CPSC-320 Intermediate Algorithm Design and Analysis Winter 2000

Homework # 4

Due Wednesday, June 18 at 6pm.

NAME:______________________________

Signature:______________________________

STD. NUM: ______________________________

General guidelines for homeworks:

Before starting on this homework, review the homework guidelines provided on the first day of class (also on the web under "Course Description"). Remember that it is encouraged to discuss the problems with others in the class, but all write-ups are to be done on your own. **Homework grades will be based not only on getting the "correct answer," but also on good writing style and clear presentation of your solution.** It is your responsibility to make sure that the graders can easily follow your line of reasoning.

Try every problem. Even if you can't solve the problem, you will receive partial credit for explaining why you got stuck on a promising line of attack. More importantly, you will get valuable feedback that will help you learn the material.

Please acknowledge the people with whom you discussed the problems and what sources you used to help you solve the problem (e.g. books from the library). This won't affect your grade but is important as academic honesty.

1. A chocolate company decides to promote its chocolate bars by including a coupon with each bar. A bar costs a dollar and with c coupons you get a new bar. How much chocolate is a dollar worth? What happens if $c = 1$?

2. Let f be the fibonacci number function:

$$f(n) = \begin{cases} 1 & n = 0 \\ 1 & n = 1 \\ f(n-1) + f(n-2) & n > 1 \end{cases} \tag{1}$$

and assume that f is non-decreasing. By using this fact alone, Prove by inducion for all $n \geq 2$ that

$$2^n \geq f(n)$$

3. Show that $1 = O(1)$ and that $1 = O(n)$.

4. Show that $5n = O(n \lg n)$ and that $7n \lg n = \Omega(n)$.

5. Suppose that algorithm A runs in time $f(n)$ and that algorithm B runs in time $g(n)$. Is A faster that B for all n if (i) $g(n) = \Omega(f(n) \lg n)$ and (ii) $g(n) = o(f(n) \lg n)$.

6. Suppose that $a > 1$ and that $f(n) = \Theta(\log_a n)$. Show that $f(n) = \Theta(lgn)$.

7. Use induction to prove that $lgn! \geq \frac{n}{2} \lg \frac{n}{2}$.

8. The following algorithm finds the largest and smallest elements in a sequence. The input is the sequence $s = \{s_i, \ldots, s_j\}$, i and j. The output is the largest element, *large*, and the smallest element, *small*.

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LARGESMALL(s, i, j, large, small)

    if i = j then
            large = s_i
            small = s_i
    m = ⌊(i + j)/2⌋
    LARGESMALL(s, i, m, large_left, small_left)
    LARGESMALL (s, m + 1, j, large_right, small_right)
    if large_left > large_right then
            large = large_left
    else
            large = large_right
    if small_left > small_right then
            small = small_right
    else
            small = small_left
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(a) Show that $c(1) = 1$ and $c(2) = 2$. Assume that comparisons (there are 2) are the things that cost us.

(b) Establish the recurrence relation.

(c) Solve the recurrence relation in case n is a power of 2.

(d) Prove by induction that $c(n) = 2n - 2$ for every positive integer n.