

# Uninformed Search Strategies

Alan Mackworth

UBC CS 322 - Search 2

January 11, 2013

Textbook §3.5

# Today's Lecture

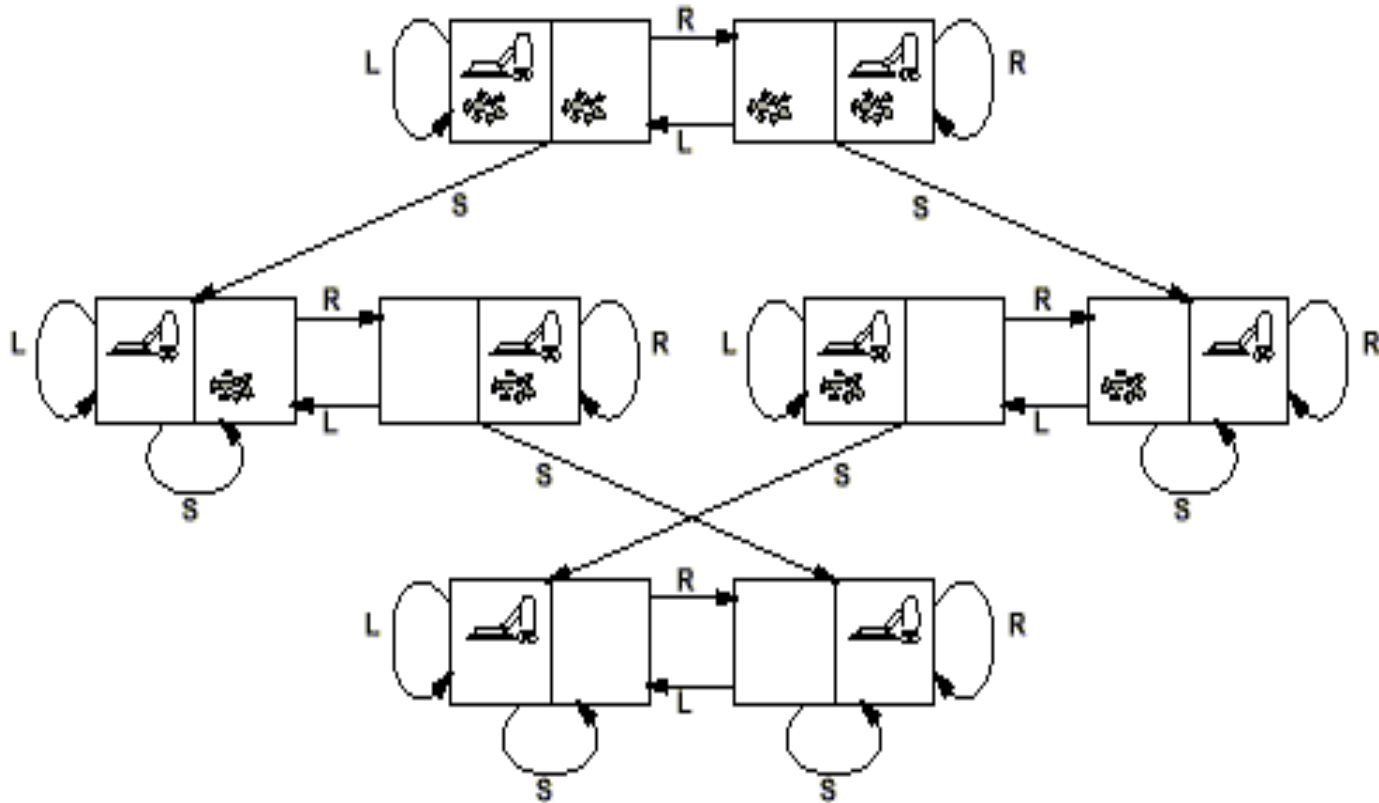
## Lecture 4 (2-Search1) Recap

- Uninformed search + criteria to compare search algorithms
  - Depth first
  - Breadth first

# Recap

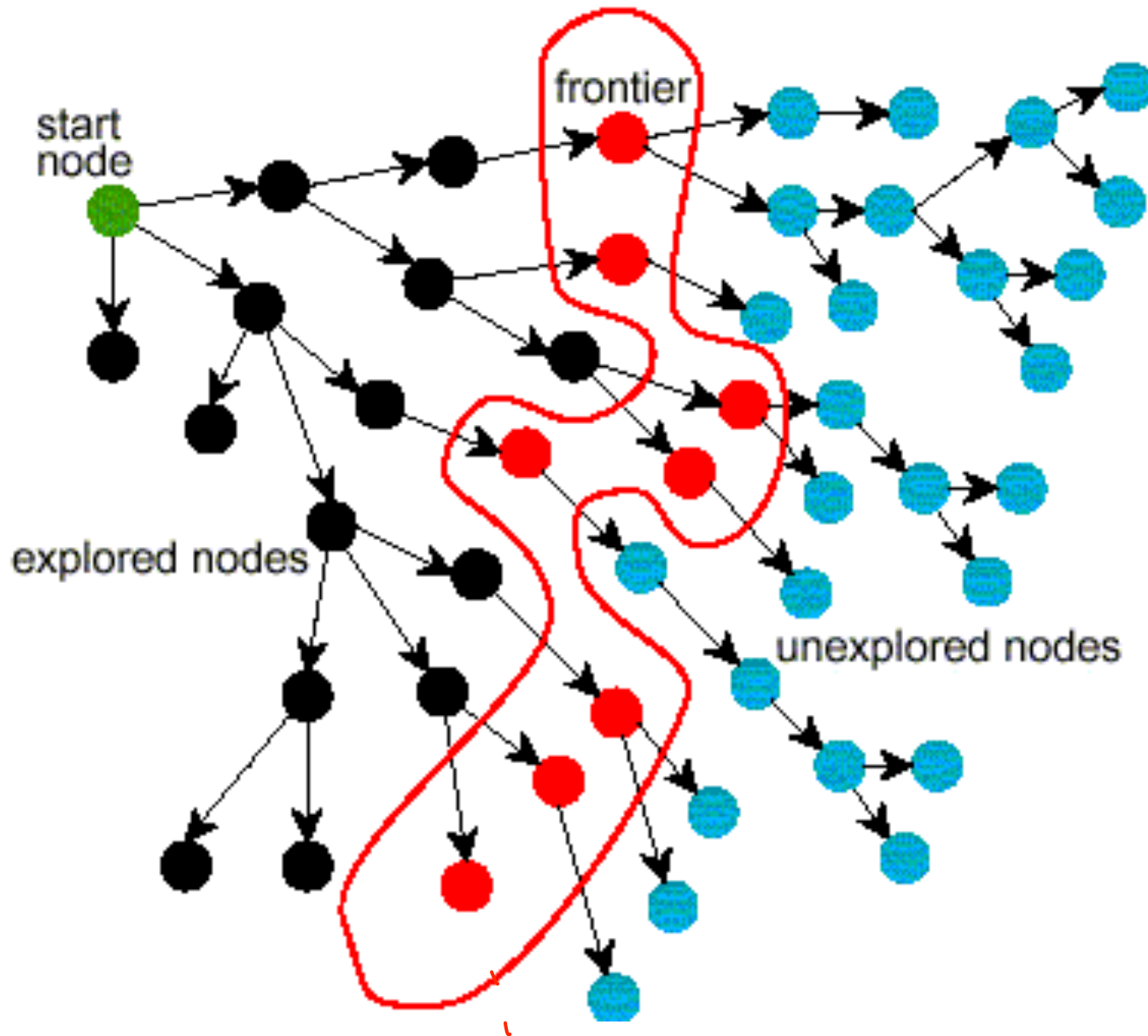
- Search is a key computational mechanism in many AI agents
- We will study the basic principles of search on the simple **deterministic goal-driven search agent model**
- Generic search approach:
  - Define a search space graph
  - Initialize the **frontier** with an empty path
  - incrementally expand frontier until goal state is reached
- Frontier:
  - The set of paths which could be explored next
- The way in which the frontier is expanded defines the search strategy

# Search Space: example



- **Operators** – *left, right, suck*
  - Successor states in the graph describe the effect of each action applied to a given state
- **Possible Goal** – no dirt

# Problem Solving by Graph Searching



# Bogus version of Generic Search Algorithm

**Input:** a graph

a set of start nodes

Boolean procedure  $\text{goal}(n)$  that tests if  $n$  is a goal node

$\text{frontier} := [\langle g \rangle : g \text{ is a goal node}]$ ;

**While** frontier is not empty:

**select** and **remove** path  $\langle n_0, \dots, n_k \rangle$  from frontier;

**If**  $\text{goal}(n_k)$

**return**  $\langle n_0, \dots, n_k \rangle$ ;

**Find** a neighbor  $n$  of  $n_k$

**add**  $\langle n \rangle$  to frontier;

**end**

- There are several bugs in this version here:  
help me find them!

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**Find** a neighbor  $n$  of  $n_k$

**add**  $\langle n \rangle$  to frontier;

**end**

- Start at the **start** node(s)
- Add **all** neighbours of  $n_k$  to the frontier
- Add **path(s)** to frontier, NOT just the node(s)

# Generic Search Algorithm

**Input:** a graph

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**Then return**  $\langle n_0, \dots, n_k \rangle$ ;

**Else**

**For every** neighbor  $n$  of  $n_k$ ,

**add**  $\langle n_0, \dots, n_k, n \rangle$  to **frontier**;

**end**



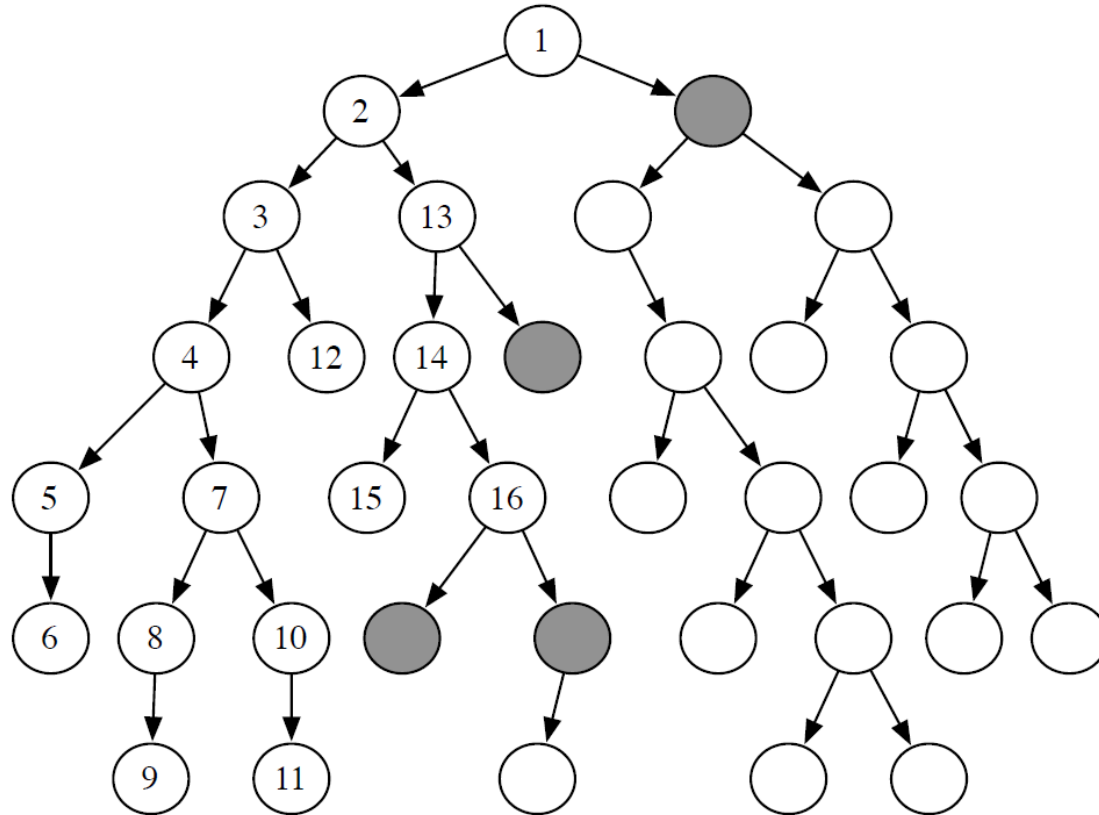
# Today's Lecture

- Lecture 4 Recap

 Uninformed search + criteria to compare search algorithms

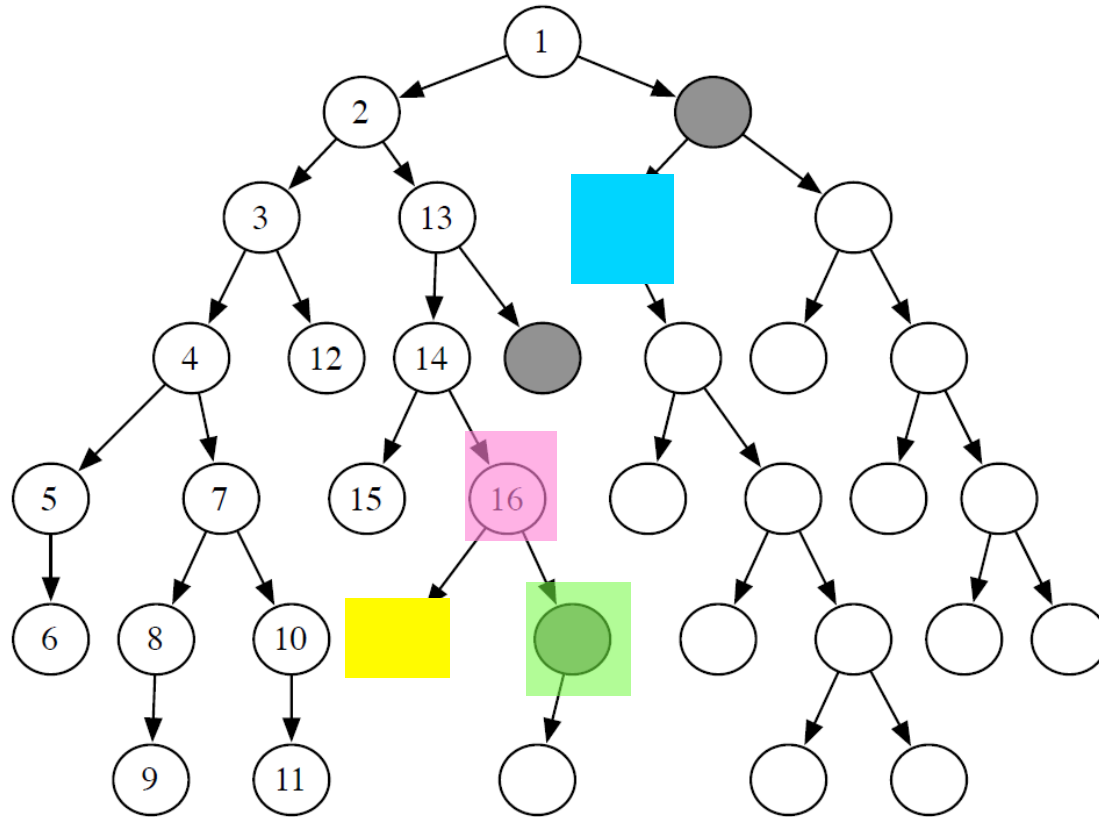
- Depth-first
- Breadth-first

# Depth-first search (DFS)



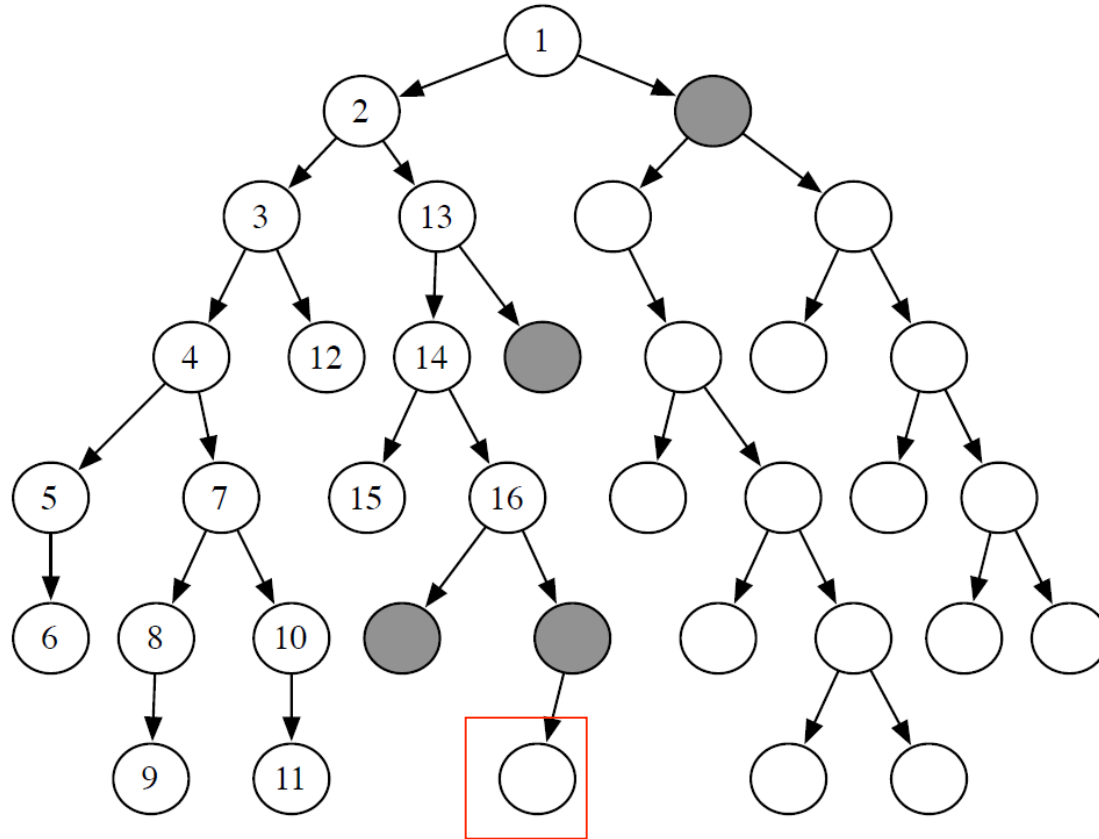
- Frontier: shaded nodes

# Depth-first search (DFS)



- Frontier: shaded nodes
- Which node will be expanded next?  
(expand = “remove path ending at node from frontier & put its successors on”)

# Depth-first search (DFS)



- Say, node in red box is a goal
- How many more nodes will be expanded?

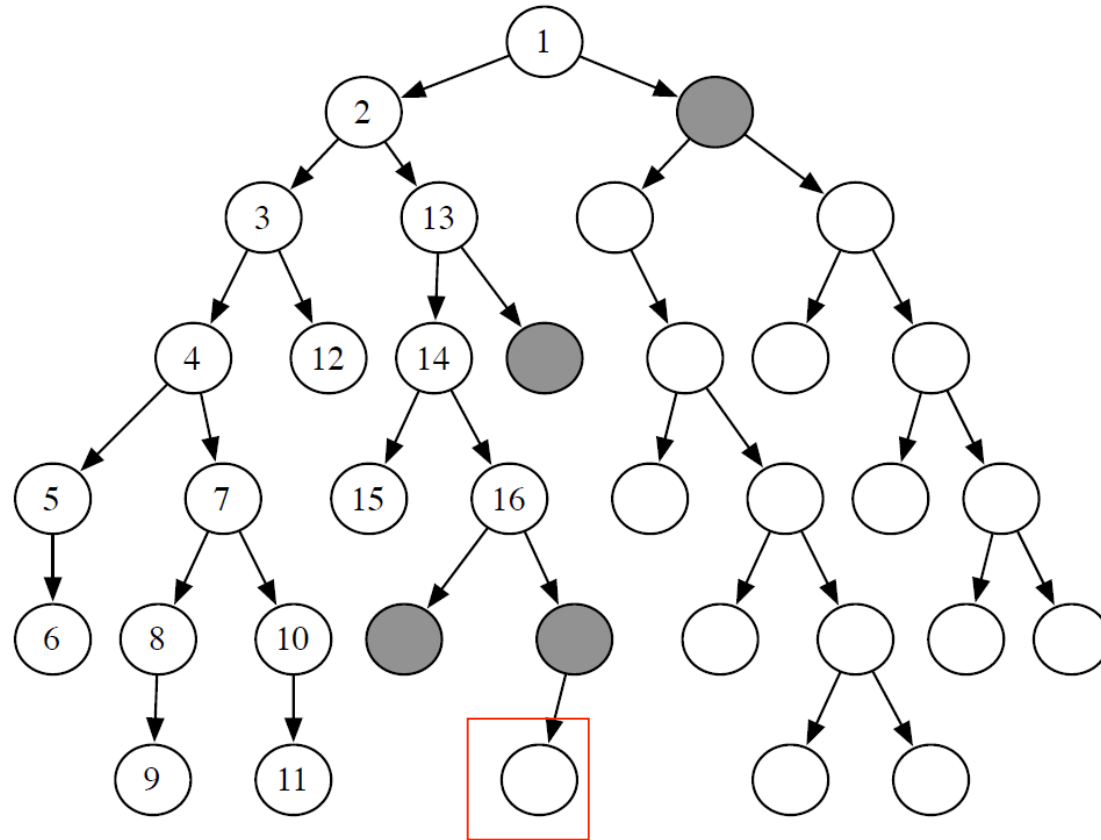
1

2

3

4

# Depth-first search (DFS)



- Say, node in red box is a goal
- How many more nodes will be expanded?
  - 3: you only return once the goal is being expanded!
  - Not when a goal is put onto the frontier!

# DFS as an instantiation of the Generic Search Algorithm

Input: a graph

a set of start nodes

Boolean procedure `goal(n)`  
testing if  $n$  is a goal node

`frontier` := [`s`:  $s$  is a start node];

While `frontier` is not empty:

    select and remove path  $\langle n_0, \dots, n_k \rangle$  from `frontier`;

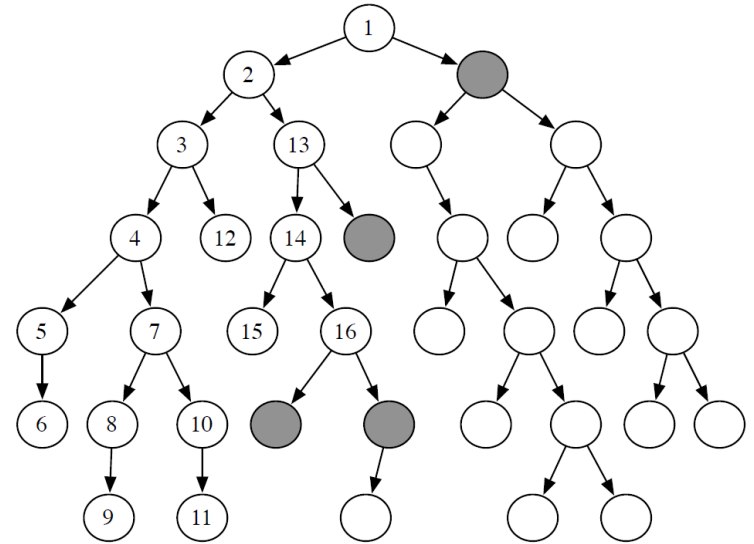
    If `goal(nk)`

        Then return  $\langle n_0, \dots, n_k \rangle$ ;

    Else

        For every neighbor  $n$  of  $n_k$ ,  
            add  $\langle n_0, \dots, n_k, n \rangle$  to `frontier`;

end



# DFS as an instantiation of the Generic Search Algorithm

Input: a graph

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    If  $goal(n_k)$

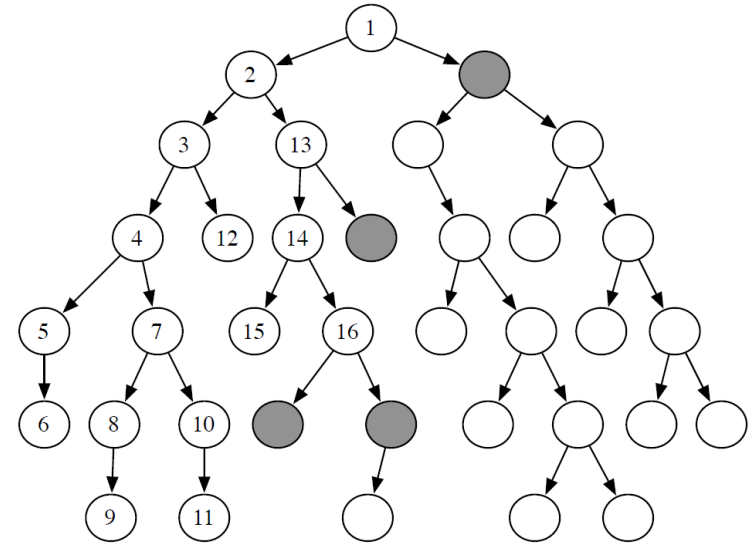
        Then return  $\langle n_0, \dots, n_k \rangle$ ;

    Else

        For every neighbor  $n$  of  $n_k$ ,

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end



In DFS, the frontier is a  
**last-in-first-out stack**

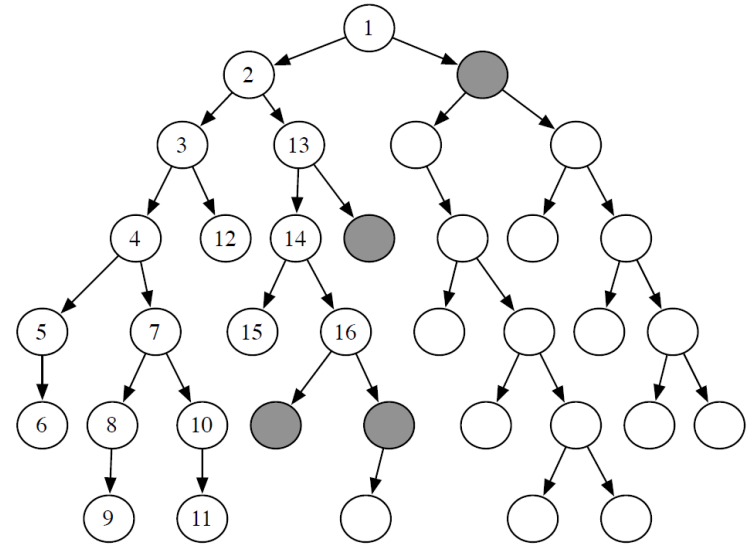
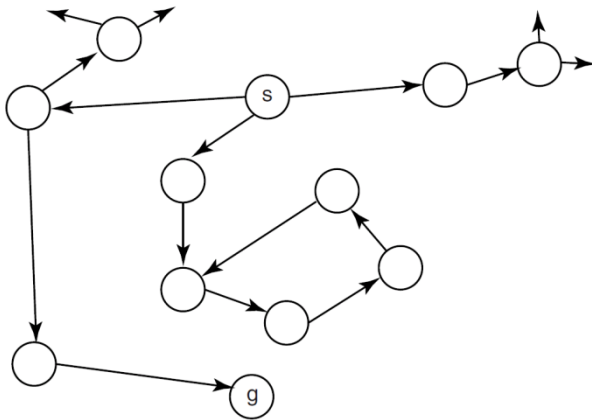
# Analysis of DFS

Def. : A search algorithm is **complete** if whenever there is at least one solution, the algorithm **is guaranteed to find it** within a finite amount of time.

Is DFS **complete**?

Yes

No





# Analysis of DFS

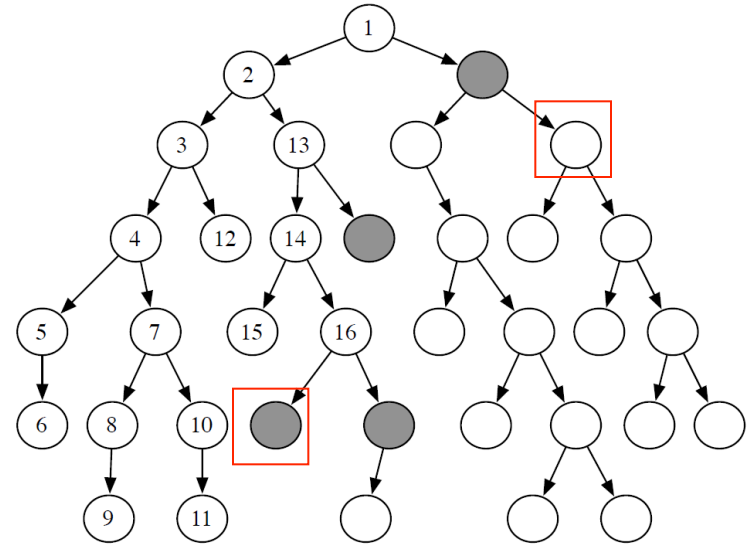
Def.: A search algorithm is **optimal** if when it finds a solution, it is **the best one**

Is DFS **optimal**?

Yes

No

- E.g., goal nodes: red boxes





# Analysis of DFS

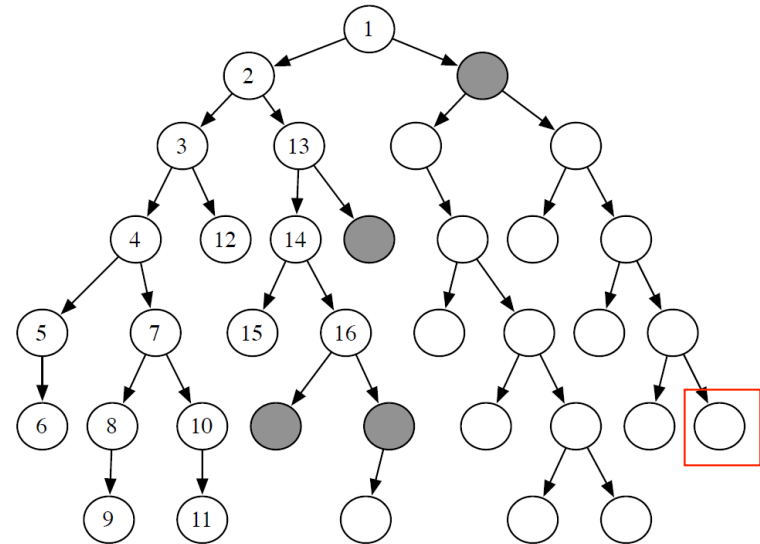
Def.: The **time complexity** of a search algorithm is the **worst-case** amount of time it will take to run, expressed in terms of

- maximum path length  $m$
- maximum forward branching factor  $b$ .

- What is DFS's **time complexity**, in terms of  $m$  and  $b$  ?

$$O(b^m)$$

- E.g., single goal node: red box



# Analysis of DFS

Def.: The **space complexity** of a search algorithm is the **worst-case** amount of memory that the algorithm will use (i.e., the maximal number of nodes on the frontier), expressed in terms of

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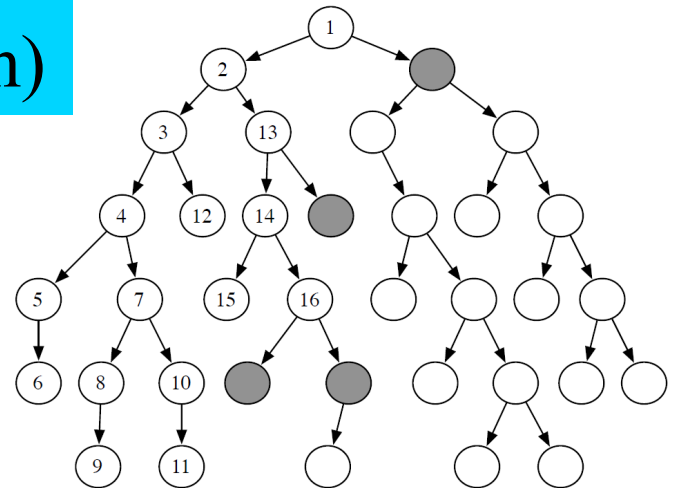
- What is DFS's **space complexity**, in terms of  $m$  and  $b$  ?

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$O(bm)$

$O(b+m)$



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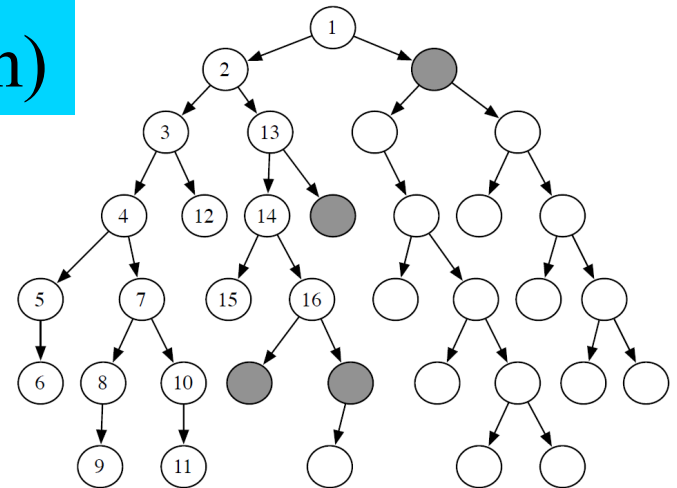
$O(b^m)$

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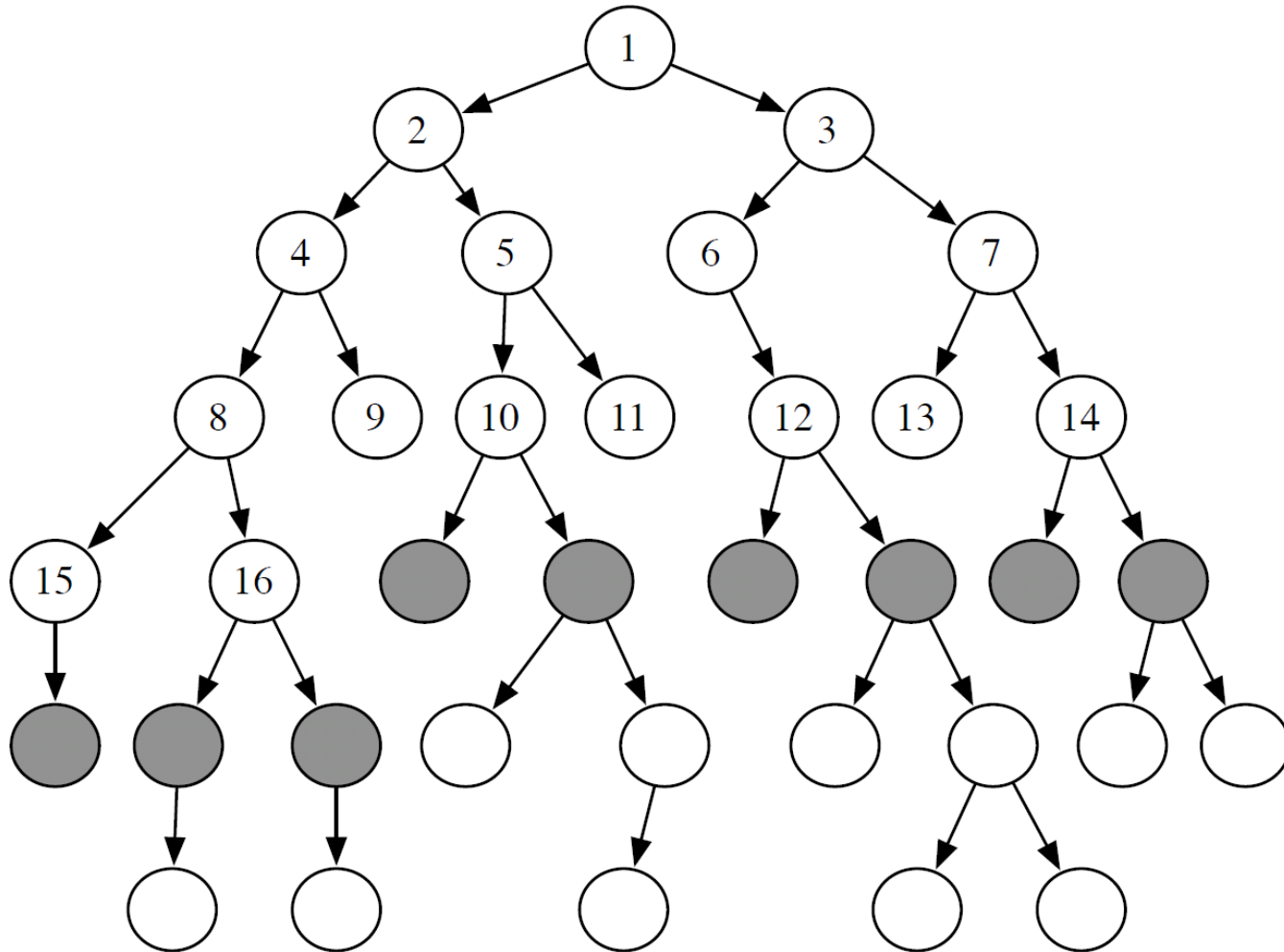
- $O(bm)$
- The longest possible path is  $m$ , and for every node in that path must maintain a fringe of size  $b$



# Today's Lecture

- Lecture 4 Recap
- Uninformed search + criteria to compare search algorithms
  - Depth first
  - ➔ Breadth first

# Breadth-first search (BFS)



# BFS as an instantiation of the Generic Search Algorithm

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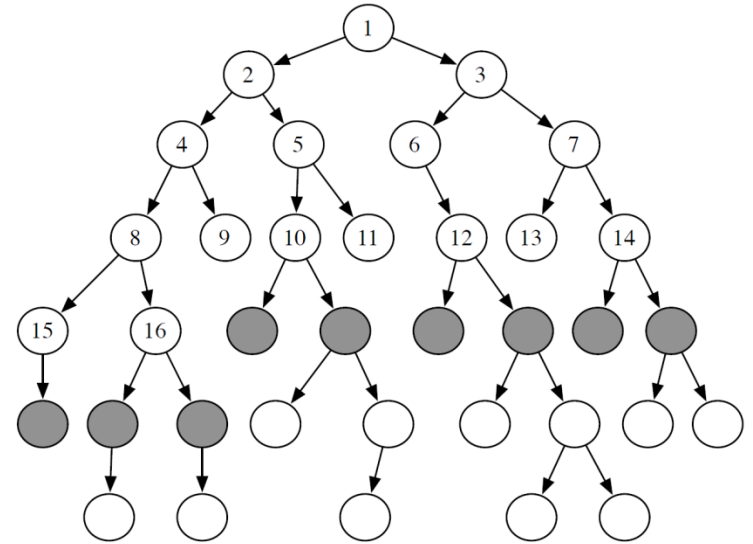
**Then return**  $\langle n_0, \dots, n_k \rangle$ ;

**Else**

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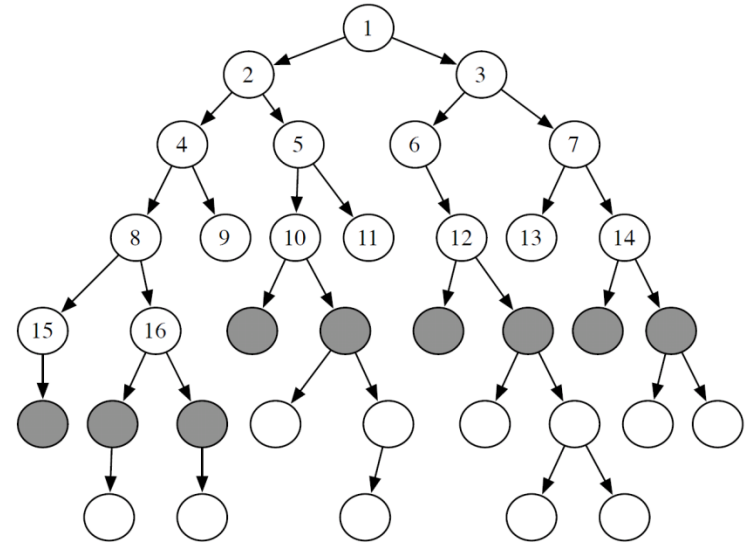
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In BFS, the frontier is a **first-in-first-out queue**

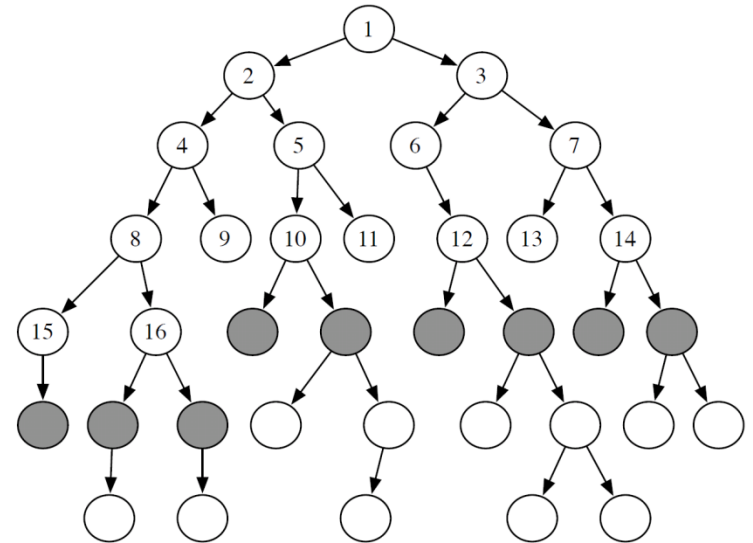
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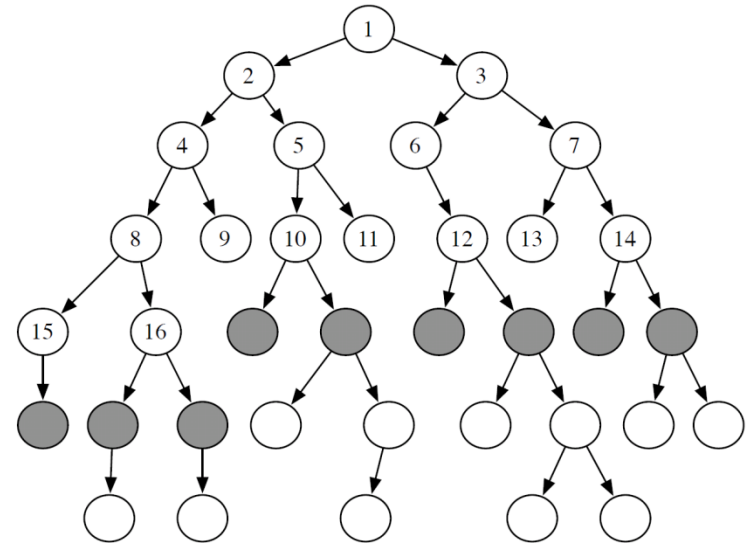
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- Proof sketch?

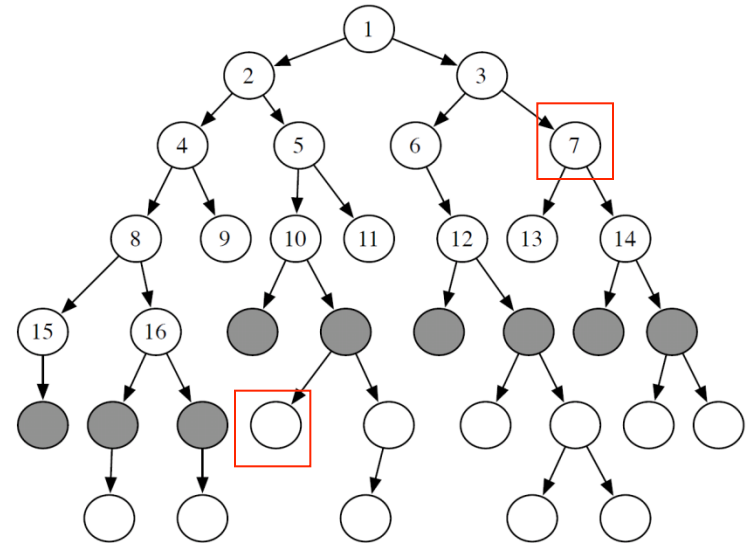
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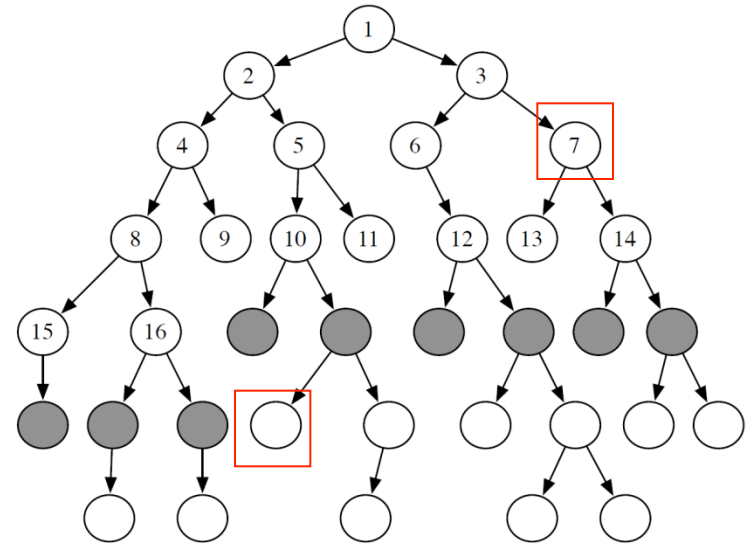
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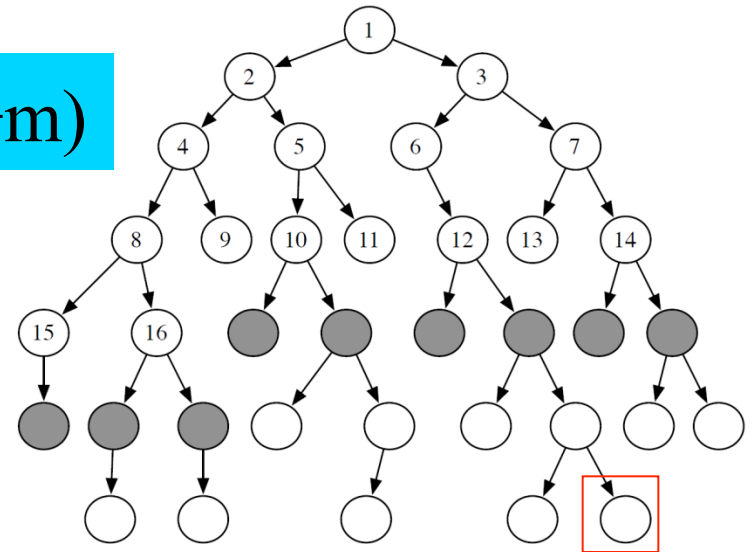
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- E.g., single goal node: red box

# Analysis of BFS

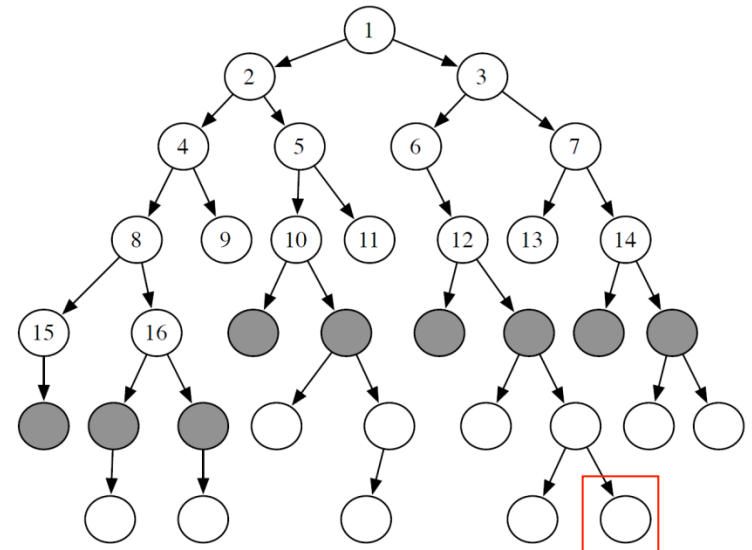
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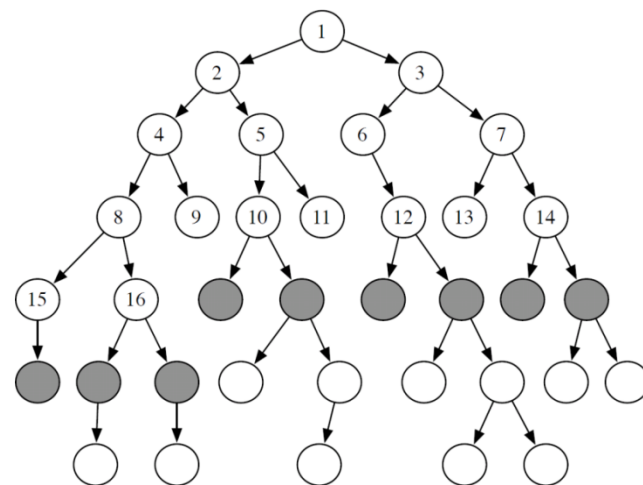
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- How many nodes at depth  $m$ ?





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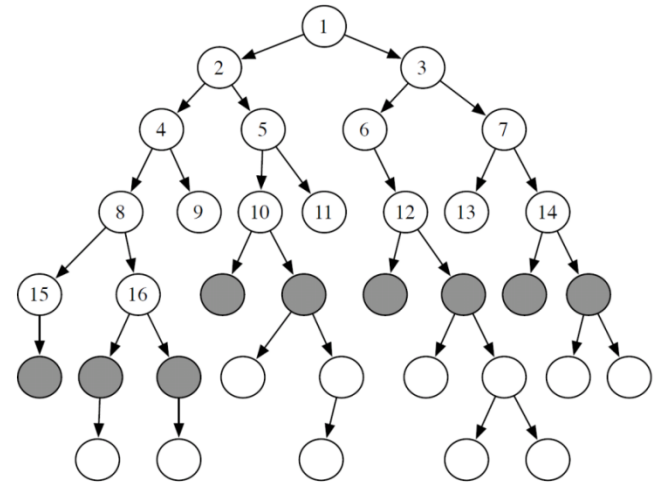
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# When to use BFS vs. DFS?

- The search graph has cycles or is infinite

**BFS**

**DFS**

- We need the shortest path to a solution

**BFS**

**DFS**

- There are only solutions at great depth

**BFS**

**DFS**

- There are some solutions at shallow depth: the other one

- No way the search graph will fit into memory

**BFS**

**DFS**

# Real Example: Solving Sudoku

Sudoku Puzzle

	9	3	6	2	8	1	4	
	6						5	
	3			1			9	
	5		8		2		7	
	4			7			6	
	8						3	
	1	7	5	9	3	4	2	

- E.g. start state on the left
- Operators:  
fill in an allowed number
- Solution: all numbers filled in,  
with constraints satisfied
- Which method would you  
rather use?

**BFS**

**DFS**

# Real Example: Eight Puzzle. DFS or BFS?

5	4	
6	1	8
7	3	2

1	2	3
8		4
7	6	5

- Which method would you rather use?

BFS

DFS

# Learning Goals for today's class

- Apply basic properties of search algorithms:
  - completeness
  - optimality
  - time and space complexity of search algorithms
- Select the most appropriate search algorithms for specific problems.
  - Depth-First Search vs. Breadth-First Search

# Coming up ...

- Read Section 3.6, Heuristic Search