Heuristic Search: A*

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UBC CS 322 – Search 4

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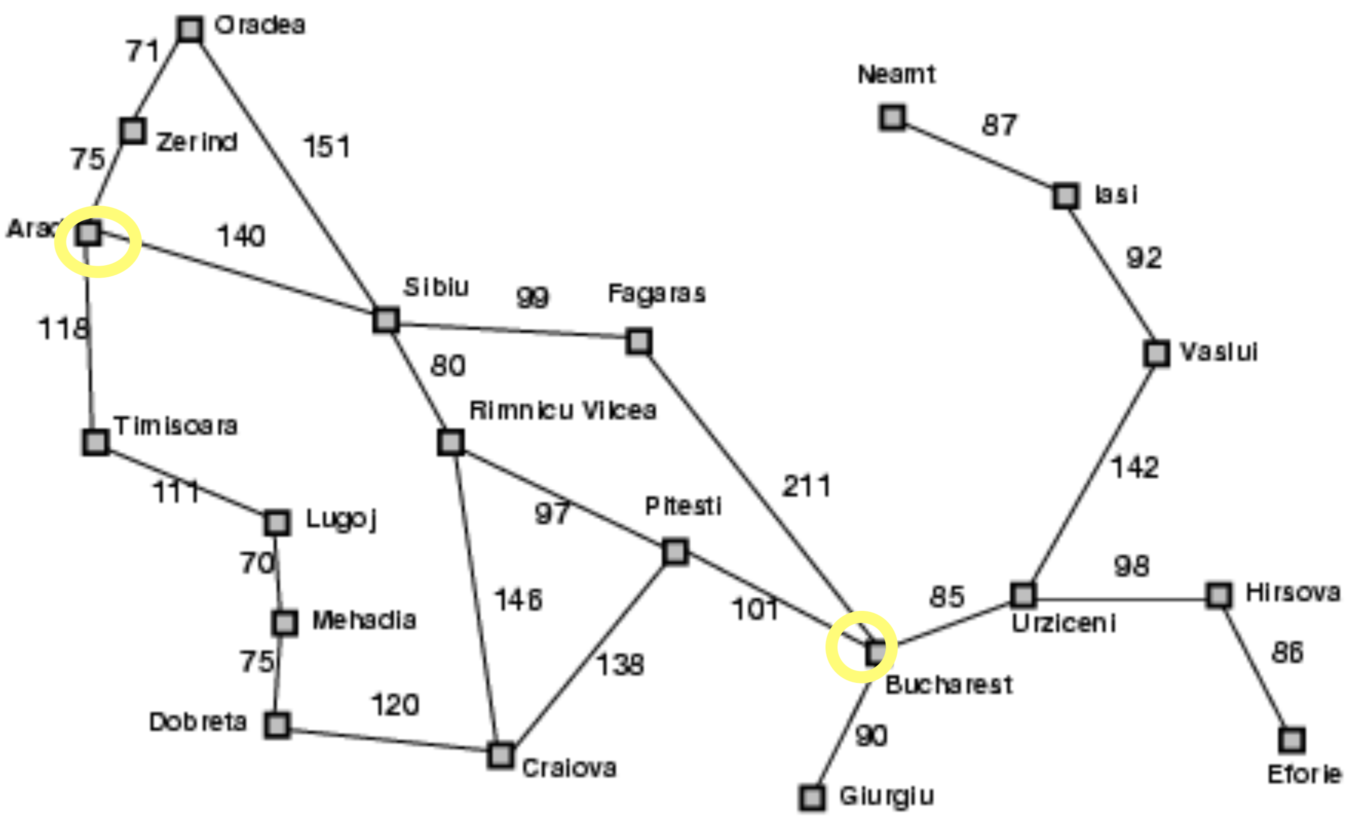
Textbook §3.6

Lecture Overview

- Recap
- Search heuristics: admissibility and examples
- Recap of BestFS
- Heuristic search: A*

Example for search with costs: finding routes



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
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Timisoara	329
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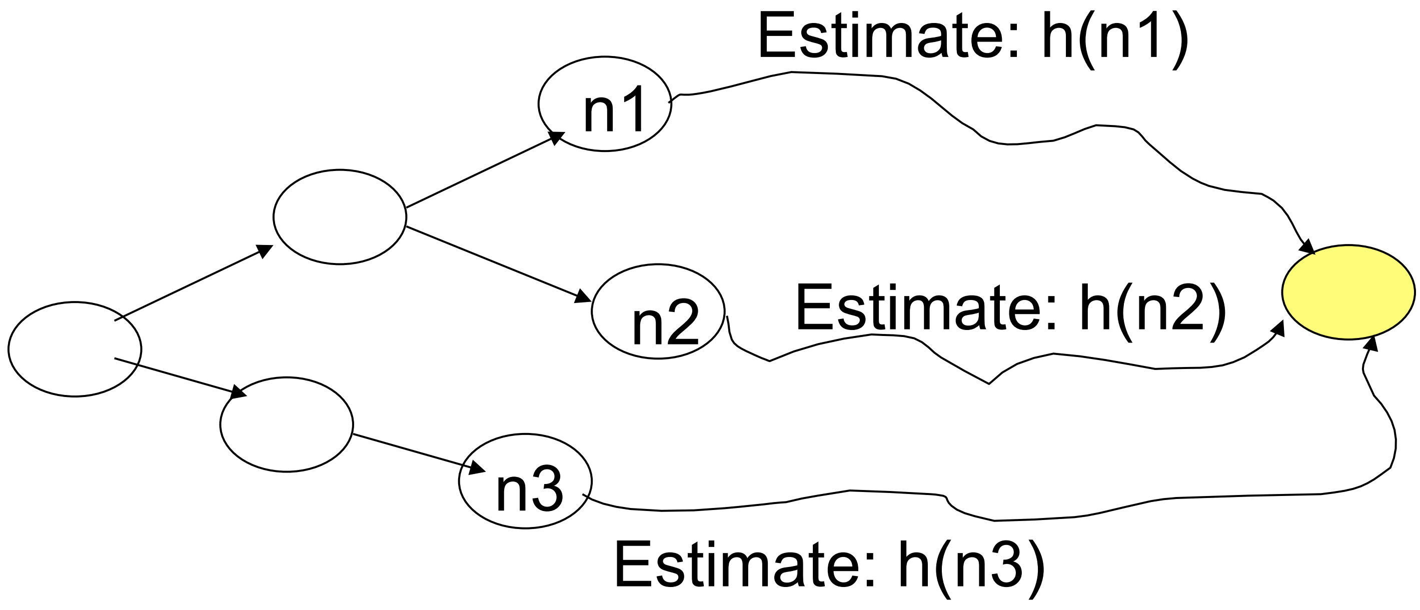
Lowest-Cost First Search (LCFS)

- Expand the path with the lowest cost
  - Generalization of Breadth-First Search
  - Implemented as priority queue of cost values
- Only complete for strictly positive arc costs
  - Otherwise: a cycle with zero cost <= 0 could be followed forever
- Only optimal for non-negative arc costs
  - Otherwise: a path that initially looks high-cost could end up getting a `refund’
- Time and space complexity: $\tilde{O}(b^m)$
  - E.g., uniform arc costs: identical to Breadth-First Search

Search heuristics

Def.:

A search heuristic *h(n)* is an estimate of the cost of the optimal (cheapest) path from node *n* to a goal node.



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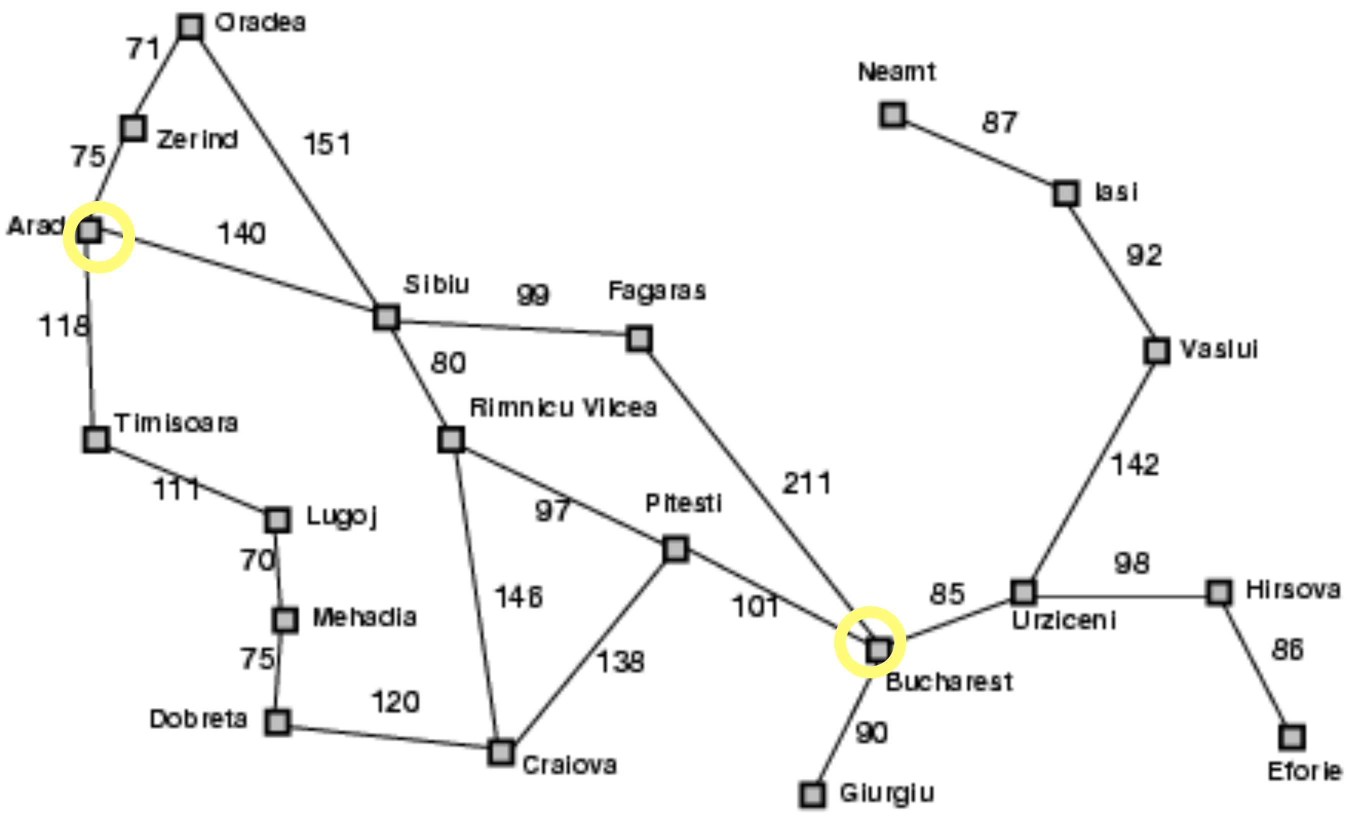
Last lecture's example: finding routes

- What could we use as h(n)? E.g., the straight-line distance between source and goal node



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Admissibility of a heuristic

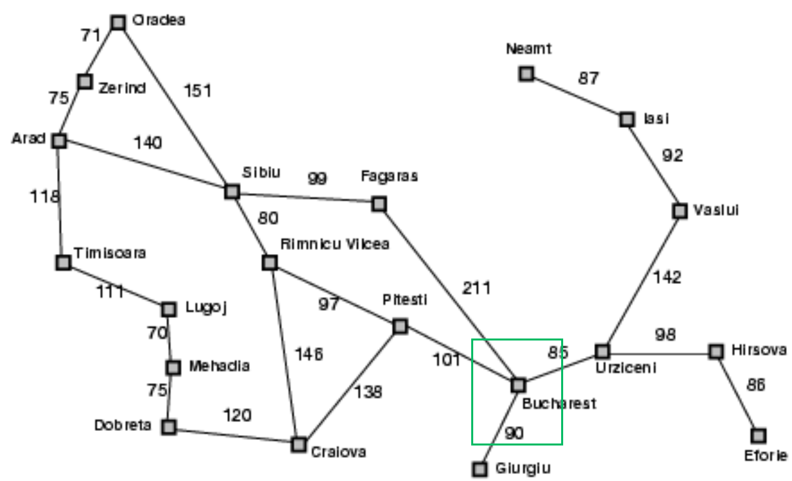
Def.:

Let c(n) denote the cost of the optimal path from node n to any goal node. A search heuristic h(n) is called admissible if h(n) ≤ c(n) for all nodes n, i.e. if for all nodes it is an underestimate of the cost to any goal.

- Example: is the straight-line distance admissible?

  YES **NO**

  - Yes! The shortest distance between two points is a line.



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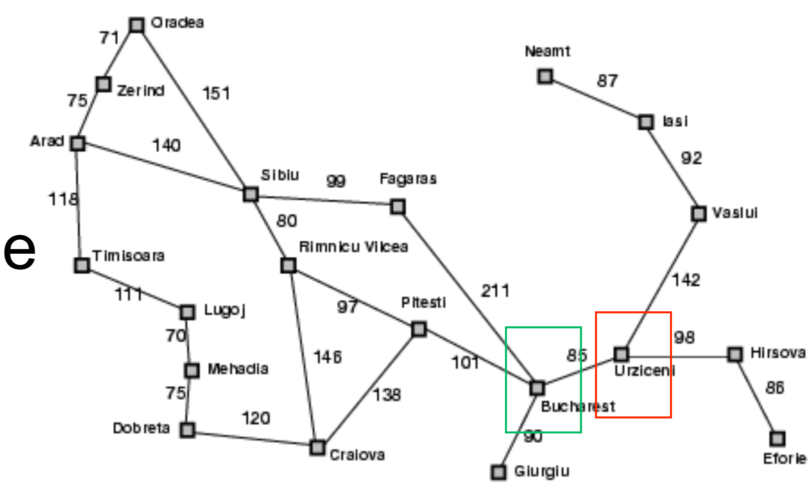
Another example:
the goal is Urzizeni (red box), but all we know is the straight-line distances to Bucharest (green box)



Straight−line distance to Bucharest

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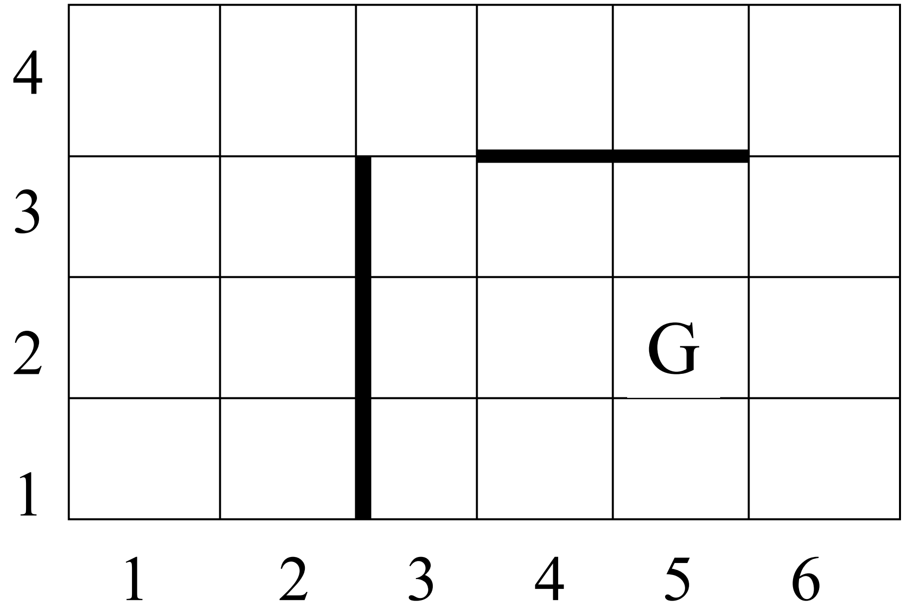
- Possible h(n) = sld(n, Bucharest) + cost(Bucharest, Urzineni)
- Admissible? **YES** **NO**

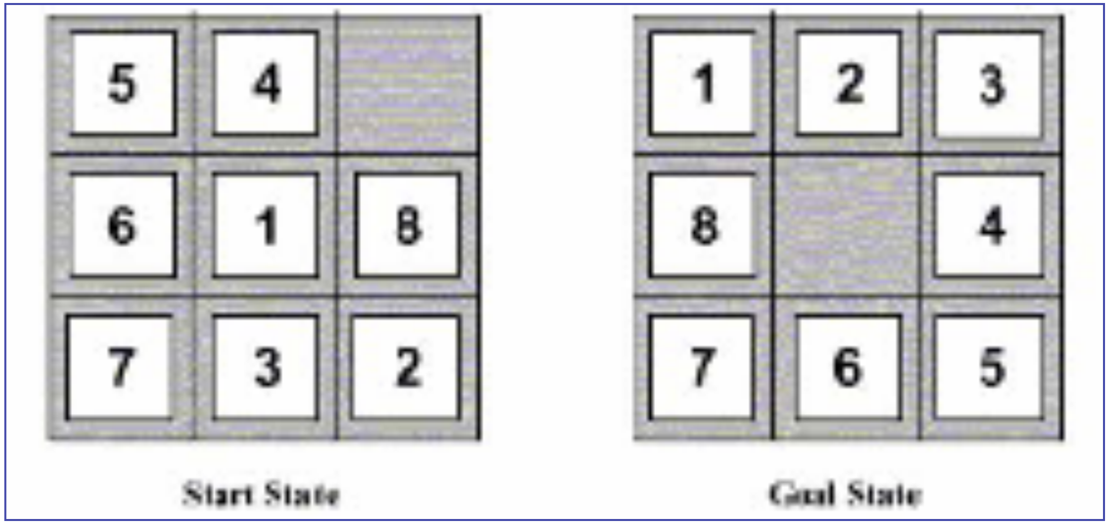
Example 2: grid world

- Search problem: robot has to find a route from start to goal location G on a grid with obstacles
- Actions: move up, down, left, right from tile to tile
- Cost : number of moves
- Possible h(n)?
  - Manhattan distance (L_1 distance) to the goal G: sum of the (absolute) difference of their coordinates
  - Admissible? **YES** **NO**



Example 3: Eight Puzzle

- One possible h(n):
  Number of Misplaced Tiles

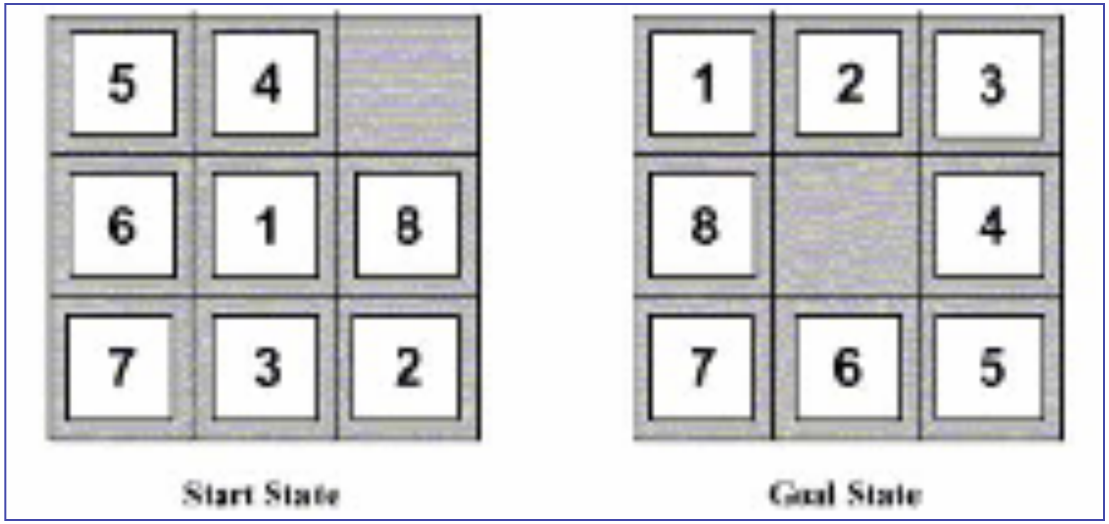


- Is this heuristic admissible?

YES **NO**

Example 3: Eight Puzzle

- Another possible h(n):
  Sum of number of moves between each tile's current position and its goal position



- Is this heuristic admissible?

YES **NO**

How to Construct an Admissible Heuristic

- Identify relaxed version of the problem:
  - where one or more constraints have been dropped
  - problem with fewer restrictions on the actions
- Grid world: the agent can move through walls
- Driver: the agent can move straight
- 8 puzzle:
  - "number of misplaced tiles":
    tiles can move everywhere and occupy same spot as others
  - "sum of moves between current and goal position":
    tiles can occupy same spot as others
- Why does this lead to an admissible heuristic?
  - The problem only gets easier!

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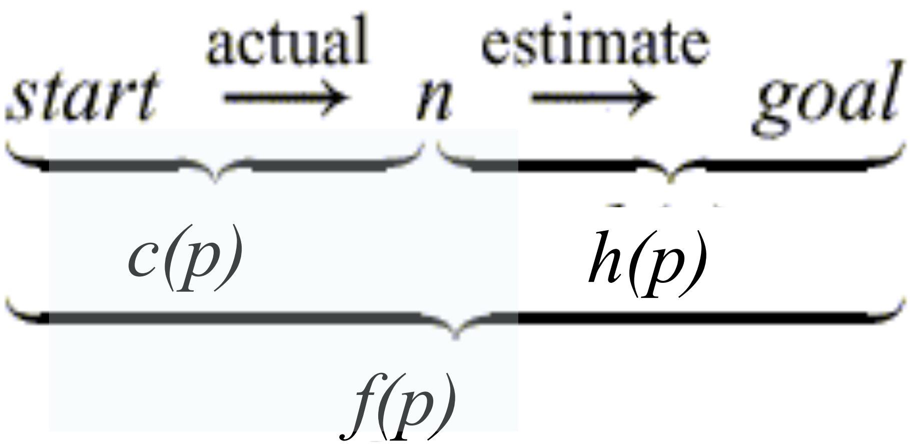
A* Search

- A* search takes into account both
  - the cost of the path to a node c(p)
  - the heuristic value of that path h(p).

- Let f(p) = c(p) + h(p).
  - f(p) is an estimate of the cost of a path from the start to a goal via p.



A* Search Algorithm

- A* combines elements of which two search algorithms?

Breadth-first | Depth-first | Best-first | Least cost first

- It treats the frontier as a priority queue ordered by f(n)

- It always chooses the path on the frontier with the lowest estimated distance from the start to a goal node constrained to go via that path.

- Let's see it in action:

A* in Infinite Mario Bros

http://www.youtube.com/watch?v=0s3d1LfjWCI

http://www.youtube.com/watch?v=DlkMs4ZHHr8

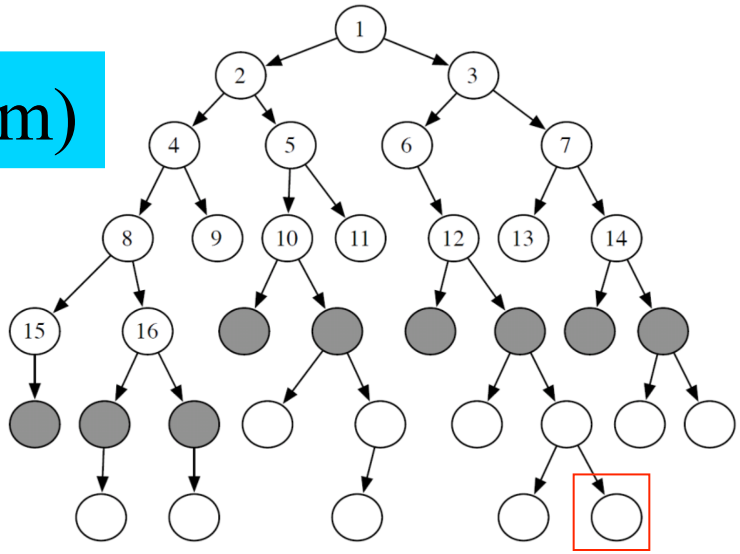
Analysis of A*

Def.: The time complexity of a search algorithm is
the worst-case amount of time it will take to run, expressed in terms of
- maximum path length *m*
- maximum forward branching factor *b*.

- What is time complexity of A* in terms of m and b ?

$\tilde{O}(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

- E.g., uniform costs and constant heuristic h(n) = 0
  - Behaves exactly like BFS



A* completeness and optimality

- A* is complete (finds a solution, if one exists) and optimal (finds the optimal path to a goal) if:
  - the branching factor is finite
  - arc costs are $> \varepsilon > 0$
  - h(n) is admissible -> an underestimate of the length of the shortest path from n to a goal node.

- This property of A* is called admissibility of A*

Learning Goals for today's class

- Construct heuristic functions for specific search problems
  Define/read/write/trace/debug different search algorithms
  - With/without cost
  - Informed/Uninformed
- Formally prove A* optimality (continued next class)