CpSc 513: OBDD Examples

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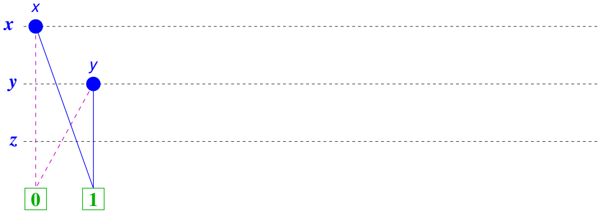
Outline:

OBDD example: majority gates

$\text{maj1}(x, y, z) = (x \wedge y) \ \vee \ (y \wedge z) \ \vee \ (z \wedge x)$



- OBDDs for x and y are simple.
- To get the OBDD for $x \wedge y$ we use Apply

OBDD example: majority gates

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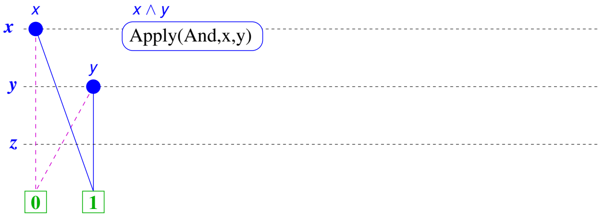

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  - x is first variable in the order

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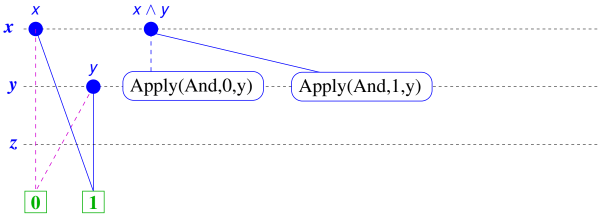

- OBDDs for x and y are simple.
- To get the OBDD for $x \wedge y$ we use Apply
  - x is first variable in the order
  - false branch at x for Apply(AND, x, y) is Apply(And$x|_{x \leftarrow 0}, y|_{x \leftarrow 0}$) which simplifies to Apply(And, 0, y). Likewise, the true branch is Apply(And, 1, y).

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  - Apply(And, 0, y) simplies to 0, and Apply(And, 1, y) simplifies to y.

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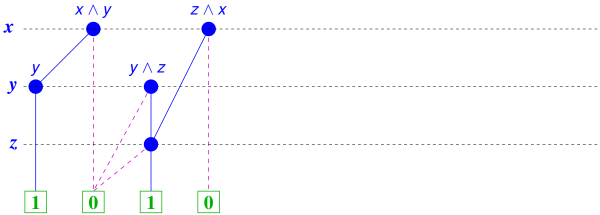

- OBDDs for x and y are simple.
- To get the OBDD for $x \wedge y$ we use Apply
- The OBDDs for $y \wedge z$ and $z \wedge x$ are similar.
- To avoid lots of crossing edges; I'll use multiple **0** and **1** leaves. To keep the OBDD canonical, all **0** leaves are actually the same node, and likewise for the **1** leaves.

OBDD example: majority gates

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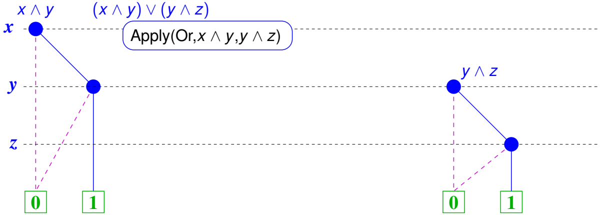

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z)$.

OBDD example: majority gates

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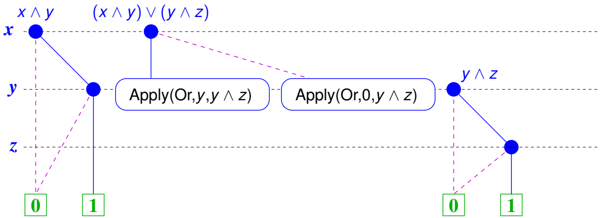

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z)$.
  - $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 0} = 0 \vee (y \wedge z)$,
    $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 1} = y \vee (y \wedge z)$.

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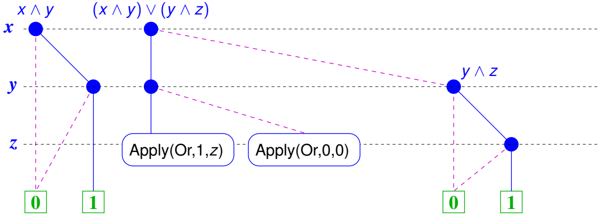

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z)$.
  - $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 0} = 0 \vee (y \wedge z)$,
    $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 1} = y \vee (y \wedge z)$.
  - $0 \vee (y \wedge z) = y \wedge z$,
    $(y \vee (y \wedge z))|_{y \leftarrow 0} = 0 \vee (0 \wedge z) = 0$,
    $(y \vee (y \wedge z))|_{y \leftarrow 1} = 1 \vee (1 \wedge z) = 1$.

OBDD example: majority gates

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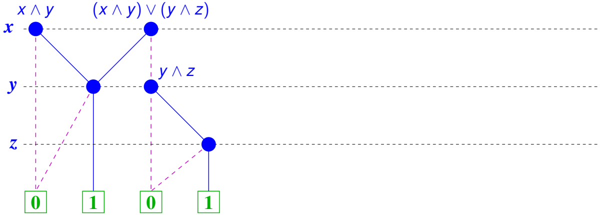

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z)$.
  - $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 0} = 0 \vee (y \wedge z)$,
    $((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 1} = y \vee (y \wedge z)$.
  - $0 \vee (y \wedge z) = y \wedge z$,
    $(y \vee (y \wedge z))|_{y \leftarrow 0} = 0 \vee (0 \wedge z) = 0$,
    $(y \vee (y \wedge z))|_{y \leftarrow 1} = 1 \vee (1 \wedge z) = 1$.
  - $\therefore ((x \wedge y) \vee (y \wedge z))|_{x \leftarrow 1} = y \vee (y \wedge z) = y$

OBDD example: majority gates

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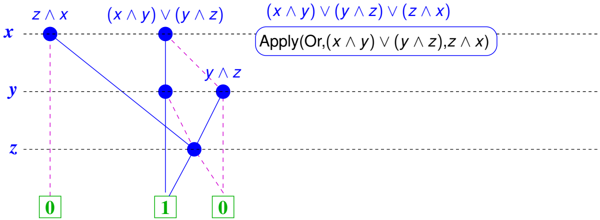

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x)$.

OBDD example: majority gates

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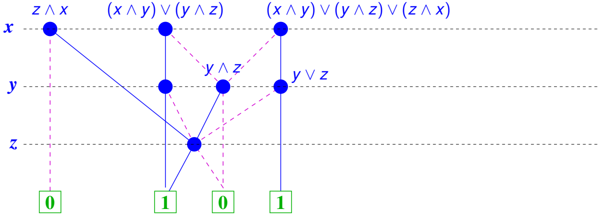

- Use apply to get OBDD for $(x \wedge y) \vee (y \wedge z) \vee (z \wedge x)$.

OBDD example: majority gates

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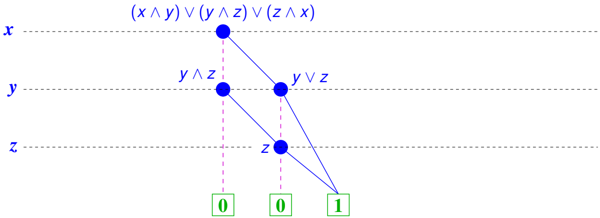
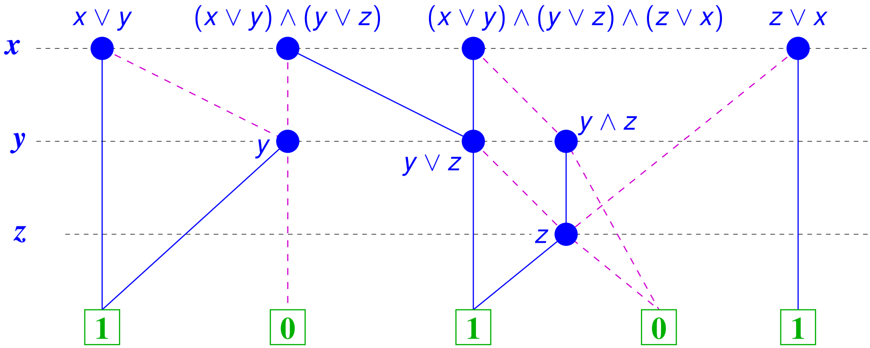


- I'll reduced the clutter and only showed the subgraph for $(x \wedge y) \vee (y \wedge z)0zx$.

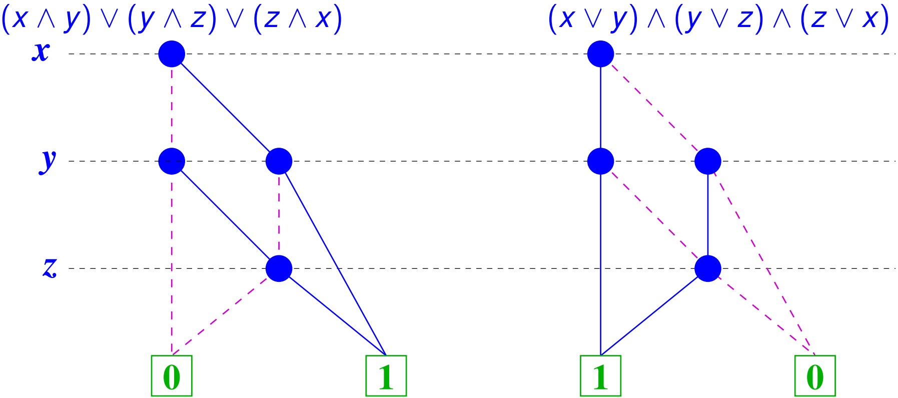
OBDD example: majority gate – the product-of-sums version

$$\text{maj2}(x, y, z) = (x \lor y) \land (y \lor z) \land (z \lor x)$$

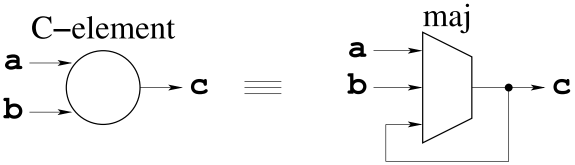


OBDD example: majority gates – are they the same?

$$\text{maj1}(x, y, z) = (x \wedge y) \ \vee \ (y \wedge z) \ \vee \ (z \wedge x)$$
$$\text{maj2}(x, y, z) = (x \vee y) \ \wedge \ (y \vee z) \ \wedge \ (z \vee x)$$



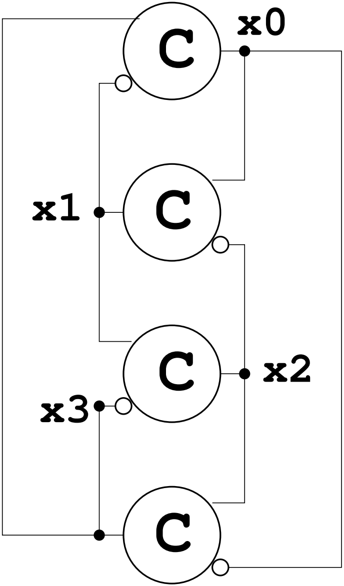
Now I C



a	b	c
0	0	0
0	1	unchanged
1	0	unchanged
1	1	1

- A C-element is a state-holding circuit – kind of like a flip-flop
- The value of `c` is the value that `a` and `b` had the last time they agreed.
- Originally described in:
  D.E. Muller and W.S. Bartky, “A Theory of Asynchronous Circuits”, *Proceedings of the International Symposium on Switching Theory*, pp. 204–243, 1959.

Fun with C-elements



Temporal Logic

- LTL: Linear time logic: properties that hold for **all** traces
  - p: The property p holds in the current state.
  - $\Box p$: **Always** – the property p holds in this state and all subsequent states.
  - $\diamond p$: **Eventually** – The property p in this state or some future state.
  - Example: $\Box(req \rightarrow \diamond ack)$ – From all states in which *req* holds, *ack* will eventually hold.
- CTL: Computational Tree Logic – traces are viewed as branching trees of all possible behaviours.