CPSC-320 Intermediate Algorithm Design and Analysis Winter 2000

Practice Homework # 5

1. Consider the following greedy algorithm for the activity selection problem. Assume for simplicity that the input is a set of n activities. Let A be the activity in the set that conflicts with with the smallest number of other activities (if there is more than one such activity, choose A arbitrarily among them). Remove from the set of activities both A and all activities that conflict with A. Repeat the algorithm to find a solution for the remaining set, and add A to form the whole solution.

   Explain why this algorithm is incorrect.

2. Suppose you want to find the longest common subsequence of *three* sequences, rather than of two sequences as described in class.

   The 3-sequence longest common subsequence problem is defined as follows: given three sequences $X = x_1, x_2, \ldots, x_n$, $Y = y_1, y_2, \ldots, y_n$, and $Z = z_1, z_2, \ldots, z_n$, find the length of the longest sequence W that is a subsequence of all three sequences. Recall that $W = w_1 w_2, \ldots, w_l$ is a subsequence of X if for some values $i_1 < i_2 < \ldots i_l$ in the range $[1, \ldots, n]$, $w_j = x_{i_j}$ for $1 \leq j \leq l$ (and similarly for Y and Z).

   Design an efficient algorithm for solving this problem. What is the running time of your algorithm?

3. The *lowest common multiple* of two numbers a and b is denoted by $lcm(a, b)$ and is defined to be the least nonnegative integer that is a multiple of both a and b. Describe an algorithm that efficiently computes $lcm(a, b)$. What is the running time of your algorithm?

4. The 2SAT problem is: given a Boolean formula in 2-conjunctive normal form, decide if the formula is satisfiable. A boolean formula is in 2-conjunctive normal form if it is the conjunction ($\wedge$) of clauses, where each clause is the disjunction ($\vee$) of two literals. For example, the following formula is in 2-conjunctive normal form.

   $$(x_1 \vee x_2) \wedge (\bar{x}_2 \vee x_3) \wedge (x_1 \vee \bar{x}_3) \wedge (x_4 \vee x_2)$$

   Find an efficient algorithm for this problem.