CPSC-320 Intermediate Algorithm Design and Analysis Winter 2000

Homework # 1

Due in class on Wednesday, September 20

General guidelines for homeworks:

Before starting on this homework, review the homework guidelines provided on the first day of class (also on the web under "Course Description"). Remember that it is encouraged to discuss the problems with others in the class, but all write-ups are to be done on your own.

Homework grades will be based not only on getting the "correct answer," but also on good writing style and clear presentation of your solution. It is your responsibility to make sure that the graders can easily follow your line of reasoning.

Try every problem. Even if you can't solve the problem, you will receive partial credit for explaining why you got stuck on a promising line of attack. More importantly, you will get valuable feedback that will help you learn the material.

Handwritten homeworks are acceptable if written clearly, but if you have poor handwriting, please type up your solutions. Ascii text is just fine.

All problems in this homework will be worth an equal number of points.

At the top of your homework, please acknowledge the people with whom you discussed the problems and what sources you used to help you solve the problem (e.g. books from the library). This won't affect your grade but is important as academic honesty.

1. Following is a brief description of an algorithm for adding two multiple digit numbers of the same length, much as you learned it in elementary school.

```
Write the two numbers one under the other, with the digits aligned
starting at the right (low order) end. Draw a line under the numbers.

Add the rightmost two digits. If the result has 1 digit, write this
result below the line, directly under the two digits just added. If
has two digits, just write the low order digit, and put the other
digit (which must be 1) over the numbers in the next column.  Call
this digit the ``carry''.

Proceed to do the same with the next column, except if there is a
carry, add that to the sum of the other two digits. Keep going until
you are at the last column of digits. In this case, if there is a
carry, put it under the line to the left of all the digits calculated
so far.
```

(a) Describe the algorithm you learned for multiplication of two multiple digit numbers when in elementary school, at the same level of detail as the above addition algorithm.

(b) Now suppose that the above addition algorithm and your multiplication algorithm are applied to two n-bit binary numbers a and b (assume the leading bit of each number is 1). As a function of n, how many individual *elementary addition steps* are done by your algorithms in the worst case? Here, an elementary addition step is where you add a single digit to something else. Explain your answer.

2. In Lecture 2, we saw that by looking at the pattern for small n, a reasonable guess for the total number of subsquares in an $n \times n$ grid is $\sum_{k=1}^{n} k^2$. Prove by induction that this is indeed the correct answer.

3. Suppose that algorithm A runs in time $f(n)$ and that algorithm B runs in time $g(n)$. Answer the following four questions for each of the following five cases. You should explain your answer in each case.

   Questions:

   (a) Is A faster than B for all n?

   (b) Is B faster than A for all n?

   (c) Is A faster than B for all n greater than some c?

   (d) Is B faster than A for all n greater than some c?

   Cases:

   (i) $g(n) = \Omega(f(n) \log_2 n)$

   (ii) $g(n) \approx f(n) \log_2 n$

   (iii) $g(n) = \Theta(f(n) \log_2 n)$

   (iv) $g(n) = O(f(n) \log_2 n)$

   (v) $g(n) = o(f(n) \log_2 n)$

4. **(Geometric Sums)**

   (a) On page 44 of the text, the following identity is given:

$$\sum_{k=0}^{n} x^k = \frac{x^{n+1} - 1}{x - 1}, \text{ if } x \neq 1.$$

   The sum on left hand side of this equation is called a geometric sum. Prove that this identity holds. (Note: you don't need to use proof by induction. Just some basic algebra.)

   (b) Derive a closed form expression for the sum

$$\sum_{k=1}^{n} kx^k, x \neq 1.$$

   Suggestion: what do you get when you take the derivative with respect to x of the equation in part (a) of this problem?

   Verify that your expression is correct for $x = 1/2$ and $n = 4, 5$.