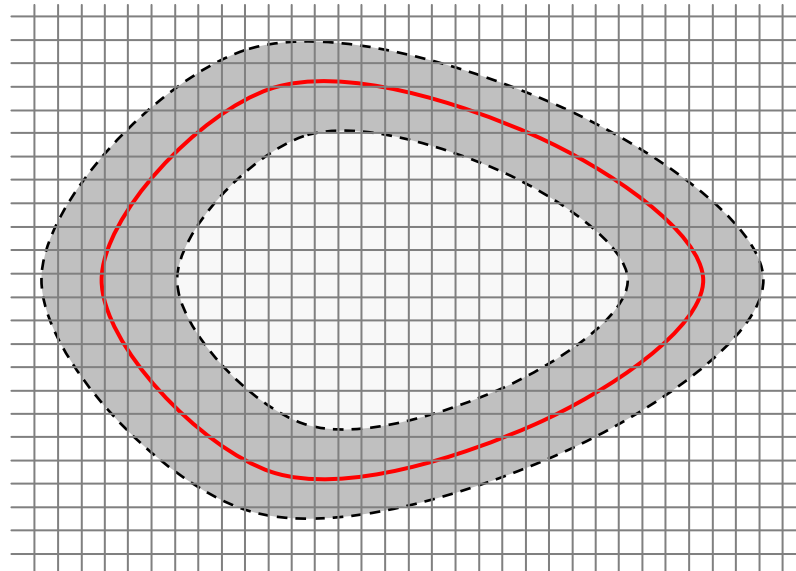


Efficiency

Narrowbanding / Local Level Set
Projections

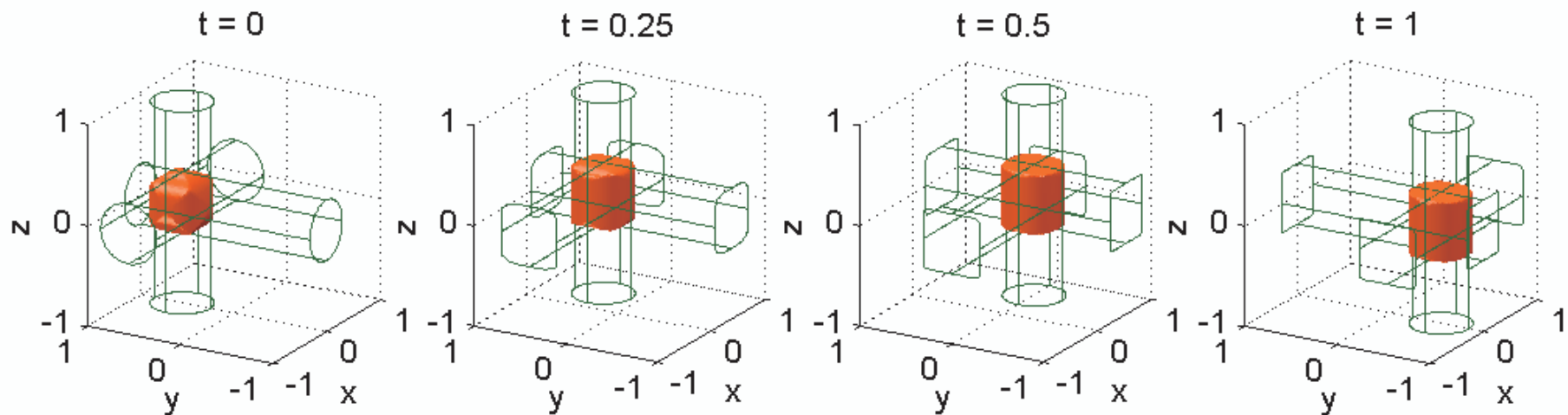
Reducing the Cost of Level Set Methods

- Solve Hamilton-Jacobi equation only in a band near interface
- Computational detail: handling stencils near edge of band
 - “Narrowbanding” uses low order accurate reconstruction whenever errors are detected
 - “Local level set” modifies Hamiltonian near edge of band
- Data structure detail: handling merging and breaking of interface



Projective Overapproximation

- Overapproximate reachable set of high dimensional system as the intersection of reachable sets for lower dimensional projections
 - [Mitchell & Tomlin, 2002]
 - Example: rotation of “sphere” about z -axis



Computing with Projections

- Forward and backward reachable sets for finite automata
 - Projecting into overlapping subsets of the variables, computing with BDDs [Govindaraju, Dill, Hu, Horowitz]
- Forward reachable sets for continuous systems
 - Projecting into 2D subspaces, representation by polygons [Greenstreet & Mitchell]
- Level set algorithms for geometric optics
 - Need multiple arrival time (viscosity solution gives first arrival time), so compute in higher dimensions and project down [Osher, Cheng, Kang, Shim & Tsai]

Hamilton-Jacobi in the Projection

- Consider x - z projection represented by level set $\phi_{xz}(x, z, t)$
 - Back projection into 3D yields a cylinder $\phi_{xz}(x, y, z, t)$
- Simple HJ PDE for this cylinder

$$D_t \phi_{xz}(x, y, z, t) + \sum_{i=1}^3 p_i f_i(x, y, z) = 0 \quad \text{where} \quad \begin{cases} p_1 &= D_x \phi_{xz}(x, y, z, t) \\ p_2 &= D_y \phi_{xz}(x, y, z, t) \\ p_3 &= D_z \phi_{xz}(x, y, z, t) \end{cases}$$

- But for cylinder parallel to y -axis, $p_2 = 0$

$$D_t \phi_{xz}(x, y, z, t) + p_1 f_1(x, y, z) + p_3 f_3(x, y, z) = 0$$

- What value to give free variable y in $f_i(x, y, z)$?
 - Treat it as a disturbance, bounded by the other projections

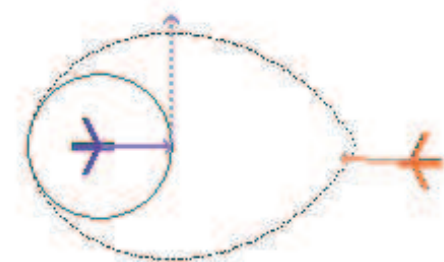
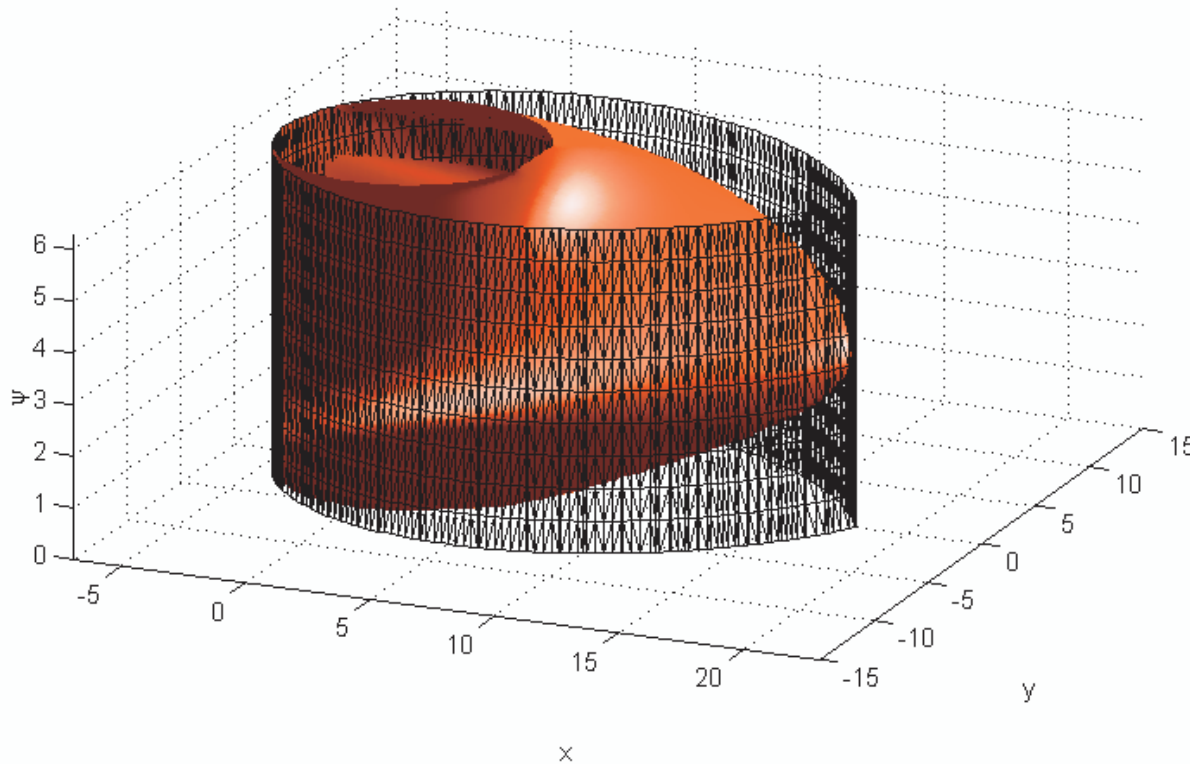
$$D_t \phi_{xz}(x, y, z, t) + \min_y [p_1 f_1(x, y, z) + p_3 f_3(x, y, z)] = 0$$

- Hamiltonian no longer depends on y , so computation can be done entirely in x - z space on $\phi_{xz}(x, z, t)$

Projective Collision Avoidance

- Work strictly in relative x - y plane
 - Treat relative heading $\psi \in [0, 2\pi]$ as a disturbance input
 - Compute time: 40 seconds in 2D vs 20 minutes in 3D
 - Compare overapproximative prism (mesh) to true set (solid)

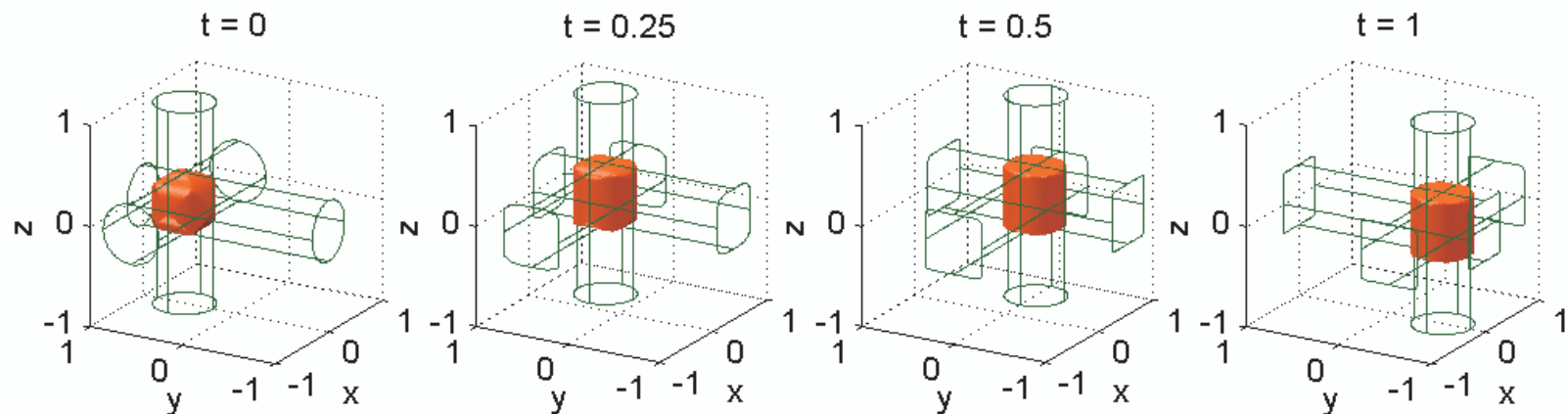
True Reachable Set (solid) vs x-y Projection Reachable Set (mesh)



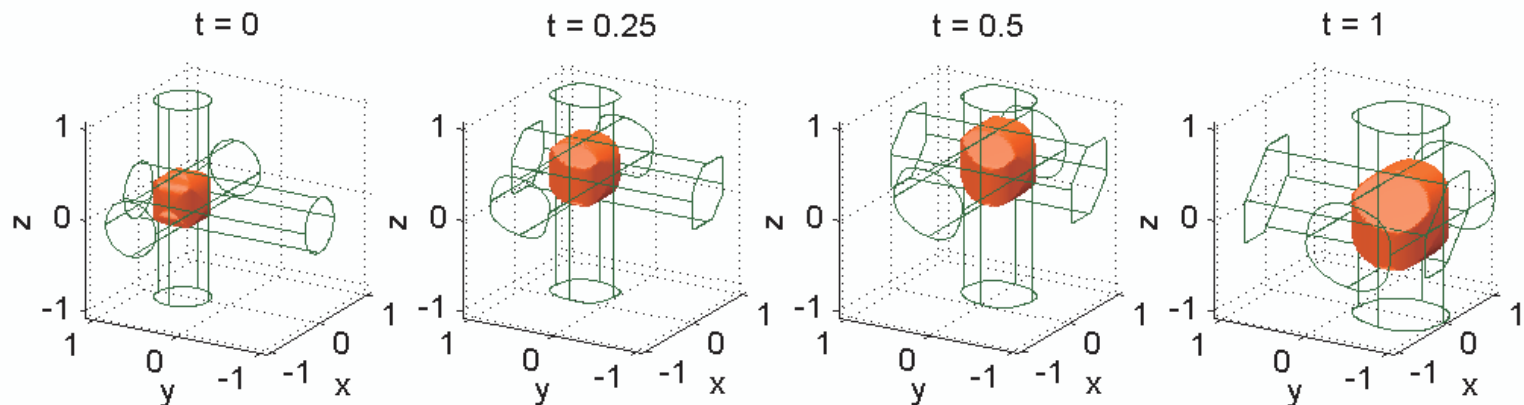
Projection Choices

- Poorly chosen projections may lead to large overapproximations
 - Projections need not be along coordinate axes
 - Number of projections is not constrained by number of dimensions

good projections



poor projections

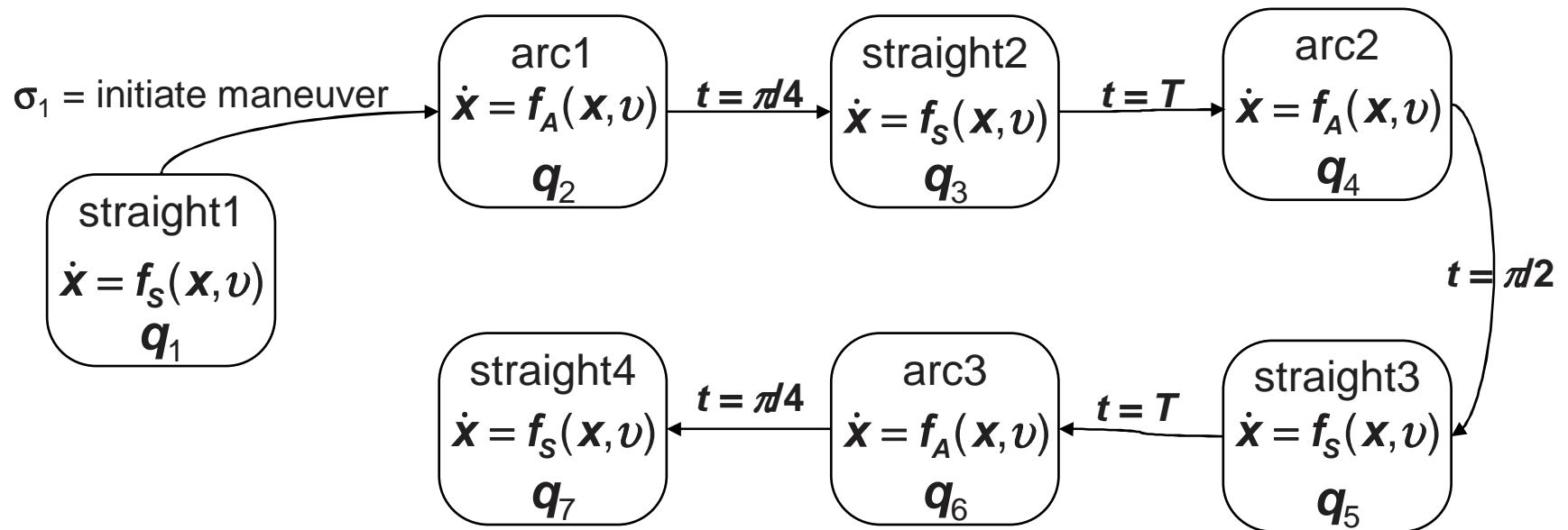
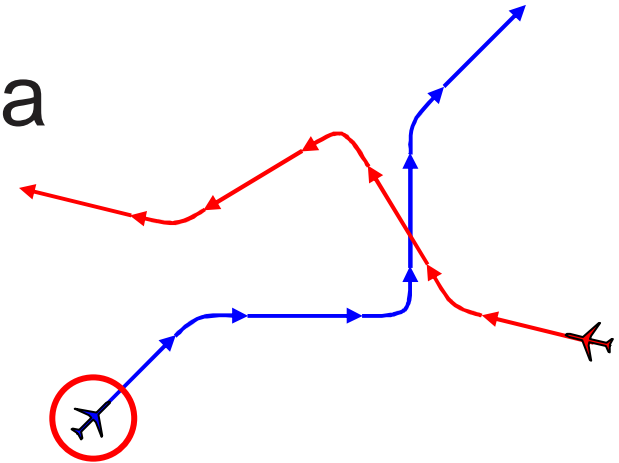


Hybrid System Reach Sets

Combining Continuous and Discrete
Evolution

Hybrid Automata

- Discrete modes and transitions
- Continuous evolution within each mode



$$f_s \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} -v + v \cos \psi \\ v \sin \psi \end{pmatrix}$$

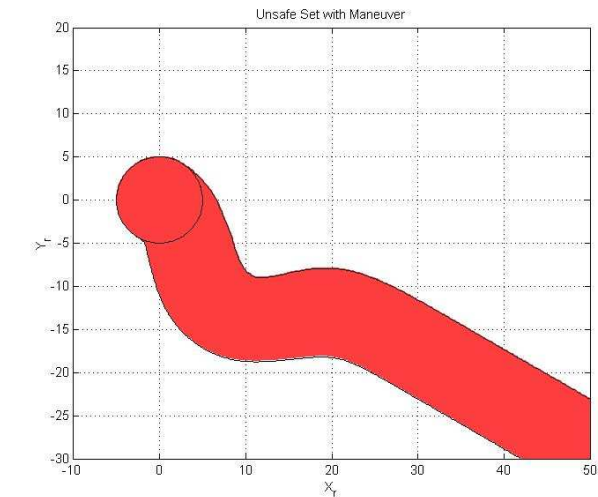
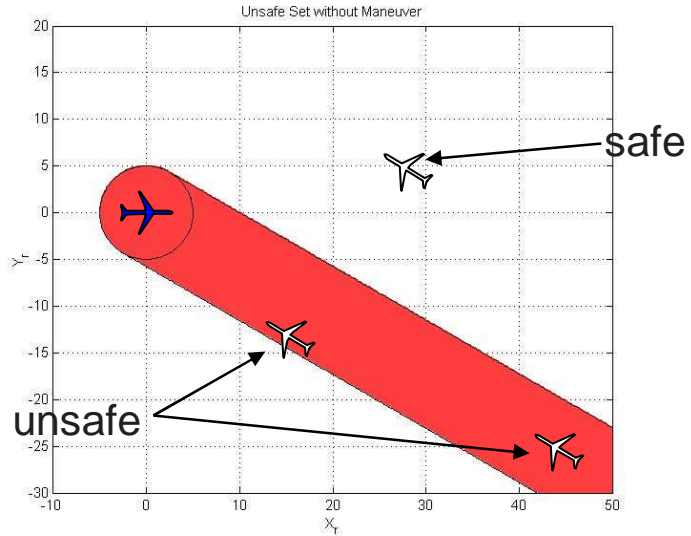
dynamics in straight modes

$$f_A \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} -v + v \cos \psi - \mathbf{x}_2 \\ v \sin \psi + \mathbf{x}_1 \end{pmatrix}$$

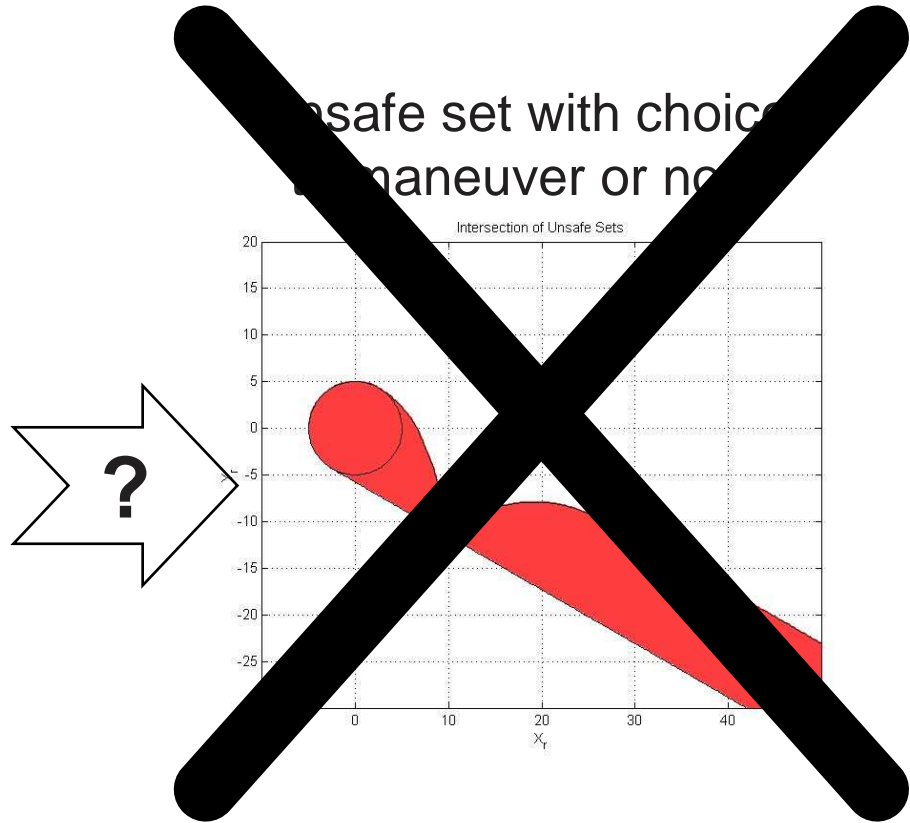
dynamics in arc modes

Seven Mode Safety Analysis

unsafe set without maneuver

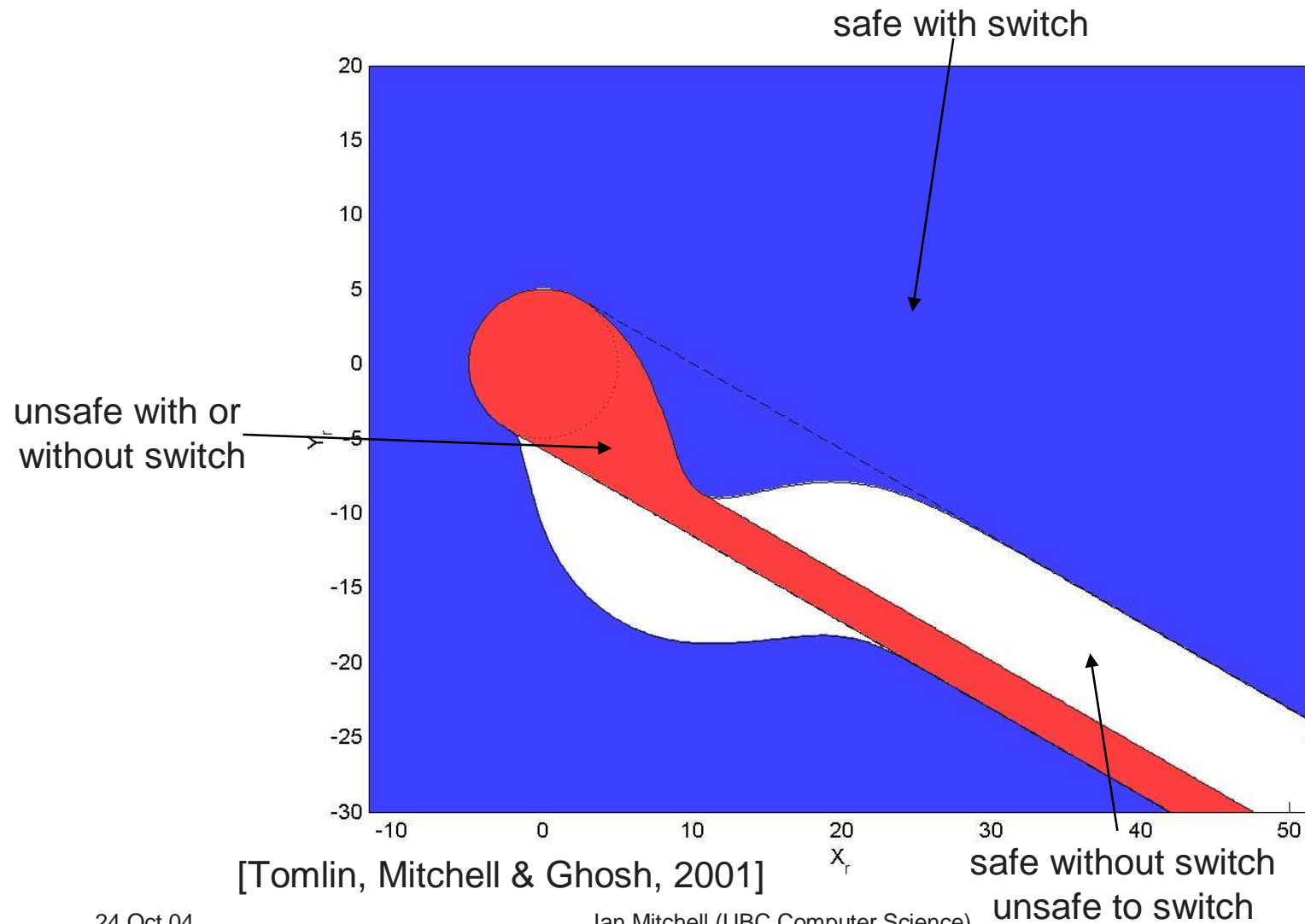


unsafe set with maneuver



Seven Mode Safety Analysis

- Ability to choose maneuver start time further reduces unsafe set

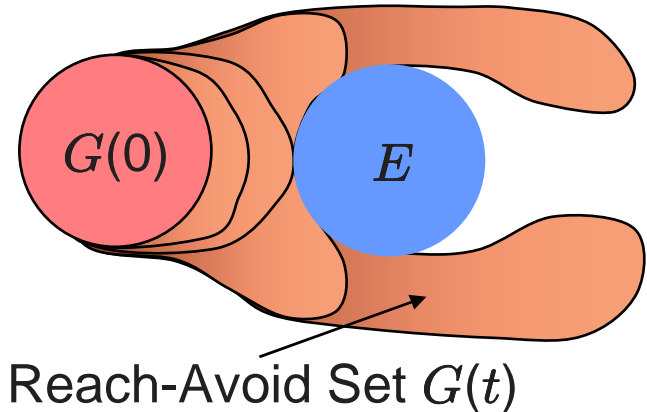


Reach-Avoid Operator

- Compute set of states which reaches $G(0)$ without entering E

$$G(t) = \{x \in \mathbb{R}^n \mid \phi_G(x, t) \leq 0\}$$

$$E = \{x \in \mathbb{R}^n \mid \phi_E(x) \leq 0\}$$



- Formulated as a constrained Hamilton-Jacobi equation or variational inequality
 - [Mitchell & Tomlin, 2000]

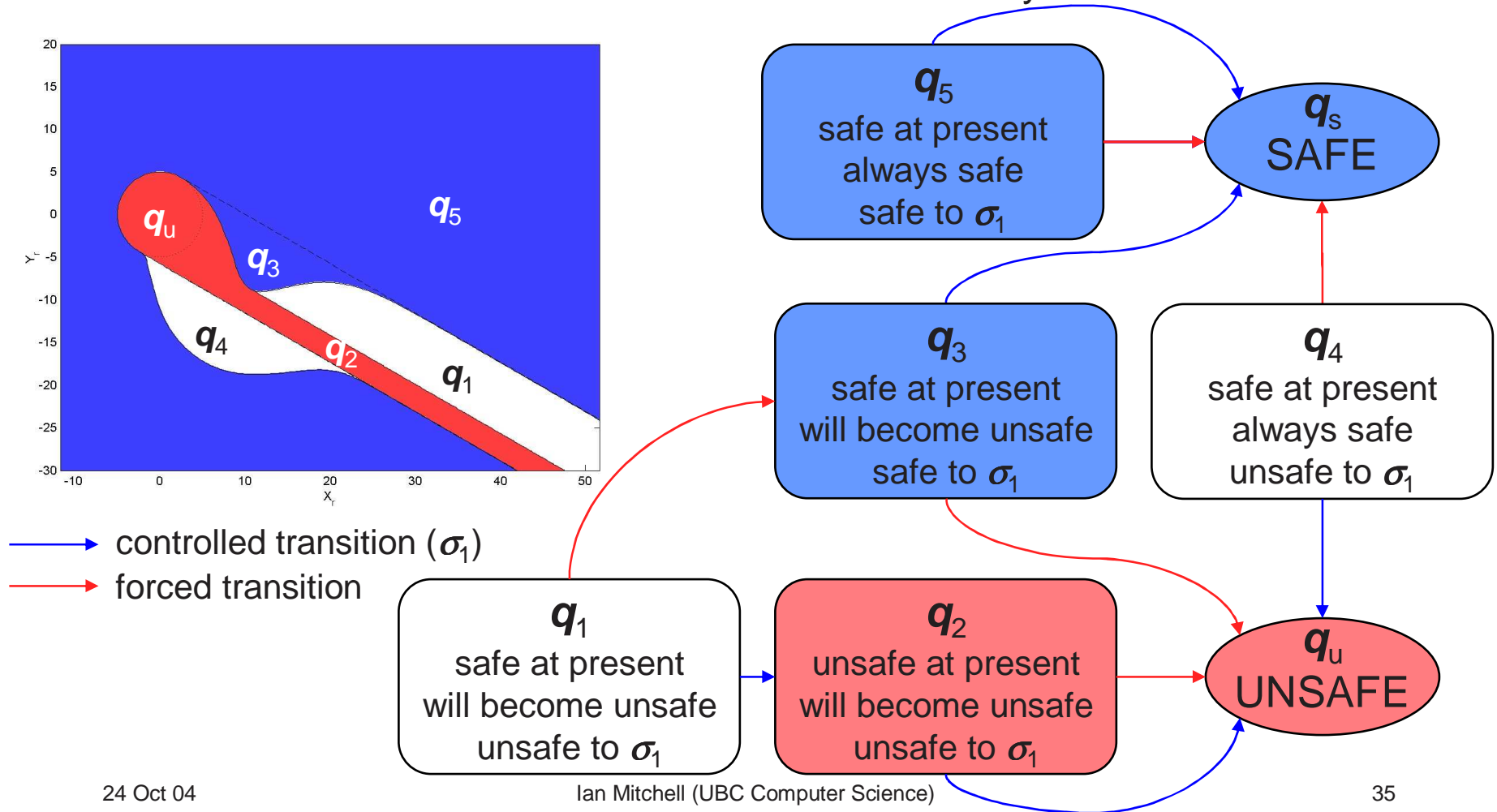
$$D_t \phi_G(x, t) + \min [0, H(x, D_x \phi_G(x, t))] = 0$$

$$\text{subject to: } \phi_G(x, t) \geq \phi_E(x)$$

- Level set can represent often odd shape of reach-avoid sets

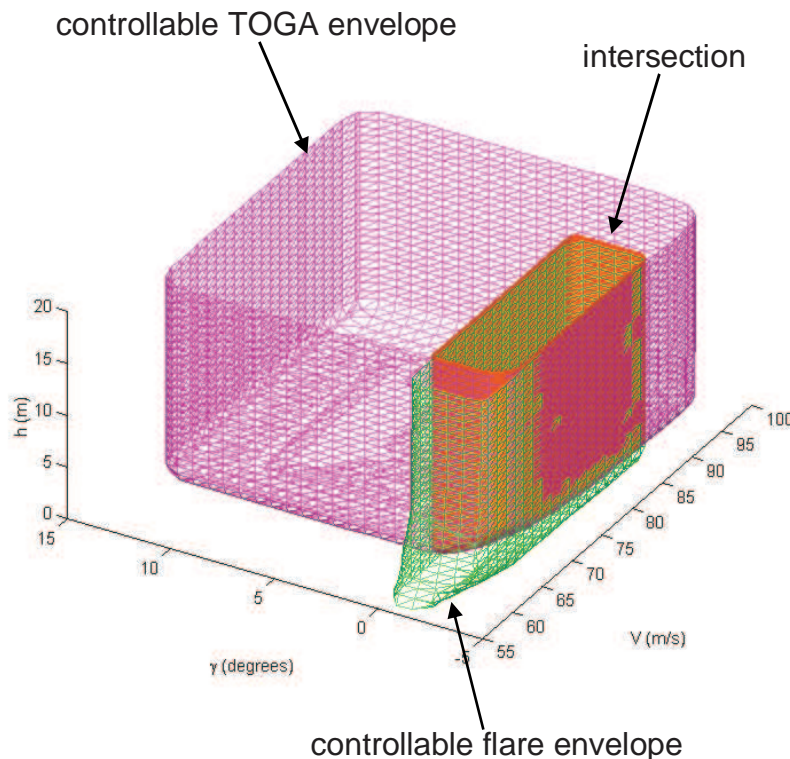
Application: Discrete Abstractions

- It can be easier to analyze discrete automata than hybrid automata or continuous systems
 - Use reachable set information to abstract away continuous details

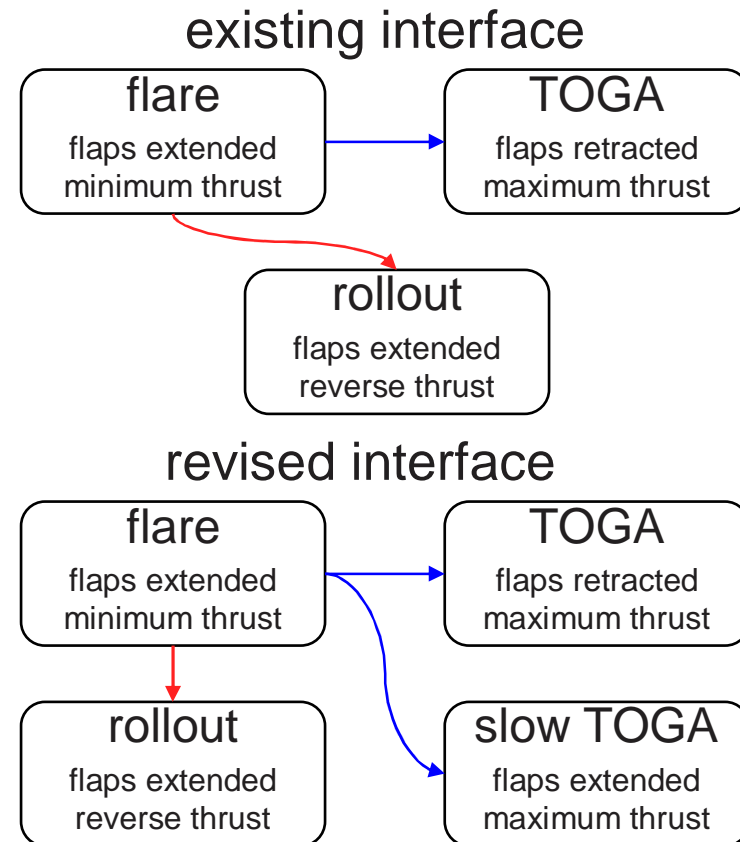


Application: Cockpit Display Analysis

- Controllable flight envelopes for landing and Take Off / Go Around (TOGA) maneuvers may not be the same
- Pilot's cockpit display may not contain sufficient information to distinguish whether TOGA can be initiated



24 Oct 04

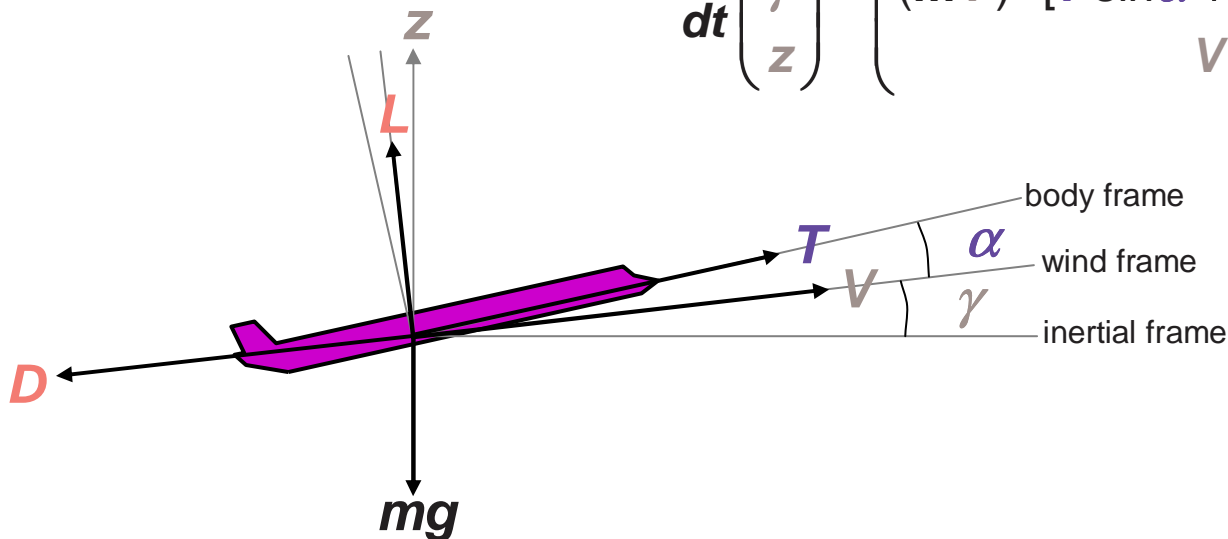


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Application: Aircraft Autolander

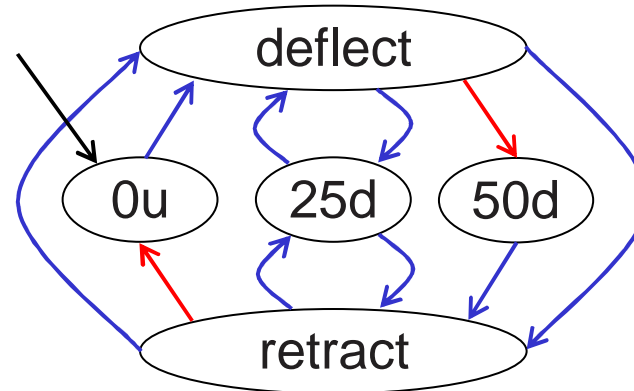
- Airplane must stay within safe flight envelope during landing
 - Bounds on velocity (V), flight path angle (γ), height (z)
 - Control over engine thrust (T), angle of attack (α), flap settings
 - Model flap settings as discrete modes of hybrid automata
 - Terms in continuous dynamics may **depend** on flap setting
 - [Mitchell, Bayen & Tomlin, 2001]

$$\frac{d}{dt} \begin{pmatrix} V \\ \gamma \\ z \end{pmatrix} = \begin{pmatrix} m^{-1}[T \cos \alpha - D(\alpha, V) - mg \sin \gamma] \\ (mV)^{-1}[T \sin \alpha + L(\alpha, V) - mg \cos \gamma] \\ V \sin \gamma \end{pmatrix}$$



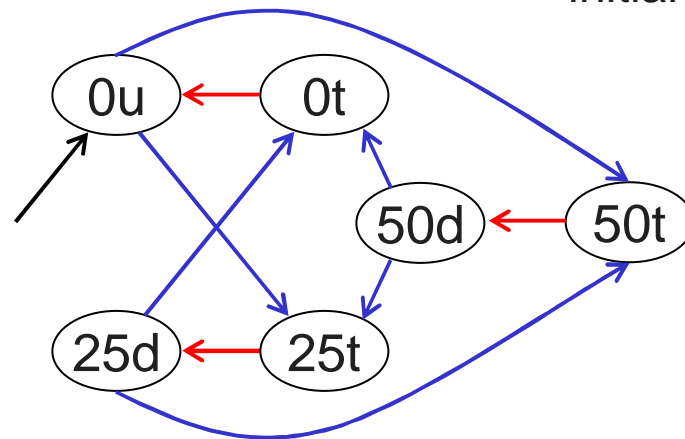
Landing Example: Discrete Model

- Flap dynamics version
 - Pilot can choose one of three flap deflections
 - Thirty seconds for zero to full deflection



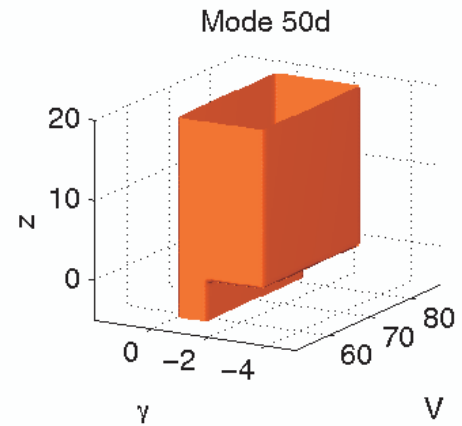
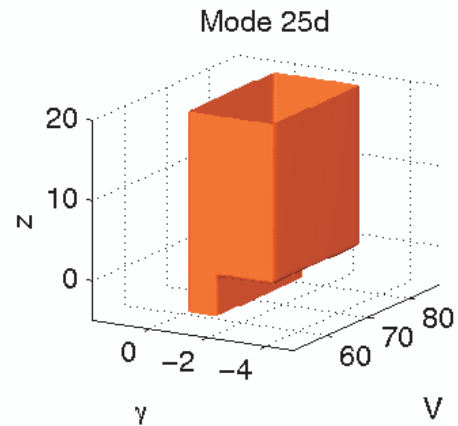
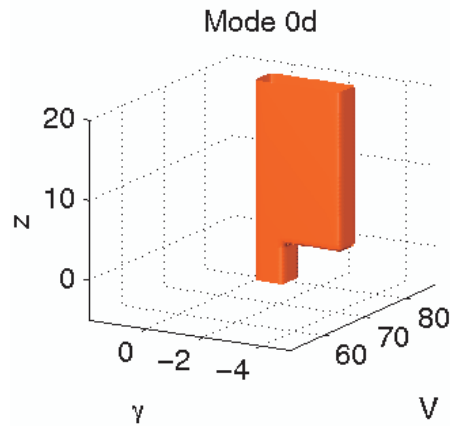
← controlled
← forced
← initial

- Implemented version
 - Instant switches between fixed deflections
 - Additional timed modes to remove Zeno behavior

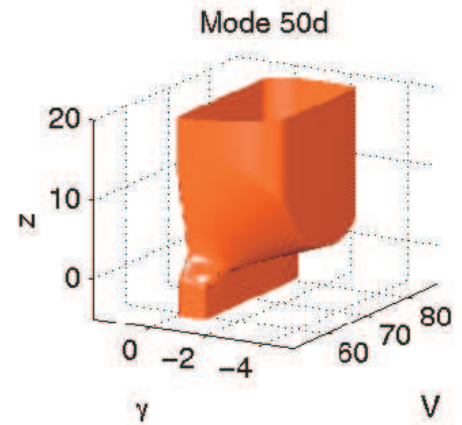
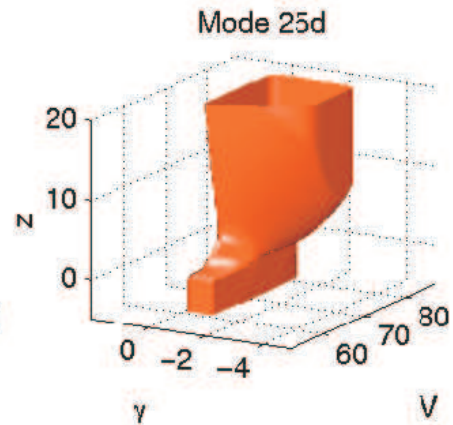
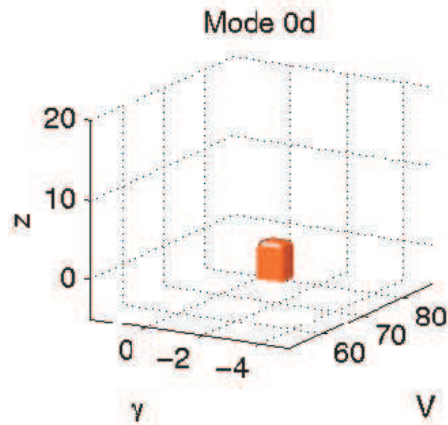


Landing Example: No Mode Switches

Envelopes

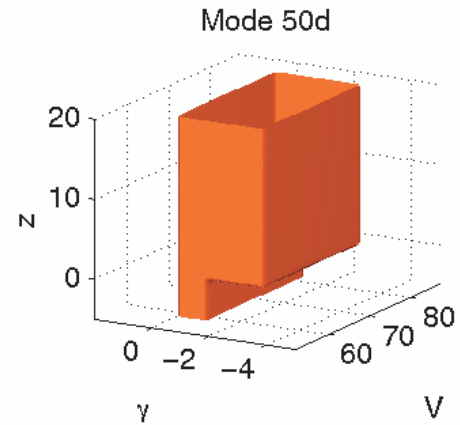
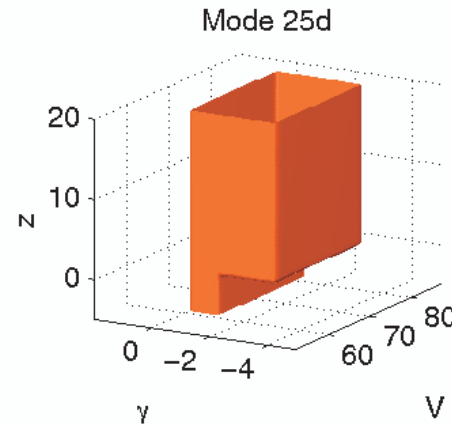
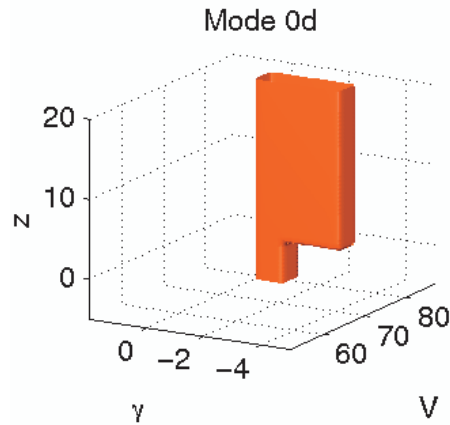


Safe sets

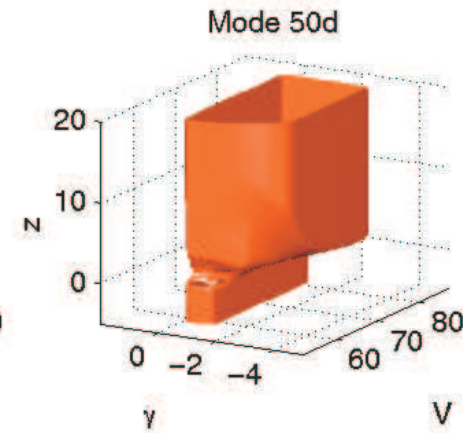
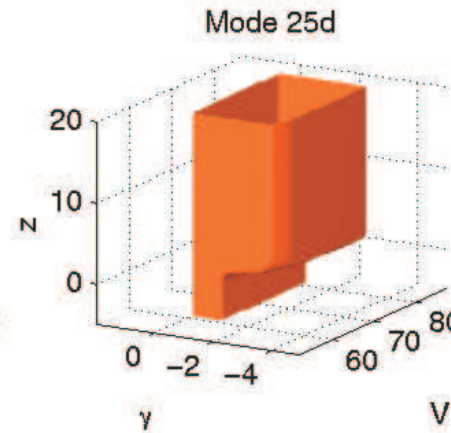
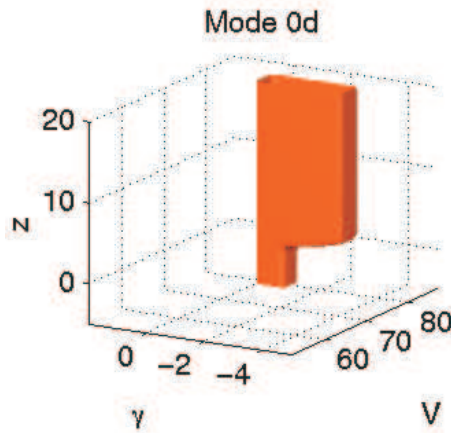


Landing Example: Mode Switches

Envelopes



Safe sets



Landing Example: Synthesizing Control

- For states at the boundary of the safe set, results of reach-avoid computation determine
 - What continuous inputs (if any) maintain safety
 - What discrete jumps (if any) are safe to perform
 - Level set values & gradients provide all relevant data

