Stat 406 Spring 2007: Homework 1

Out Mon 8 Jan, back Mon 15 Jan

1 Bayes rule

After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of tetsing negative given that you don't have the disease). The good news is that this is a rare diseas, striking only one in 10,000 people. What are the chances that you actually have the disease? (Show your calculations as well as giving the final result.)

2 Bernoulli distributions

Let $X \in \{0, 1\}$ be a binary random variable (e.g., a coin toss). Suppose $p(X = 1) = \theta$. Then

$$p(x|\theta) = \text{Be}(X|\theta) = \theta^x (1 - \theta)^{1-x} \tag{1}$$

is called a **Bernoulli** distribution. Prove the following facts:

$$\begin{aligned} E[X] &= p(X = 1) = \theta & (2) \\ \text{Var}\,[X] &= \theta(1 - \theta) & (3) \end{aligned}$$

3 The Monty Hall problem

On a game show, a contestant is told the rules as follows:

> There are three doors, labelled 1, 2, 3. A single prize has been hidden behind one of them. You get to select one door. Initially your chosen door will *not* be opened. Instead, the gameshow host will open one of the other two doors, and *he will do so in such a way as not to reveal the prize.* For example, if you first choose door 1, he will then open one of doors 2 and 3, and it is guaranteed that he will choose which one to open so that the prize will not be revealed.
>
> At this point, you will be given a fresh choice of door: you can either stick with your first choice, or you can switch to the other closed door. All the doors will then be opened and you will receive whatever is behind your final choice of door.

Imagine that the contestant chooses door 1 first; then the gameshow host opens door 3, revealing nothing behind the door, as promised. Should the contestant (a) stick with door 1, or (b) switch to door 2, or (c) does it make no difference? You may assume that initially, the prize is equally likely to be behind any of the 3 doors. Hint: use Bayes rule.