CPSC-320 Intermediate Algorithm Design and Analysis Winter 2000

Homework # 1

Due Friday, May 16 at 1pm.

NAME:____

Signature:____

STD. NUM: ____

General guidelines for homeworks:

Before starting on this homework, review the homework guidelines provided on the first day of class (also on the web under "Course Description"). Remember that it is encouraged to discuss the problems with others in the class, but all write-ups are to be done on your own. **Homework grades will be based not only on getting the "correct answer," but also on good writing style and clear presentation of your solution.** It is your responsibility to make sure that the graders can easily follow your line of reasoning.

Try every problem. Even if you can't solve the problem, you will receive partial credit for explaining why you got stuck on a promising line of attack. More importantly, you will get valuable feedback that will help you learn the material.

Please acknowledge the people with whom you discussed the problems and what sources you used to help you solve the problem (e.g. books from the library). This won't affect your grade but is important as academic honesty.

1. Prove by induction that, for all natural numbers n, $n^2 + n$ is divisible by 2. Can you prove this directly? Explain.

2. (a) Show that the *telescoping series* $\sum_{k=1}^{n}\left[(k+1)^4 - k^4\right]$ converges to $(n+1)^4 - 1$.

(b) Show that the series $\sum_{k=1}^{n}\left[(k+1)^4 - k^4\right]$ simplifies to $\sum_{k=1}^{n}\left[4k^3 + 6k^2 + 4k + 1\right]$.

(c) Using parts (a), (b) and the material in the lecture notes, prove that $\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$.

3. The maximum n for which a computer can solve an n^3 problem is $n = 1000$. If we double the speed of this computer, can we expect to solve the n^3 problem with $n = 2000$? Explain.

4. (a) By expanding $n!$ and using sensible upper and lower bounds, show that:

$$n^n \geq n! \geq \left(\frac{n}{2}\right)^{\frac{n}{2}}$$

(b) Use this result to show that $\lg(n!) = \Theta(n \lg n)$.

5. Is $2^{n+1} = O(2^n)$?. Is $2^{2n} = O(2^n)$? Explain why.

6. (a) Can we prove that $f(n)$ is polynomially bounded by showing that $\lg(f(n)) = O(\lg n)$?

(b) Show that $\lceil \lg n \rceil = \Theta(\lg n)$.

(c) Using (a), (b) and 4(a), are the functions $\lceil \lg n \rceil!$ and $\lceil \lg \lg n \rceil!$ polynomially bounded?