LOGIC AND DEPICTION

Logic-based tools have been widely used in artificial intelligence. Many cognitive areas, for example, language understanding, robot planning, commonsense reasoning, and problem solving (qv) have benefited from various uses of logic. However, the perceptual areas, such as computational vision, have not generally been seen as amenable to logic-based approaches. In this article, a theory of depiction is outlined within a framework for image interpretation tasks (Reiter and Mackworth, 1989). The theory has two sets of goals: scientific and engineering. The scientific goals include understanding the concept of an interpretation of an image and understanding the role constraint satisfaction (qv) plays in image interpretation. The engineering goals include the provision of tools for specifying the behavior of image interpretation systems and tools for verifying that a system meets its specification. Potential benefits include the advantages of a common framework for vision and graphics systems and the provision of more modular and portable systems.

The methods proposed are based on a two-domain theory of perception. For any perceptual task at least two domains must be distinguished: the signal domain and the referent domain (or, for deconstructionists, the signer and the signified). For vision the image domain and the scene domain are initially distinguished. All objects are either image objects or scene objects. Given those domains axioms can be written down in, say, first-order logic, constraining the image and scene objects. For a given application there are three classes of general axioms: image axioms \( I \), scene axioms \( S \), and mapping axioms \( M \). Axioms in \( I \) mention only image domain objects and their attributes and relations. Similarly, axioms in \( S \) are confined to describing legitimate scenes. Each axiom in \( M \) mentions objects in both domains; it may use a reserved predicate \( \Delta(i,s) \) signifying that image object \( i \) depicts scene object \( s \). If the theory is to be used for image interpretation axioms that describe the particular image to be interpreted, \( I_0 \) are also required. The theory states that an interpretation of an image corresponds to a logical model of the set of axioms \( I_0 \cup I \cup S \cup M \). This provides a formal task specification for image interpretation. This specification is then refined by model-preserving transformations to a provably correct implementation that computes all or some of the interpretations of the image.

The theory is illustrated with a specification in first-order logic of a simple sketch map interpretation task.

Consider the sketch maps shown in Figure 1. For this task each region must depict a land area or water area and each chain of line segments must depict a road, a river, or a shore. Roads and rivers appear only on land; shores separate land and water. Rivers must flow into other rivers or shores. Given that background knowledge the image in Figure 1a depicts one of three possible scenes. Either regions \( r_1 \) and \( r_2 \) both depict land while chain \( c_1 \) depicts a road; \( r_1 \) depicts land (an island), \( r_2 \) depicts water, and \( c_1 \) depicts a shore; or finally, \( r_1 \) depicts water (a lake), \( r_2 \) depicts land, and \( c_1 \) depicts a shore. For this application \( I \) consists of taxonomy axioms (eg, “each image object is a chain or a region”). \( I_0 \) consists of a description of the image in terms of primitive predicates (“chain \( c_1 \) bounds region \( r_1 \)”) and closure axioms (eg, “\( c_1 \) is the only chain”). \( S \) consists of taxonomy axioms (“each linear-scene-object is a road, a river, or a shore”), and general scene knowledge (“the inside area of a shoreline is land if and only if its outside is water” and “rivers lead to other rivers or shores”). The mapping knowledge \( M \) includes axioms such as “each image object \( i \) depicts a unique scene object \( \sigma(i) \), “depiction holds only between image and scene objects,” “a chain depicts a linear-scene-object,” and the like. Given that specification it is possible to refine it to an equivalent formula in propositional logic by eliminating the quantifiers over finite domains and various other database-oriented transformations. To find all the visual interpretations it is necessary only to find all the logical models of that formula using standard SAT or CSP techniques (see CONSTRAINT SATISFACTION)

For the map domain these models all share in common fixed extensions of all the image, scene, and mapping predicates except ROADC(), RIVERC(), SHOREC(), LANDC() and WATERC(). For the example in Figure 1a the three models correspond to the descriptions:

\[
\text{LANDC}(\sigma(r_1)) \land \text{LANDC}(\sigma(r_2)) \land \text{ROADC}(\sigma(c_1))
\]

\[
\text{WATERC}(\sigma(r_1)) \land \text{WATERC}(\sigma(r_2)) \land \text{SHOREC}(\sigma(c_1))
\]

\[
\text{LANDC}(\sigma(r_1)) \land \text{WATERC}(\sigma(r_2)) \land \text{SHOREC}(\sigma(c_1))
\]

For the map shown in Figure 1b there are four possible interpretations corresponding to:

\[
\text{LANDC}(\sigma(r_1)) \land \text{LANDC}(\sigma(r_2)) \land \text{ROADC}(\sigma(c_1)) \land \text{ROADC}(\sigma(c_2)) \land \text{ROADC}(\sigma(c_3))
\]

Figure 1. Two simple maps.
LOGIC, CONDITIONAL

Conditional logic examines the proof theory and semantics for ordinary conditionals in natural language. Contemporary work in this area is motivated by the so-called paradoxes of material implication and by the apparent non-truth-functionality of many ordinary conditionals. A standard formal language for representing the logical structure of conditionals has been developed, and several conditional logics have gained widespread attention. Both possible worlds and probabilistic semantics have been proposed as alternatives to the classic truth functional account of conditionals. Within the artificial intelligence community there have been several efforts to develop nonmonotonic reasoning systems based on conditional logic (see REASONING, NONMONOTONIC).

PROBLEMS WITH MATERIAL IMPLICATION

The typical conditional has the structure “If A, then C” where A is called the antecedent and C the consequent of the conditional. The classic treatment of conditionals translates ordinary language conditionals into material conditionals. A material conditional, represented $A \supset C$, is a compound expression of which the truth value is a function of the truth values of its antecedent and consequent as defined by Table 1. $A \supset C$ is true whenever A is false or C is true, and this is the source of the so-called paradoxes of implication. Where A is the false sentence “Shakespeare didn’t write Hamlet” and C is the sentence “Someone other than Shakespeare wrote Hamlet,” both the material conditional $A \supset C$ and the corresponding English conditional

1. If Shakespeare didn’t write Hamlet, then someone else wrote Hamlet

are true. But if the mood of sentence 1 is changed from indicative to subjunctive, the resulting English conditional

2. If Shakespeare had not written Hamlet, then someone else would have written Hamlet.

is at least improbable. Perhaps indicative conditionals can be represented as material conditionals, but most conditionals in the subjunctive mood cannot. The problem is not that the material conditional is the wrong truth function for representing English subjunctive conditionals; these conditionals cannot be represented by any truth function. Consider the following four conditionals:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>C</th>
<th>$A \supset C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>t</td>
<td>f</td>
<td>t</td>
</tr>
<tr>
<td>2</td>
<td>f</td>
<td>t</td>
<td>t</td>
</tr>
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<td>f</td>
<td>f</td>
<td>t</td>
</tr>
<tr>
<td>4</td>
<td>f</td>
<td>t</td>
<td>t</td>
</tr>
</tbody>
</table>

BIBLIOGRAPHY


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