EE E6820: Speech & Audio Processing & Recognition

Lecture 10: ASR: Sequence Recognition

1. **Signal template matching**
2. **Statistical sequence recognition**
3. **Acoustic modeling**
4. **The Hidden Markov Model (HMM)**

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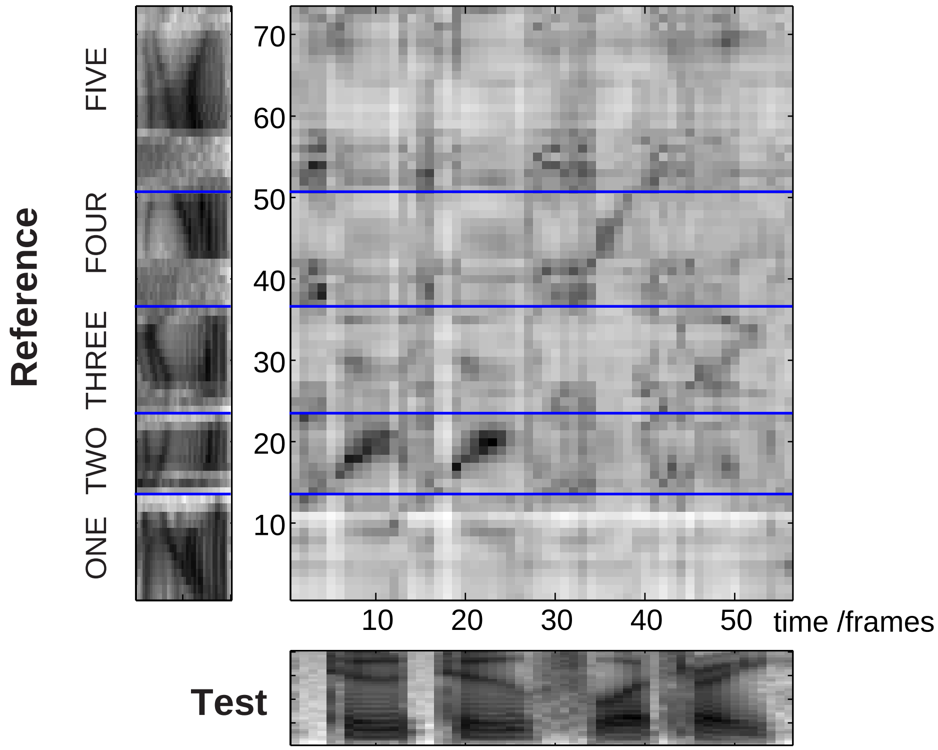
1 Signal template matching

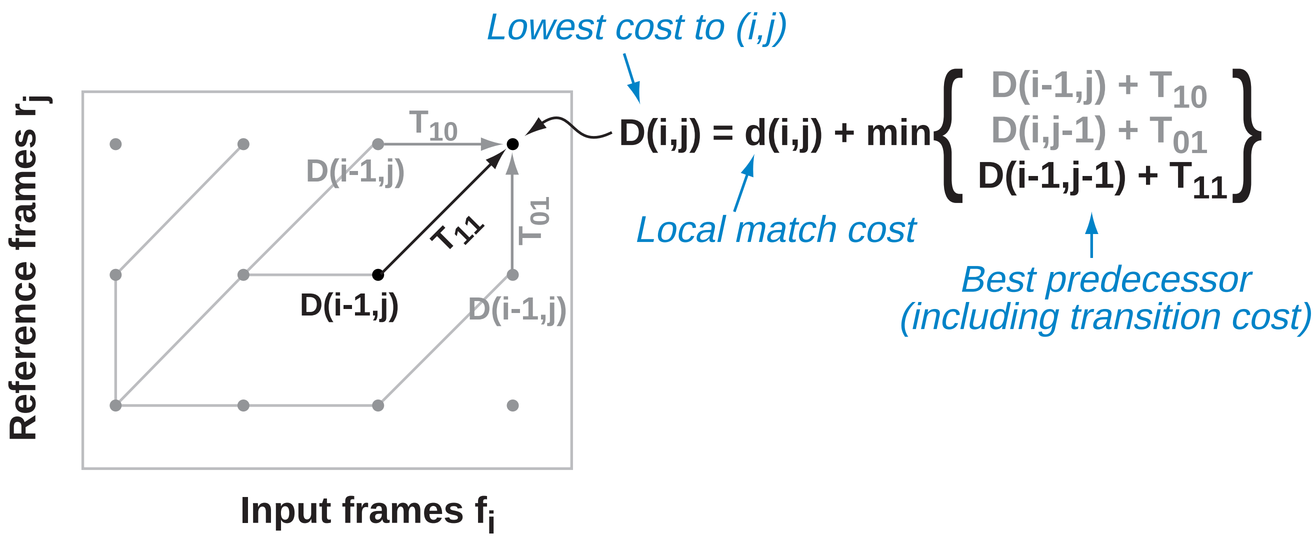
- **Framewise comparison of unknown word and stored templates:**



  - distance metric?
  - comparison between templates?
  - constraints?

Dynamic Time Warp (DTW)

- **Find lowest-cost constrained path:**
  - matrix d(i,j) of distances between input frame f_i and reference frame r_j
  - allowable predecessors & transition costs T_{xy}

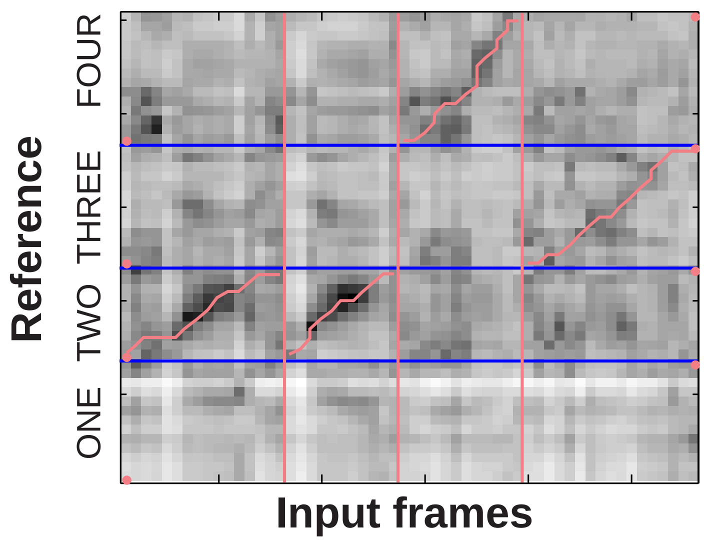


$$D(i,j) = d(i,j) + \min\left\{\begin{array}{l} D(i-1,j) + T_{10} \\ D(i,j-1) + T_{01} \\ D(i-1,j-1) + T_{11} \end{array}\right\}$$

- **Best path via traceback from final state**
  - have to store predecessors for (almost) every (i,j)

DTW-based recognition

- **Reference templates for each possible word**
- **Isolated word:**
  - mark endpoints of input word
  - calculate scores through each template (+prune)
  - choose best
- **Continuous speech**
  - one matrix of template slices;
    special-case constraints at word ends



DTW-based recognition (2)

+ **Successfully handles timing variation**
+ **Able to recognize speech at reasonable cost**
- **Distance metric?**
  - pseudo-Euclidean space?
- **Warp penalties?**
- **How to choose templates?**
  - several templates per word?
  - choose ‘most representative’?
  - align and average?

$\rightarrow$ **need a *rigorous* foundation...**

Outline

1. Signal template matching
2. **Statistical sequence recognition**
   - state-based modeling
3. Acoustic modeling
4. The Hidden Markov Model (HMM)

2 Statistical sequence recognition

- **DTW limited because it's hard to optimize**
  - interpretation of distance, transition costs?
- **Need a theoretical foundation: Probability**
- **Formulate as MAP choice among models:**

$$M^* = \underset{M_j}{\operatorname{argmax}}\ p(M_j | X, \Theta)$$

  - X = observed features
  - M_j = word-sequence models
  - Θ = all current parameters

Statistical formulation (2)

- **Can rearrange via Bayes' rule (& drop $p(X)$):**

$$M^* = \underset{M_j}{\operatorname{argmax}}\ p(M_j | X, \Theta)$$

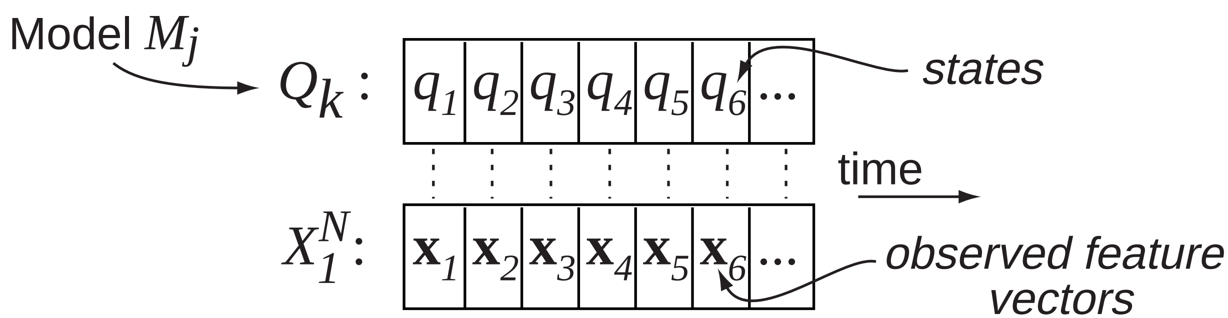
$$= \underset{M_j}{\operatorname{argmax}}\ p(X | M_j, \Theta_A) p(M_j | \Theta_L)$$

  - $p(X \mid M_j)$ = likelihood of obs'v'ns under model
  - $p(M_j)$ = prior probability of model
  - Θ_A = acoustics-related model parameters
  - Θ_L = language-related model parameters

- **Questions:**
  - what form of model to use for $p(X | M_j, \Theta_A)$?
  - how to find Θ_A (training)?
  - how to solve for M_j (decoding)?

State-based modeling

- **Assume discrete-state model for the speech:**
  - observations are divided up into time frames
  - model → states → observations:



- **Probability of observations given model is:**

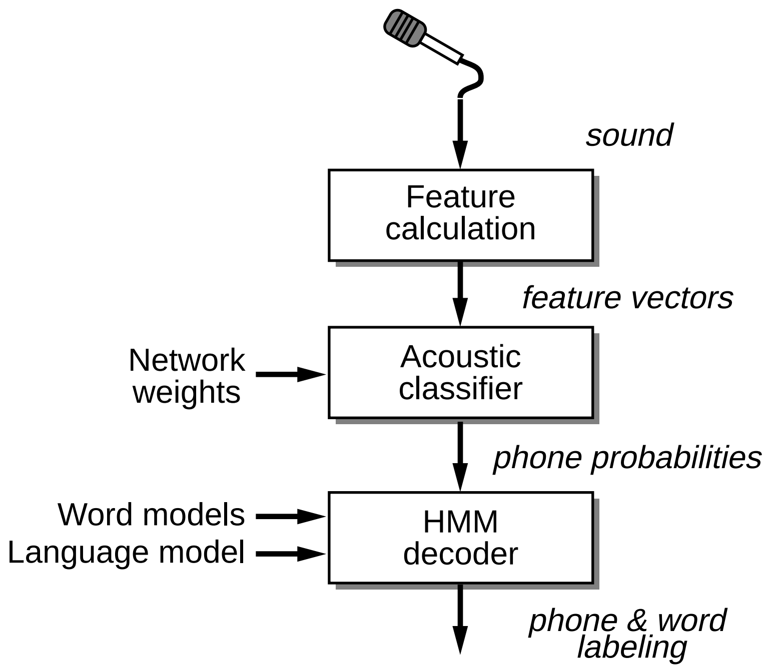
$$p(X|M_j) = \sum_{\text{all } Q_k} p(X_1^N \big| Q_k, M_j) \cdot p(Q_k \big| M_j)$$

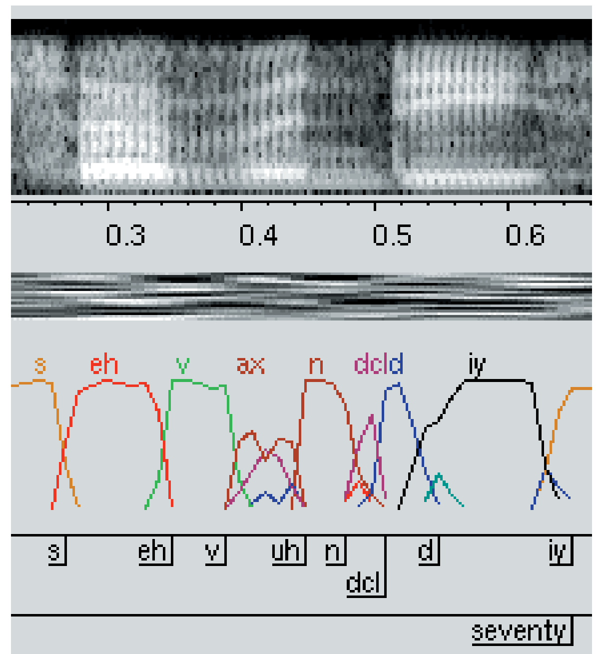
  - sum over all possible state sequences Q_k

- **How do observations depend on states?**
  How do state sequences depend on model?

The speech recognition chain

- **After classification, still have problem of classifying the sequences of frames:**





- **Questions**
  - what to use for the acoustic classifier?
  - how to represent 'model' sequences?
  - how to score matches?

Outline

1. **Signal template matching**
2. **Statistical sequence recognition**
3. **Acoustic modeling**
   - defining targets
   - neural networks & Gaussian models
4. **The Hidden Markov Model (HMM)**

3 Acoustic Modeling

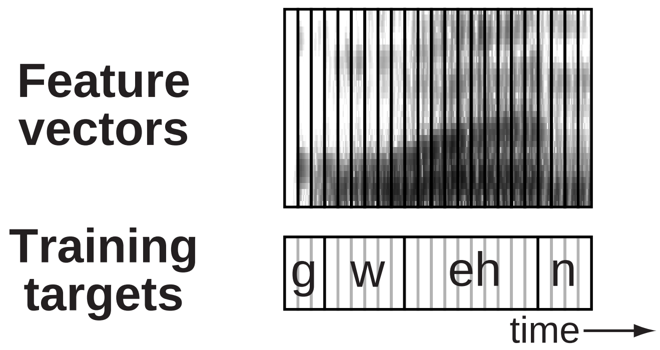
- **Goal: Convert features into probabilities of particular labels:**

  i.e find $p(q_n^i | X_n)$ **over some state set {**q^i**}**

  - conventional statistical classification problem
- **Classifier construction is *data-driven***
  - assume we can get examples of known good Xs for each of the q^is
  - calculate model parameters by standard training scheme
- **Various classifiers can be used**
  - GMMs model distribution under each state
  - Neural Nets directly estimate posteriors
- **Different classifiers have different properties**
  - features, labels limit ultimate performance

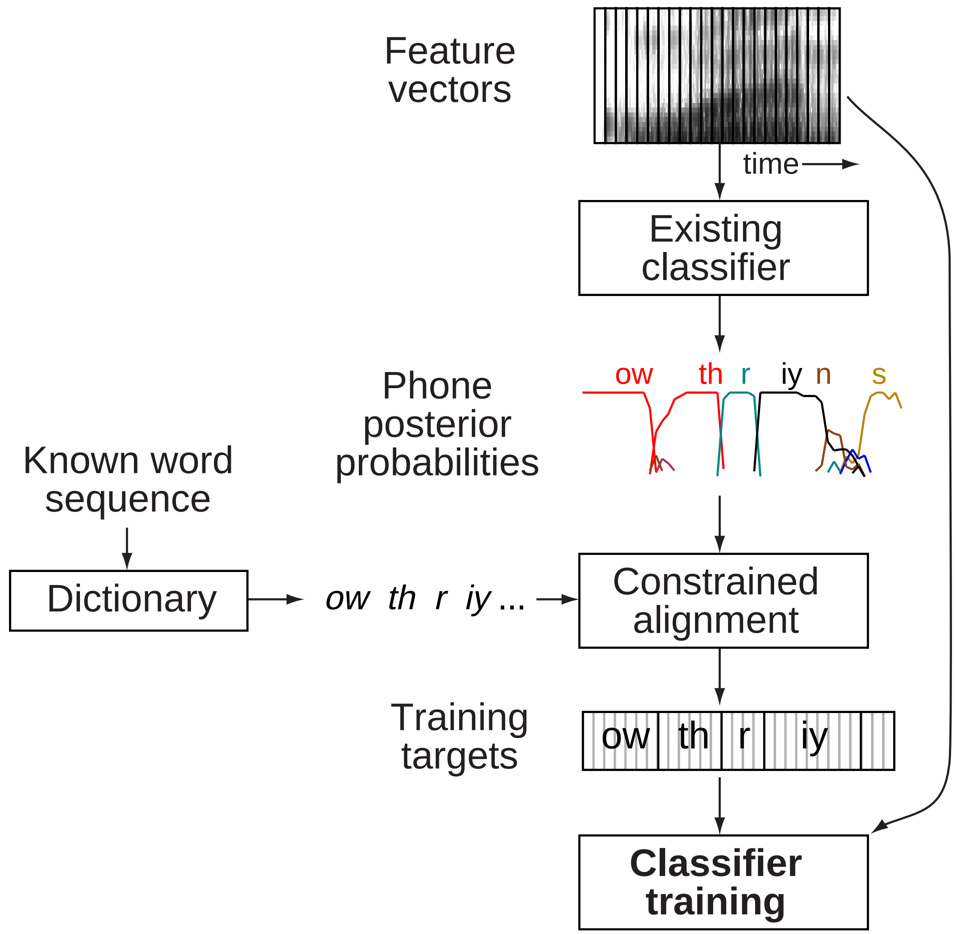
Defining classifier targets

- **Choice of {q^i} can make a big difference**
  - must support recognition task
  - must be a practical classification task
- **Hand-labeling is one source...**
  - ‘experts’ mark spectrogram boundaries
- **...Forced alignment is another**
  - ‘best guess’ with existing classifiers, given words
- **Result is *targets* for each training frame:**



Forced alignment

- **Best labeling given existing classifier constrained by known word sequence**



Gaussian Mixture Models vs. Neural Nets

- **GMMs fit distribution of features under states:**
  - *separate* 'likelihood' model for each state q^i

$$p(\mathbf{x}|q^k) = \frac{1}{(\sqrt{2\pi})^d |\Sigma_k|^{1/2}} \cdot \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_k)^T \Sigma_k^{-1}(\mathbf{x}-\mu_k)\right]$$

  - match any distribution given enough data

- **Neural nets estimate posteriors directly**

$$p(q^k|\mathbf{x}) = F[\sum_j w_{jk} \cdot F[\sum_j w_{ij} x_i]]$$

  - parameters set to *discriminate* classes

- **Posteriors & likelihoods related by Bayes' rule:**

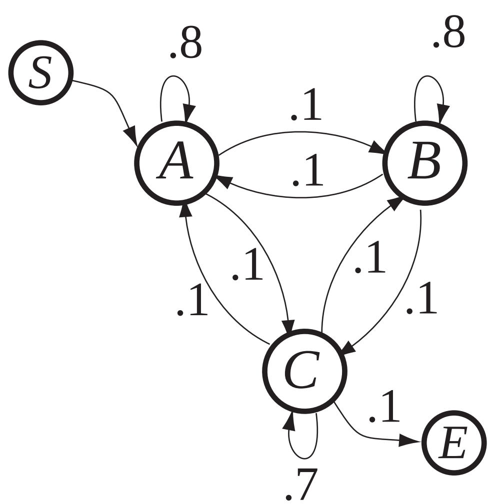
$$p(q^k|\mathbf{x}) = \frac{p(\mathbf{x}|q^k) \cdot Pr(q^k)}{\sum_j p(\mathbf{x}|q^j) \cdot Pr(q^j)}$$

Outline

1. **Signal template matching**
2. **Statistical sequence recognition**
3. **Acoustic classification**
4. **The Hidden Markov Model (HMM)**
   - generative Markov models
   - hidden Markov models
   - model fit likelihood
   - HMM examples

Markov models

- **A (first order) Markov model is a finite-state system whose behavior depends *only on the current state***
- **E.g. *generative* Markov model:**



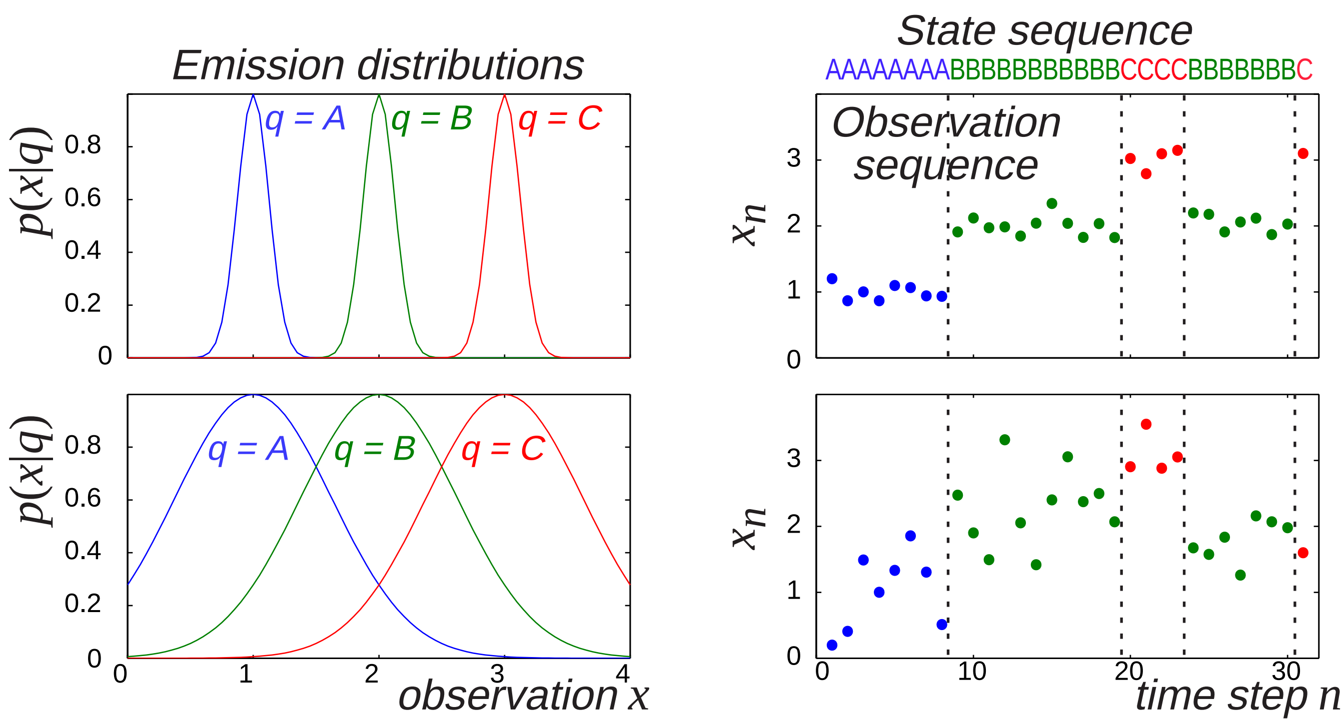
$p(q_{n+1}|q_n)$

q_n \ q_{n+1}	S	A	B	C	E
S	0	1	0	0	0
A	0	.8	.1	.1	0
B	0	.1	.8	.1	0
C	0	.1	.1	.7	.1
E	0	0	0	0	1

S A A A A A A A A B B B B B B B B B C C C C B B B B B B C E

Hidden Markov models

- **Markov models where state sequence** $Q = \{q_n\}$ **is not directly observable (= 'hidden')**
- **But, observations** X ***do*** **depend on** Q**:**
  - x_n is rv that depends on current state: $p(x|q)$



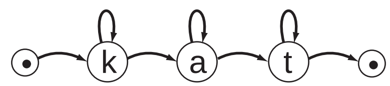
  - can still tell *something* about state seq...

(Generative) Markov models (2)

- **HMM is specified by:**
  - transition probabilities $p(q_n^j | q_{n-1}^i) \equiv a_{ij}$
  - (initial state probabilities $p(q_1^i) \equiv \pi_i$)
  - emission distributions $p(x | q^i) \equiv b_i(x)$

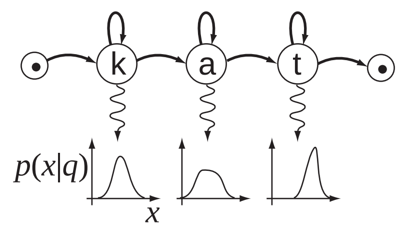
- states q^i

- transition probabilities a_{ij}



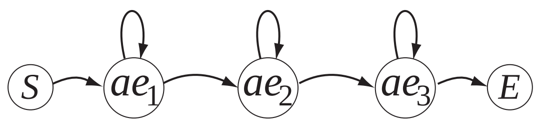
	k	a	t	•
•	1.0	0.0	0.0	0.0
k	0.9	0.1	0.0	0.0
a	0.0	0.9	0.1	0.0
t	0.0	0.0	0.9	0.1

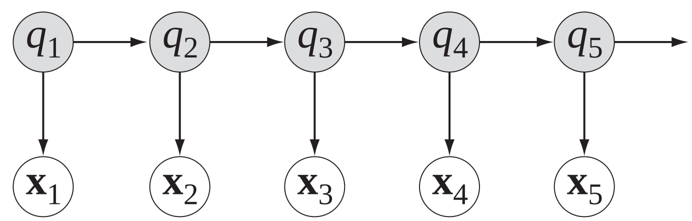
- emission distributions $b_i(x)$



Markov models for speech

- **Speech models** M_j
  - typ. left-to-right HMMs (sequence constraint)
  - observation & evolution are conditionally independent of rest given (hidden) state q_n





  - *self-loops* for time dilation

Markov models for sequence recognition

- **Independence of observations:**
  - observation x_n depends only current state q_n

$$p(X|Q) = p(x_1, x_2, \ldots x_N | q_1, q_2, \ldots q_N)$$

$$= p(x_1|q_1) \cdot p(x_2|q_2) \cdot \ldots p(x_N|q_N)$$

$$= \prod_{n=1}^{N} p(x_n|q_n) = \prod_{n=1}^{N} b_{q_n}(x_n)$$

- **Markov transitions:**
  - transition to next state q_{i+1} depends only on q_i

$$p(Q|M) = p(q_1, q_2, \ldots q_N | M)$$

$$= p(q_N|q_1 \ldots q_{N-1}) p(q_{N-1}|q_1 \ldots q_{N-2}) \ldots p(q_2|q_1) p(q_1)$$

$$= p(q_N|q_{N-1}) p(q_{N-1}|q_{N-2}) \ldots p(q_2|q_1) p(q_1)$$

$$= p(q_1) \prod_{n=2}^{N} p(q_n|q_{n-1}) = \pi_{q_1} \prod_{n=2}^{N} a_{q_{n-1} q_n}$$

Model fit calculation

- **From 'state-based modeling':**

$$p(X|M_j) = \sum_{\text{all } Q_k} p(X_1^N \big| Q_k, M_j) \cdot p(Q_k \big| M_j)$$

- **For HMMs:**

$$p(X|Q) = \prod_{n=1}^{N} b_{q_n}(x_n)$$

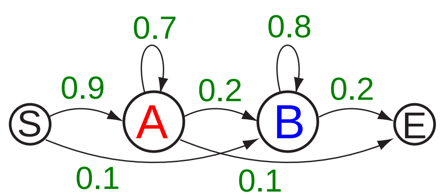
$$p(Q|M) = \pi_{q_1} \cdot \prod_{n=2}^{N} a_{q_{n-1}q_n}$$

- **Hence, solve for** M^* **:**
  - calculate $p(X|M_j)$ for each available model, scale by prior $p(M_j) \rightarrow p(M_j|X)$

- **Sum over *all*** Q_k **???**

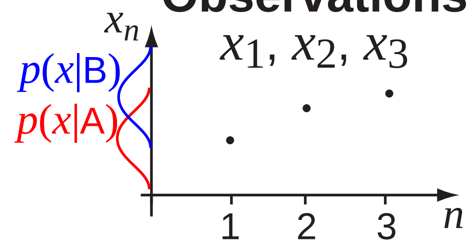
Summing over all paths

Model M_1



	S	A	B	E
S	•	0.9	0.1	•
A	•	0.7	0.2	0.1
B	•	•	0.8	0.2
E	•	•	•	1

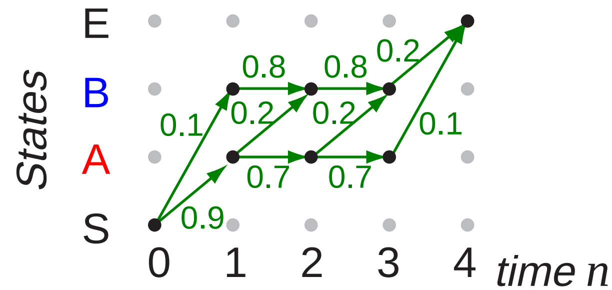
Observations



Observation likelihoods

$p(x\|q)$		x_1	x_2	x_3
q	A	2.5	0.2	0.1
	B	0.1	2.2	2.3

Paths



All possible 3-emission paths Q_k from S to E

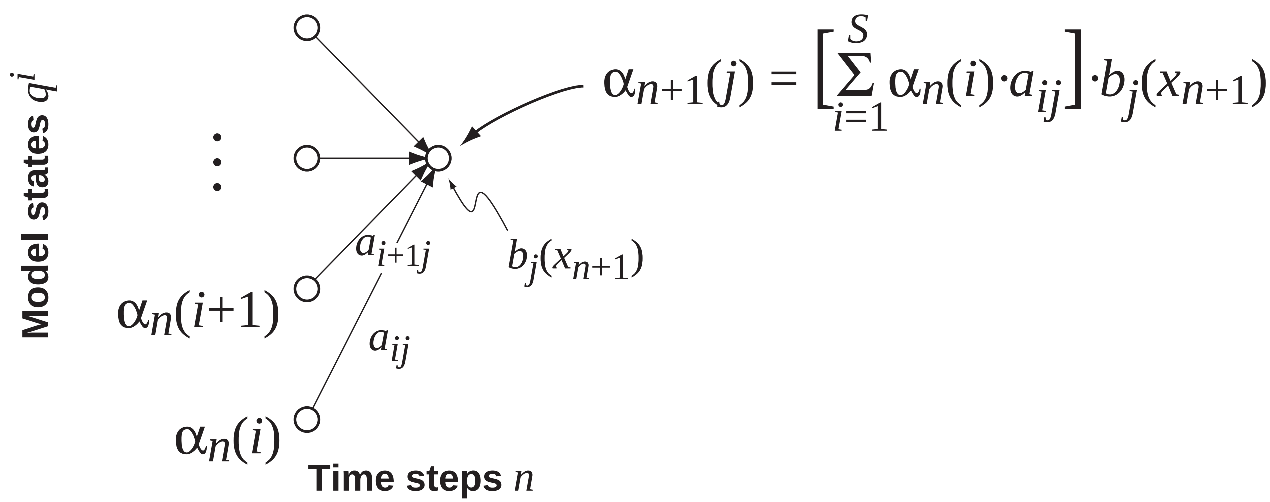
q_0	q_1	q_2	q_3	q_4	$p(Q \mid M) = \prod_n p(q_n \mid q_{n-1})$	$p(X \mid Q,M) = \prod_n p(x_n \mid q_n)$	$p(X,Q \mid M)$
S	A	A	A	E	.9 x .7 x .7 x .1 = **0.0441**	2.5 x 0.2 x 0.1 = 0.05	0.0022
S	A	A	B	E	.9 x .7 x .2 x .2 = 0.0252	2.5 x 0.2 x 2.3 = 1.15	0.0290
S	A	B	B	E	.9 x .2 x .8 x .2 = 0.0288	2.5 x 2.2 x 2.3 = 12.65	**0.3643**
S	B	B	B	E	.1 x .8 x .8 x .2 = 0.0128	0.1 x 2.2 x 2.3 = 0.506	0.0065
					Σ = 0.1109		Σ = $p(X \mid M)$ = **0.4020**

The 'forward recursion'

- **Dynamic-programming-like technique to calculate sum over all** Q_k
- **Define** $\alpha_n(i)$ **as the probability of *getting to* state** q^i **at time step** n **(by any path):**

$$\alpha_n(i) = p(x_1, x_2, \ldots x_n, q_n = q^i) \equiv p(X_1^n, q_n^i)$$

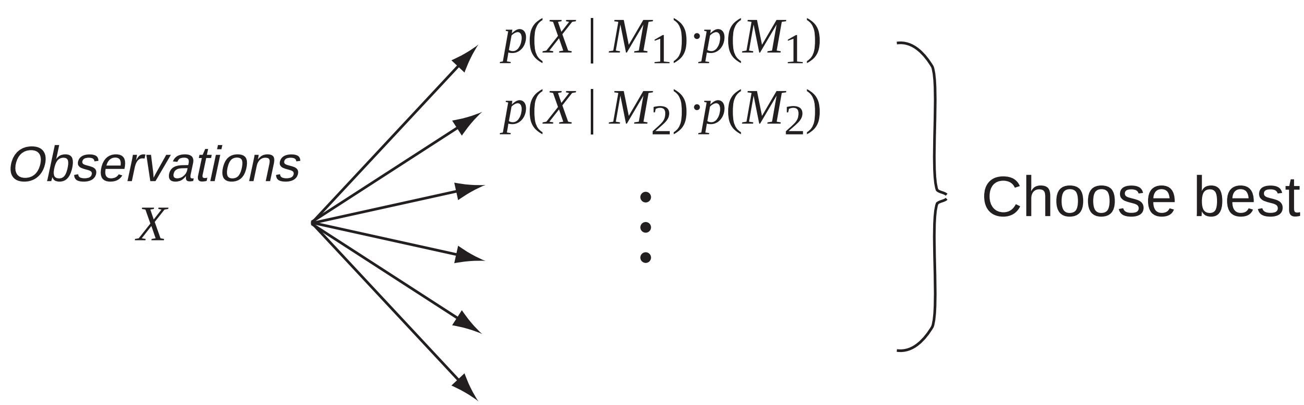
- **Then** $\alpha_{n+1}(j)$ **can be calculated recursively:**



Forward recursion (2)

- **Initialize** $\alpha_1(i) = \pi_i \cdot b_i(x_1)$
- **Then total probability** $p(X_1^N | M) = \sum_{i=1}^{S} \alpha_N(i)$

$\rightarrow$ **Practical way to solve for** $p(X | M_j)$ **and hence perform recognition**



Optimal path

- **May be interested in actual q_n assignments**
  - which state was 'active' at each time frame
  - e.g. phone labelling (for training?)
- **Total probability is over *all* paths...**
- **... but can also solve for single *best* path = "Viterbi" state sequence**
- **Probability along best path to state q_{n+1}^{j}:**

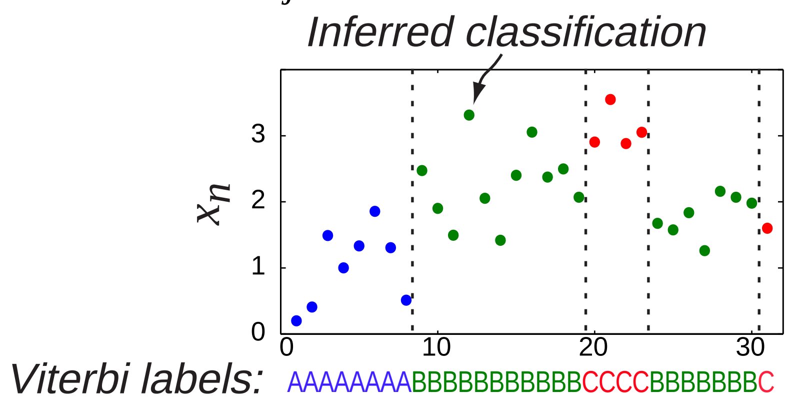
$$\alpha_{n+1}^{*}(j) = \left[\max_{i} \{\alpha_{n}^{*}(i) a_{ij}\}\right] \cdot b_j(x_{n+1})$$

  - backtrack from final state to get best path
  - final probability is product only (no sum) $\rightarrow$ log-domain calculation just summation
- **Total probability often dominated by best path:**

$$p(X, Q^{*} | M) \approx p(X | M)$$

Interpreting the Viterbi path

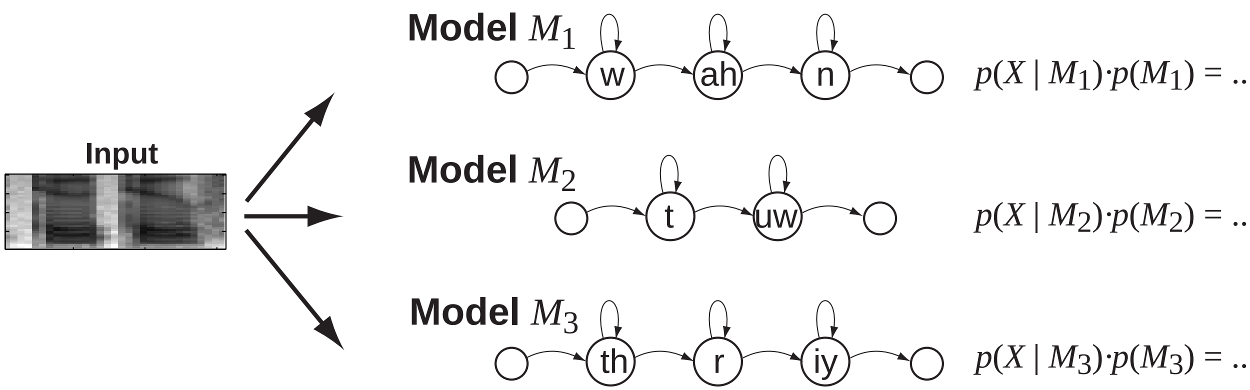
- **Viterbi path assigns each x_n to a state q^i**
  - performing classification based on $b_i(x)$
  - ... at the same time as applying transition constraints a_{ij}



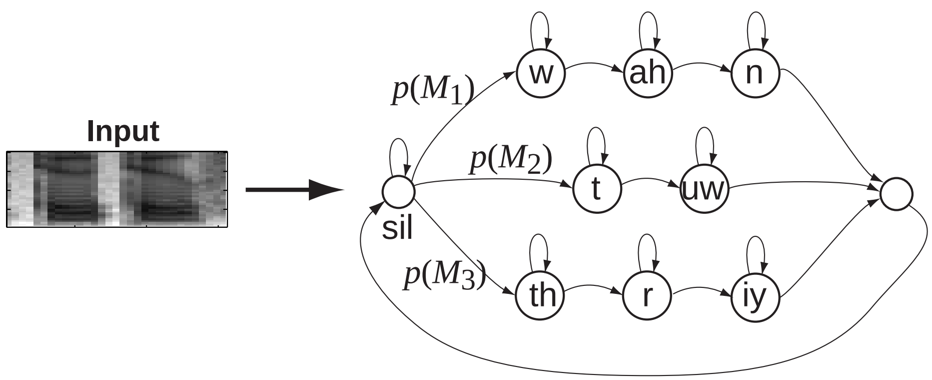
- **Can be used for segmentation**
  - train an HMM with 'garbage' and 'target' states
  - decode on new data to find 'targets', boundaries
- **Can use for (heuristic) training**
  - e.g. train classifiers based on labels...

Recognition with HMMs

- **Isolated word**
  - choose best $p(M|X) \propto p(X|M)p(M)$

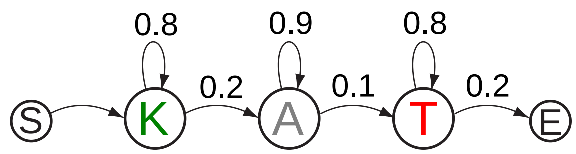


- **Continuous speech**
  - Viterbi decoding of one large HMM gives words

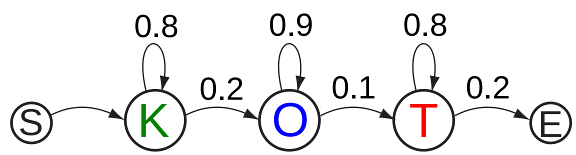


HMM example: Different state sequences

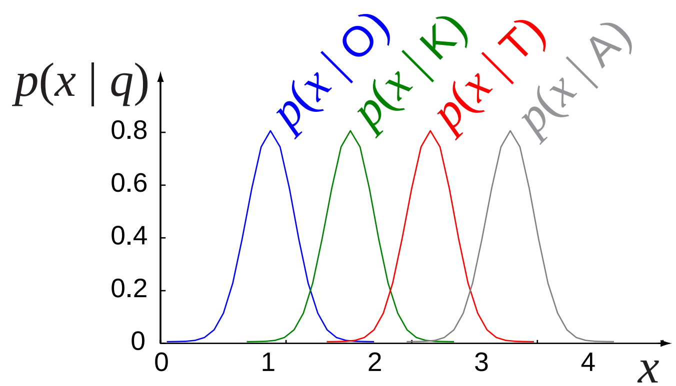
Model M_1



Model M_2

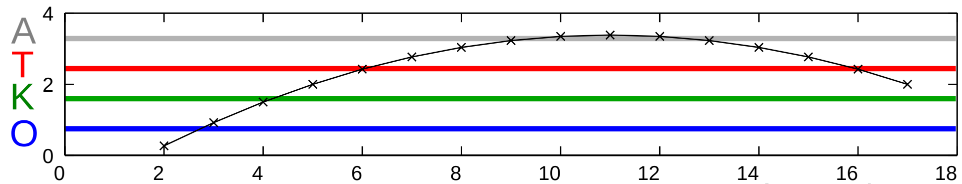


Emission distributions



Model inference: Emission probabilities

Observation sequence
x_n

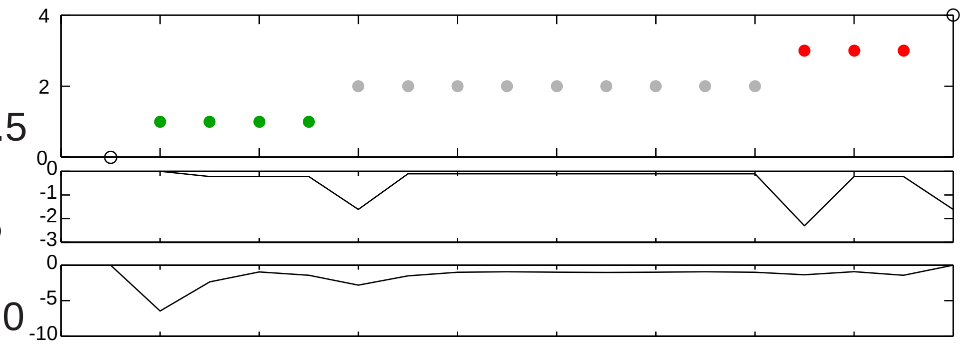


Model M_1
log $p(X \mid M)$ = -32.1

state alignment
log $p(X,Q^* \mid M)$ = -33.5

log trans.prob
log $p(Q^* \mid M)$ = -7.5

log obs.l'hood
log $p(X \mid Q^*,M)$ = -26.0

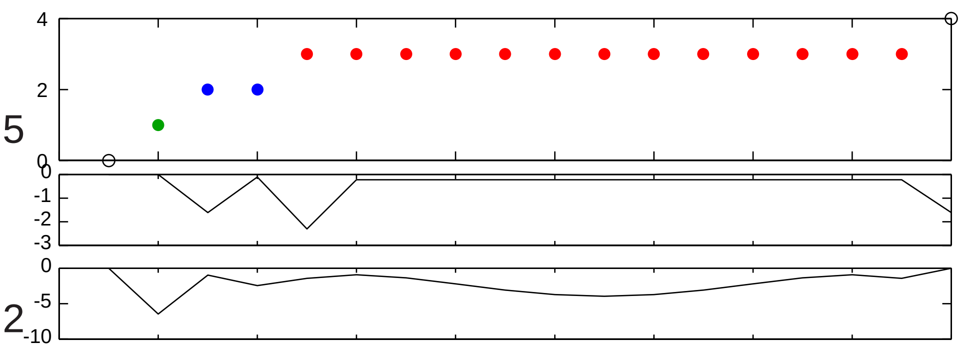


Model M_2
log $p(X \mid M)$ = -47.0

state alignment
log $p(X,Q^* \mid M)$ = -47.5

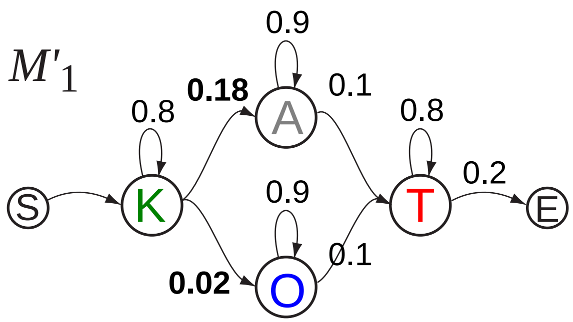
log trans.prob
log $p(Q^* \mid M)$ = -8.3

log obs.l'hood
log $p(X \mid Q^*,M)$ = -39.2

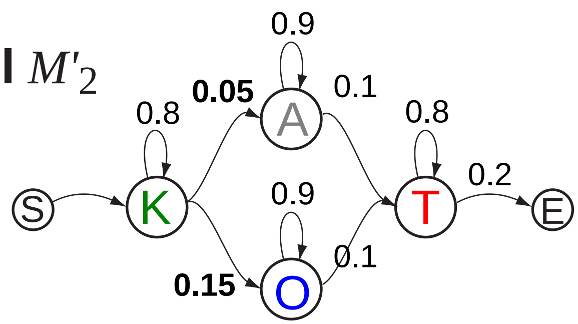


Model inference: Transition probabilities

Model M'_1



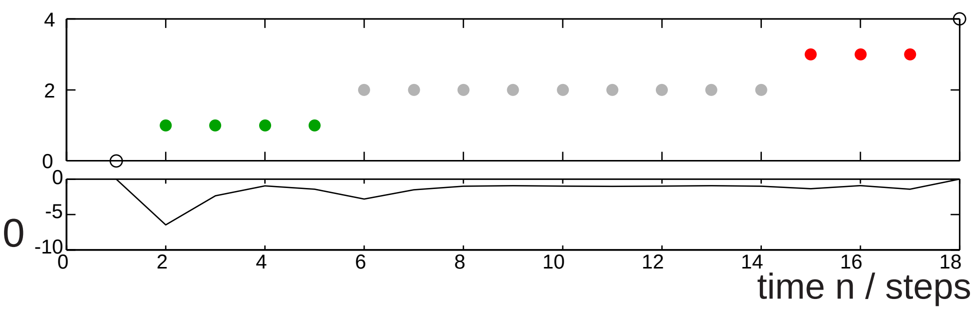
Model M'_2



state alignment

log obs.l'hood

$\log p(X \mid Q^*, M) = -26.0$

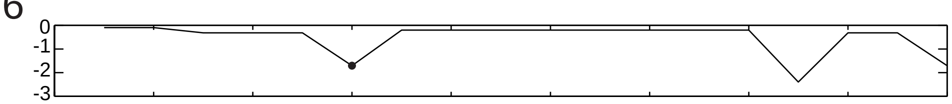


Model M'_1 $\log p(X \mid M) = -32.2$

$\log p(X, Q^* \mid M) = -33.6$

log trans.prob

$\log p(Q^* \mid M) = -7.6$

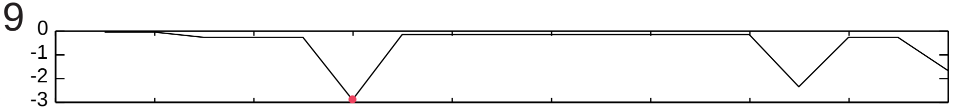


Model M'_2 $\log p(X \mid M) = -33.5$

$\log p(X, Q^* \mid M) = -34.9$

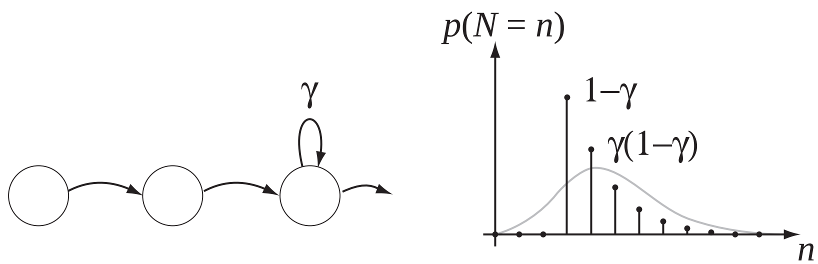
log trans.prob

$\log p(Q^* \mid M) = -8.9$

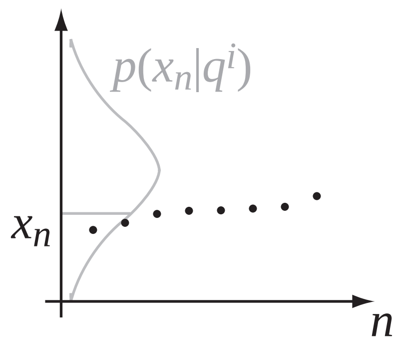


Validity of HMM assumptions

- **Key assumption is *conditional independence*:**
  Given q^i, future evolution & obs. distribution are independent of previous events
  - duration behavior: self-loops imply exponential distribution

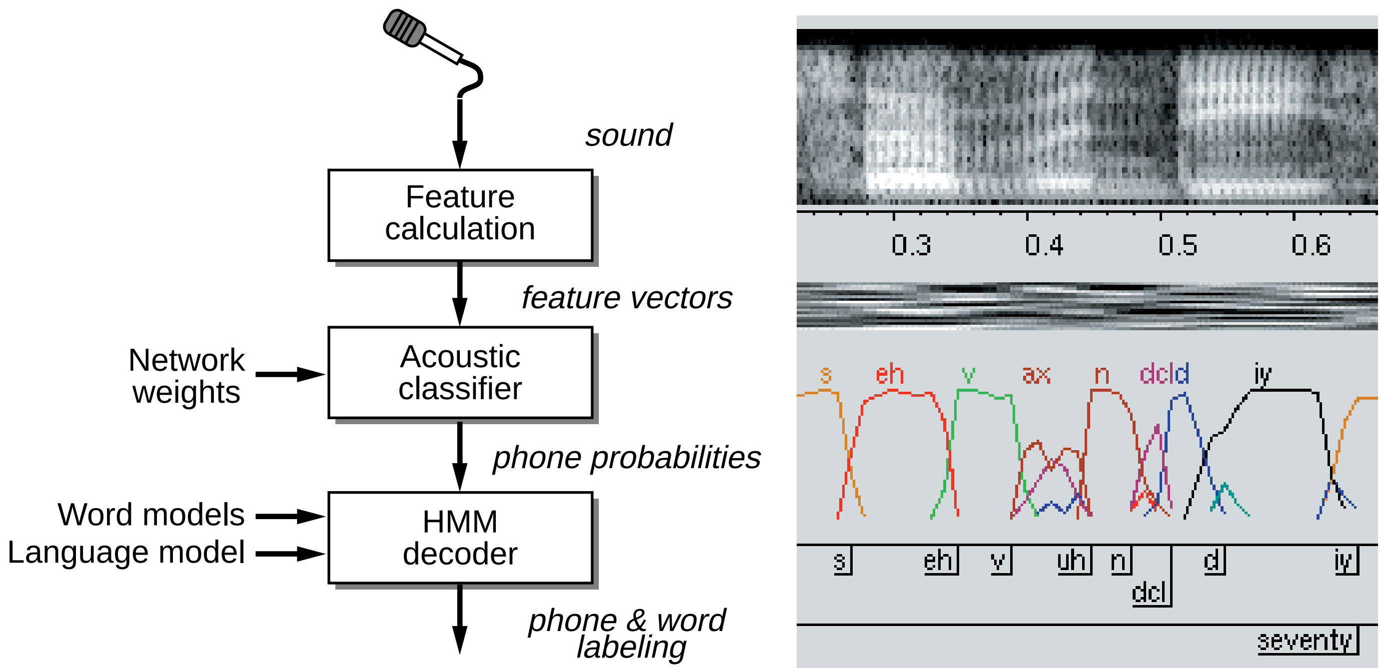


  - independence of successive x_ns



$$p(X) = \prod p(x_n|q^i) \ ?$$

Recap: Recognizer Structure



- **Know how to execute each state**
- **.. training HMMs?**
- **.. language/word models**

Summary

- **Speech is modeled as a *sequence* of features**
  - need temporal aspect to recognition
  - best time-alignment of templates = DTW
- **Hidden Markov models are rigorous solution**
  - self-loops allow temporal dilation
  - exact, efficient likelihood calculations