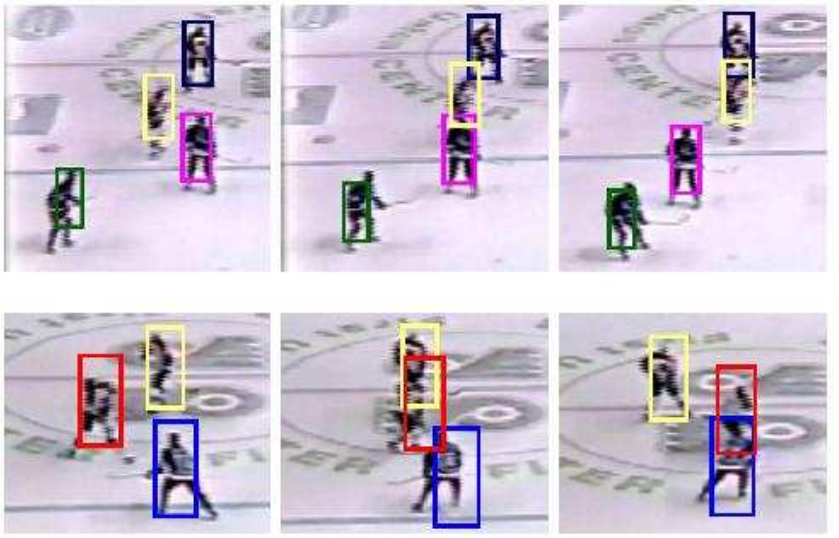
Relevant feature

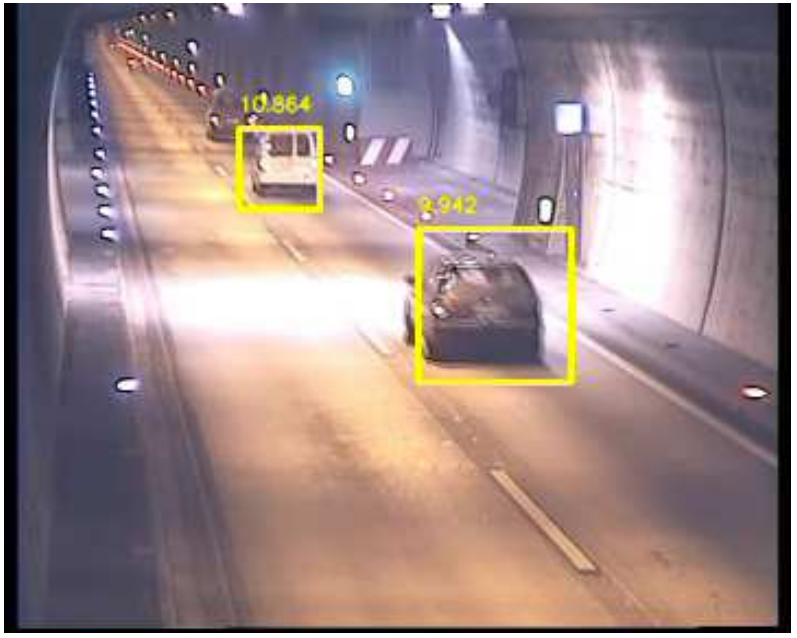


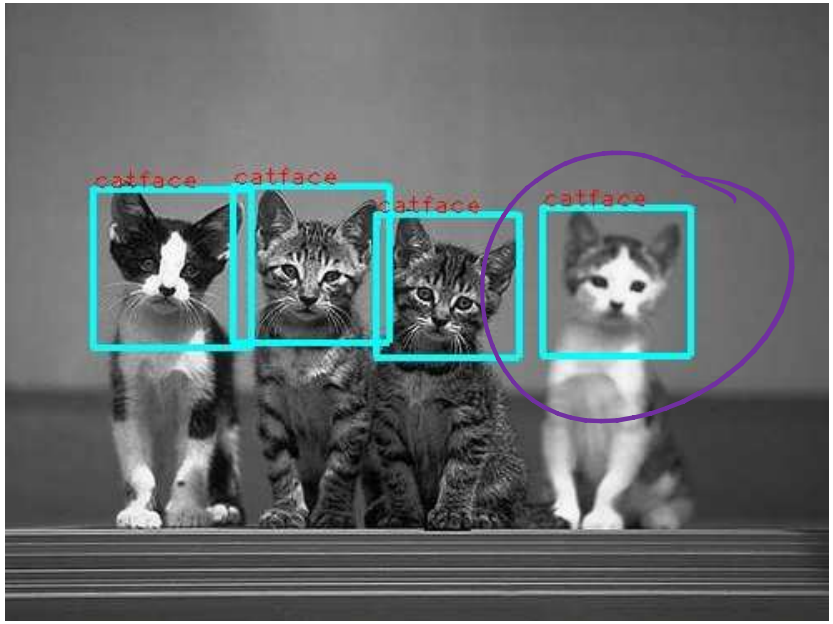
Irrelevant feature

Object detection

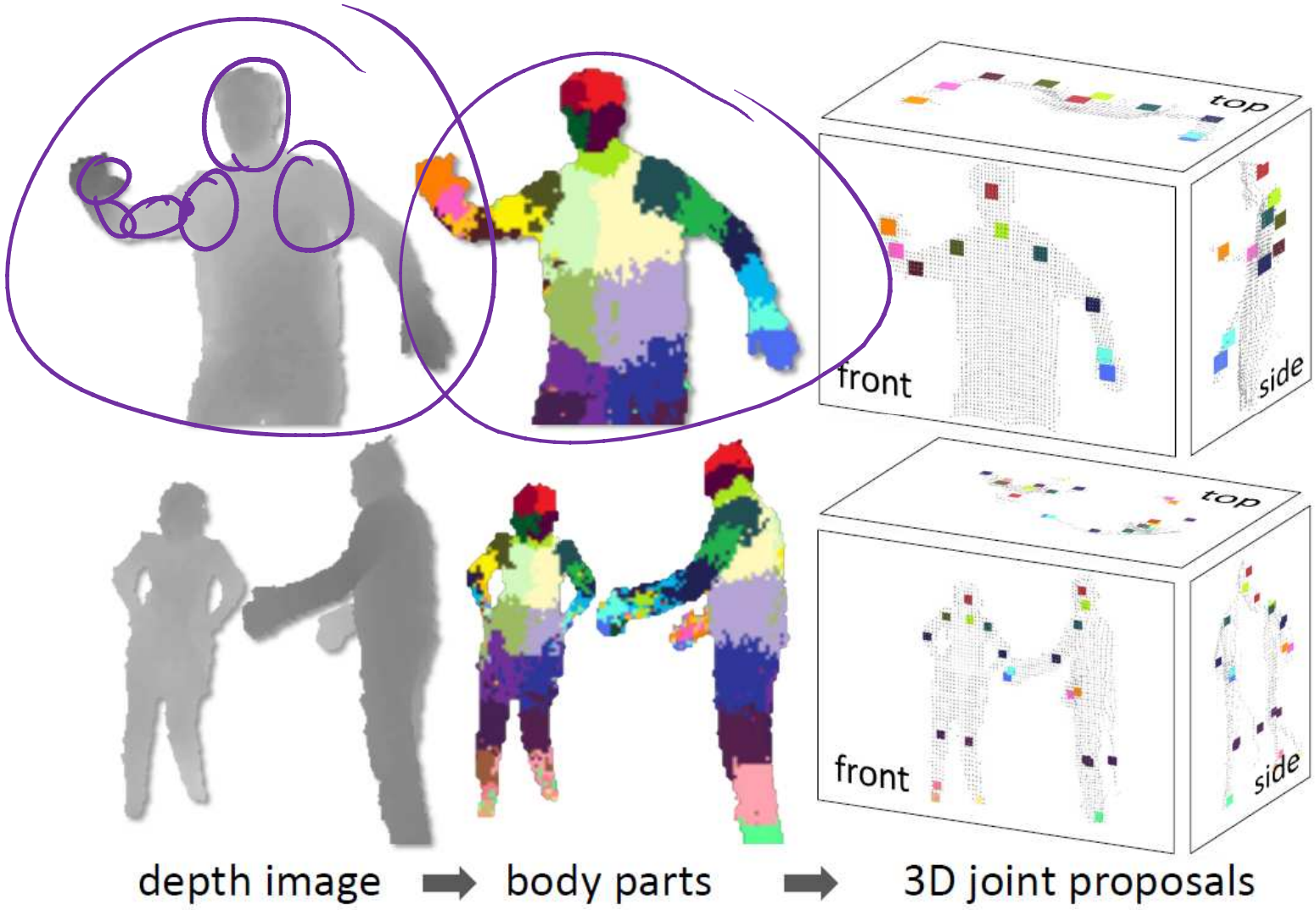








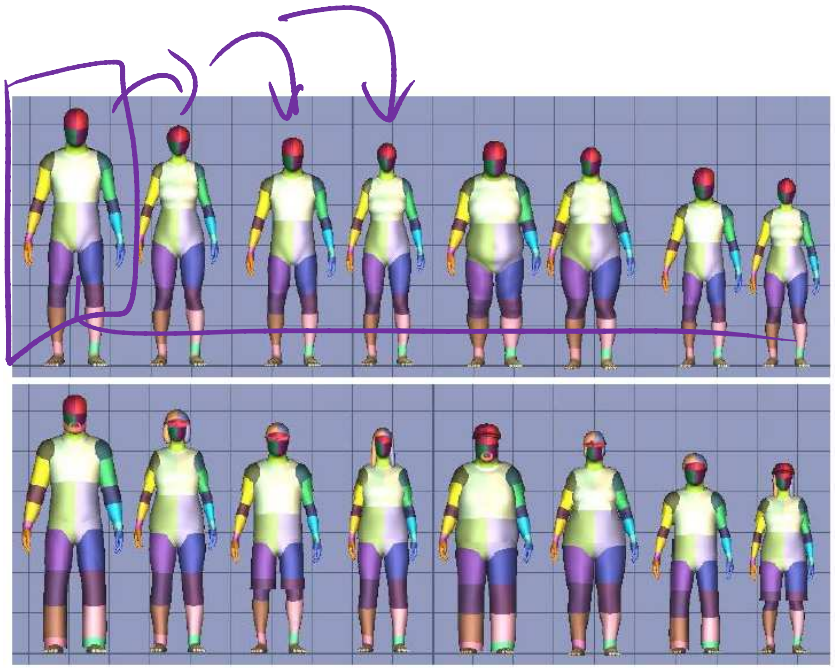
Random Forests and the Kinect

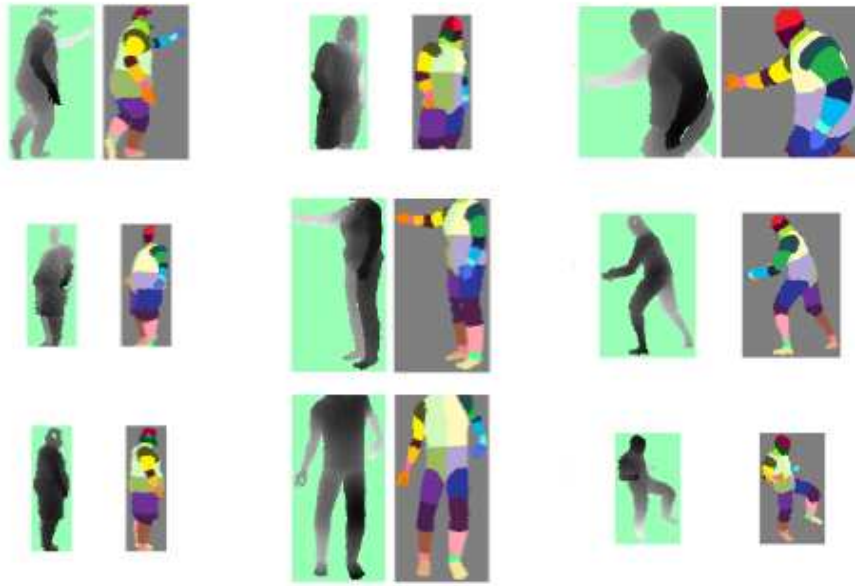


[Jamie Shotton et al 2011]

Random Forests and the Kinect

Lesson 1: Use computer graphics to generate plenty of data.

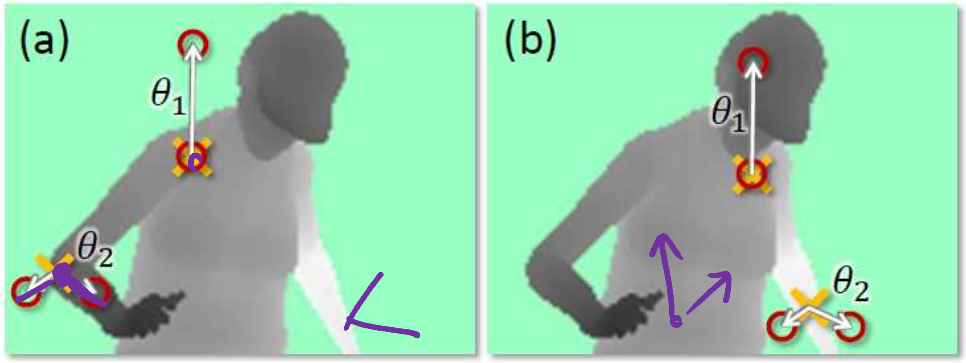




[Jamie Shotton et al 2011]

Random Forests and the Kinect

Lesson 2: Use simple depth features within random forests algorithm.



- For each pixel *x*, compute the feature:

$$f_\theta(I, \mathbf{x}) = d_I\left(\mathbf{x} + \frac{\mathbf{u}}{d_I(\mathbf{x})}\right) - d_I\left(\mathbf{x} + \frac{\mathbf{v}}{d_I(\mathbf{x})}\right)$$

$d_I(x)$ is the depth at pixel *x* in image *I*
Parameters $\theta = (u; v)$ describe offsets u and v.

- The normalization of the offsets ensures the features are depth invariant: At a given point on the body, a fixed world space offset will result whether the pixel is close or far from the camera.

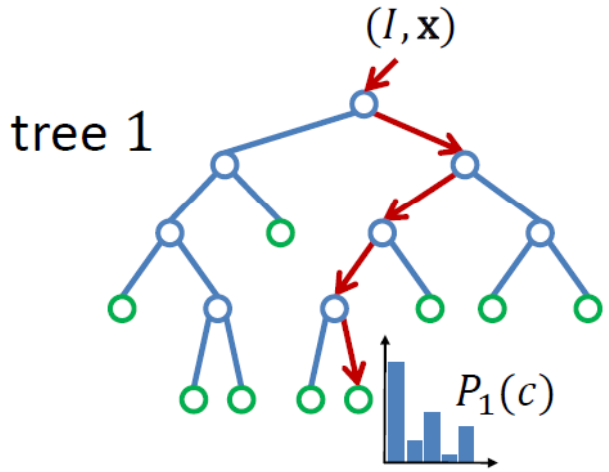
[Jamie Shotton et al 2011]

Tree algorithm

1. Randomly propose a set of splitting candidates $\phi = (\theta, \tau)$ (feature parameters θ and thresholds τ).

2. Partition the set of examples $Q = \{(I, \mathbf{x})\}$ into left and right subsets by each ϕ:

$$\begin{aligned} Q_{\mathrm{l}}(\phi) &= \{\, (I, \mathbf{x}) \mid f_\theta(I, \mathbf{x}) < \tau \,\} \\ Q_{\mathrm{r}}(\phi) &= Q \setminus Q_{\mathrm{l}}(\phi) \end{aligned}$$



3. Compute the ϕ giving the largest gain in information:

$$\begin{aligned} \phi^\star &= \operatorname*{argmax}_\phi G(\phi) \\ G(\phi) &= H(Q) - \sum_{s \in \{\mathrm{l},\mathrm{r}\}} \frac{|Q_s(\phi)|}{|Q|} H(Q_s(\phi)) \end{aligned}$$

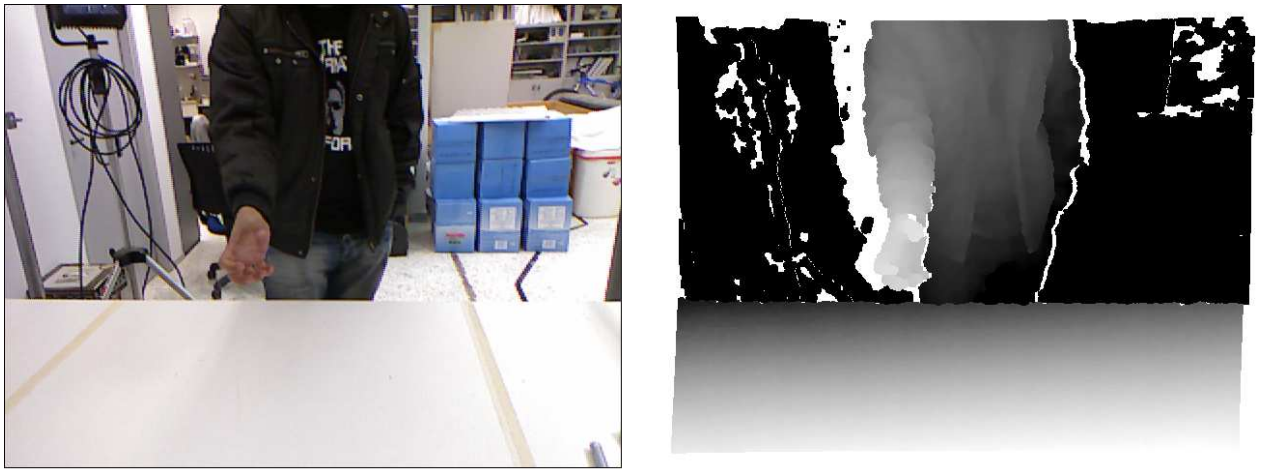
4. If the largest gain $G(\phi^\star)$ is sufficient, and the depth in the tree is below a maximum, then recurse for left and right subsets $Q_{\mathrm{l}}(\phi^\star)$ and $Q_{\mathrm{r}}(\phi^\star)$.

[Jamie Shotton et al 2011]

Performance on train and test data

[Jamie Shotton et al 2011]

Applications: Interfaces



[Iason Oikonomidis et al 2011]

Next lecture

There is no next lecture. Congrats!