CPSC-340 Machine Learning and Data Mining 2005

Homework # 2
Due Friday, Sep 23 4pm.

NAME:_____

Signature:_____

STD. NUM: _____

General guidelines for homeworks:

You are encouraged to discuss the problems with others in the class, but all write-ups are to be done on your own.
Homework grades will be based not only on getting the "correct answer," but also on good writing style and clear presentation of your solution. It is your responsibility to make sure that the graders can easily follow your line of reasoning.
Try every problem. Even if you can't solve the problem, you will receive partial credit for explaining why you got stuck on a promising line of attack. More importantly, you will get valuable feedback that will help you learn the material.
Please acknowledge the people with whom you discussed the problems and what sources you used to help you solve the problem (e.g. books from the library). This won't affect your grade but is important as academic honesty.
When dealing with Matlab exercises, please attach a printout with all your code and show your results clearly.

1. **(Learning Matlab)**

   (i) Write a MATLAB m file that plots the following 2D Gaussian density function:

   $$p(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}$$

   for x and y in the range [-3,:0.1:3]. You will need to learn the commands `help`, `figure`, `clf`, `meshgrid` and `surf`. Your answer should consist of the `surf` plot and the code.

   (ii) Load the special MATLAB function called `peaks`. You can do this by typing `[x,y,z] = peaks;`. Use the commands `contour` and `clabel` to generate a countour plot of the function `peaks`. Repeat the exercise using the functions `contourf` and `contour3`. You should hand in your code and a single figure with 4 subplots (you'll need to learn the command `subplot`) consisting of the `surf` plot and the 3 countour plots.

   (iii) Now we'll try to visualise a function in 4D! In particular, use the command `slice` to generate a plot of the 3D contours of the following Gaussian:

   $$p(x, y, z) = \frac{1}{(2\pi)^{3/2}} e^{-\frac{1}{2}(x^2+y^2+z^2)}$$

   for x, y and z defined in the range [-3,:0.1:3].Use the commands `xlabel`, `ylabel` and `zlabel` to label the axes appropriately. Hand in the code and plot.

2. **(Eigenproblems)**

   (i) Suppose the matrix $\mathbf{A} \in \mathbb{R}^{\mathbf{n}\times\mathbf{n}}$ has n linearly independent eigenvectors $\mathbf{x_1}, \ldots, \mathbf{x_n}$. Define the matrix $\mathbf{Q}$ having these vectors as columns, i.e. $\mathbf{Q} = [\mathbf{x_1} \quad \mathbf{x_2} \quad \ldots \quad \mathbf{x_n}]$. Let $\mathbf{D}$ be a diagonal matrix with the eigenvalues λ_i in the diagonal. Show that

   $$\mathbf{A} = \mathbf{QDQ^{-1}}$$

(ii) Prove the spectral mapping theorem.

(iii) Compute the eigenvalues and eigenvectors of the following matrix by hand and using matlab:

$$\mathbf{A} = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$$