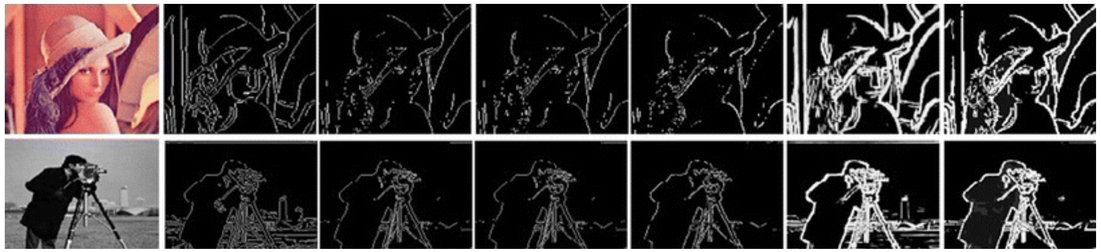
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 10: Edge Detection (cont.)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today

Topics:

— Corner **Detection**

— Image **Structure**

— **Harris Corner** Detection

Readings:

— **Today's** Lecture: Szeliski 7.1-7.2, Forsyth & Ponce 5.3.0 - 5.3.1

Reminders:

— **Assignment 2**: Scaled Representations, Face Detection and Image Blending (due Monday **Feb 13** 23:59)

—**Midterm**: **February 27th 3:30pm** in class

Lecture 9: Re-cap **Edge Detection**

Goal: Identify sudden changes in image intensity

This is where most shape information is encoded

Example: artist's line drawing (but artist also is using object-level knowledge)

Lecture 9: Re-cap **Edge Detection**

Good detection: minimize probability of false positives/negatives (spurious/missing) edges

Good localization: found edges should be as close to true image edge as possible

Single response: minimize the number of edge pixels around a single edge

	Approach	Detection	Localization	Single Resp	Limitations
Sobel	Gradient Magnitude Threshold	Good	Poor	Poor	Results in Thick Edges
Marr / Hildreth	Zero-crossings of 2nd Derivative (LoG)	Good	Good	Good	Smooths Corners
Canny	Local extrema of 1st Derivative	Best	Good	Good	

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

−1	1

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x

$$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

y-Derivative

0	5	0	0	0	0	0	0	0
0	-5	0	0	64	128	196	0	0
0	0	0	64	64	68	0	0	0
0	0	70	64	68	0	0	0	0
0	64	58	68	0	0	0	0	0
0	-64	68	0	0	-66	-66	0	0
0	0	0	0	0	66	66	0	0
0	0	0	0	0	0	0	0	0
x	x	x	x	x	x	x	x	x

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

y-Derivative

0	5	0	0	0	0	0	0	0
0	-5	0	0	64	128	196	0	0
0	0	0	64	64	68	0	0	0
0	0	70	64	68	0	0	0	0
0	64	58	68	0	0	0	0	0
0	-64	68	0	0	-66	-66	0	0
0	0	0	0	0	66	66	0	0
0	0	0	0	0	0	0	0	0
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\sqrt{64^2 + 70^2} = \sqrt{4096 + 4900} = \sqrt{8996} = 94.847$$

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

y-Derivative

0	5	0	0	0	0	0	0	0
0	-5	0	0	64	128	196	0	0
0	0	0	64	64	68	0	0	0
0	0	70	64	68	0	0	0	0
0	64	58	68	0	0	0	0	0
0	-64	68	0	0	-66	-66	0	0
0	0	0	0	0	66	66	0	0
0	0	0	0	0	0	0	0	0
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\sqrt{64^2 + 70^2} = \sqrt{4096 + 4900} = \sqrt{8996} = 94.847$$

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x

y-Derivative

0	5	0	0	0	0	0	0	0
0	-5	0	0	64	128	196	0	0
0	0	0	64	64	68	0	0	0
0	0	70	64	68	0	0	0	0
0	64	58	68	0	0	0	0	0
0	-64	68	0	0	-66	-66	0	0
0	0	0	0	0	66	66	0	0
0	0	0	0	0	0	0	0	0
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	48	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

x-Derivative

0	0	0	0	0	0	196	0	x
5	-5	0	0	0	0	196	0	x
0	0	0	64	64	68	0	0	x
0	0	64	64	68	0	0	0	x
0	70	58	68	0	0	0	0	x
64	64	68	0	0	0	0	0	x
0	196	0	0	-66	0	66	0	x
0	196	0	0	0	0	0	0	x
0	196	0	0	0	0	0	0	x

y-Derivative

0	5	0	0	0	0	0	0	0
0	-5	0	0	64	128	196	0	0
0	0	0	64	64	68	0	0	0
0	0	70	64	68	0	0	0	0
0	64	58	68	0	0	0	0	0
0	-64	68	0	0	-66	-66	0	0
0	0	0	0	0	66	66	0	0
0	0	0	0	0	0	0	0	0
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	48	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

Sobel (threshold = 100)

								x
								x
								x
								x
								x
								x
								x
								x
x	x	x	x	x	x	x	x	x

Sobel (threshold = 50)

								x
								x
								x
								x
								x
								x
								x
								x
x	x	x	x	x	x	x	x	x

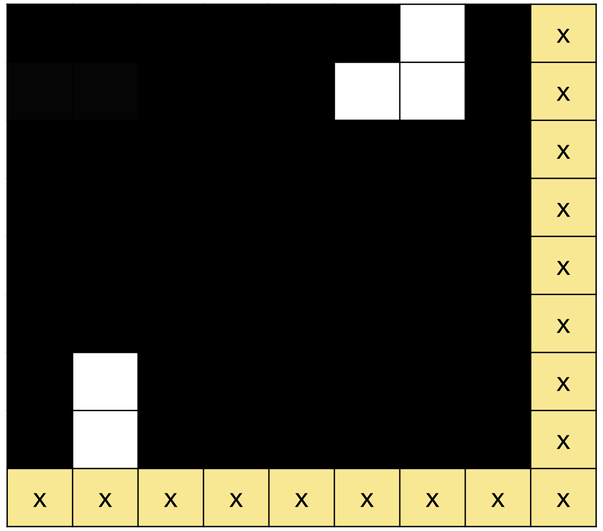
Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

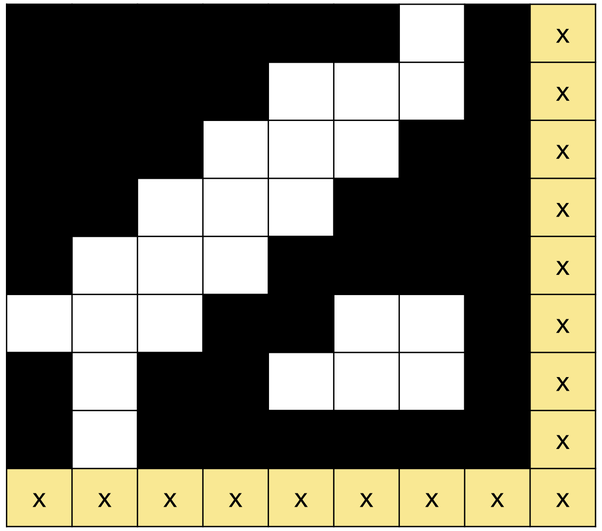
Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

Sobel (threshold = 100)



Sobel (threshold = 50)



Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Sobel issues:

- Brittle = result depends on threshold
- Thick edges = poor localization

Canny Edge Detector

3. **Non-maximum** suppression
   — thin multi-pixel wide “ridges” down to single pixel width

4. **Linking** and thresholding
   — Low, high edge-strength thresholds
   — Accept all edges over low threshold that are connected to edge over high threshold

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

No longer considered as possible edge points

Can still be edge points

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Goal:

— Identify local maxima, which can be edge points

— Thin edges, so we can improve localization

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$\text{gradient magnitude} > \mathbf{k}_{high}$ =100

$\mathbf{k}_{low} < \text{gradient magnitude} < \mathbf{k}_{high}$

$\text{gradient magnitude} < \mathbf{k}_{low}$ =50

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$\text{gradient magnitude} > \mathbf{k}_{high}$ =100

$\mathbf{k}_{low} < \text{gradient magnitude} < \mathbf{k}_{high}$

$\text{gradient magnitude} < \mathbf{k}_{low}$ =50

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\mathbf{k}_{low} < \text{gradient magnitude} < \mathbf{k}_{high}$$

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	~~277~~	0	x
0	0	0	91	91	~~96~~	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\mathbf{k}_{low} < \text{gradient magnitude} < \mathbf{k}_{high}$$

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	~~277~~	0	x
0	0	0	91	91	~~96~~	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

$$\mathbf{k}_{low} < \text{gradient magnitude} < \mathbf{k}_{high}$$

Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

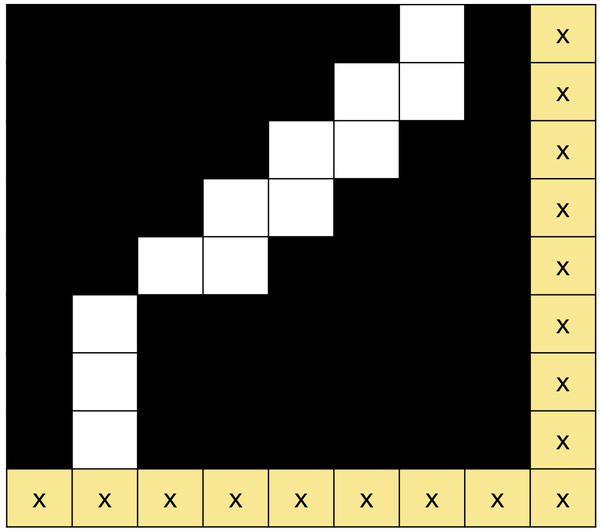
Canny **Non-Maxima Suppression**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Linking Edge Points

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

Canny Edge Detector



Gradient **Magnitude**

0	5	0	0	0	0	196	0	x
5	7	0	0	64	128	277	0	x
0	0	0	91	91	96	0	0	x
0	0	95	91	96	0	0	0	x
0	95	82	96	0	0	0	0	x
64	91	96	0	0	66	66	0	x
0	196	0	0	66	66	93	0	x
0	196	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

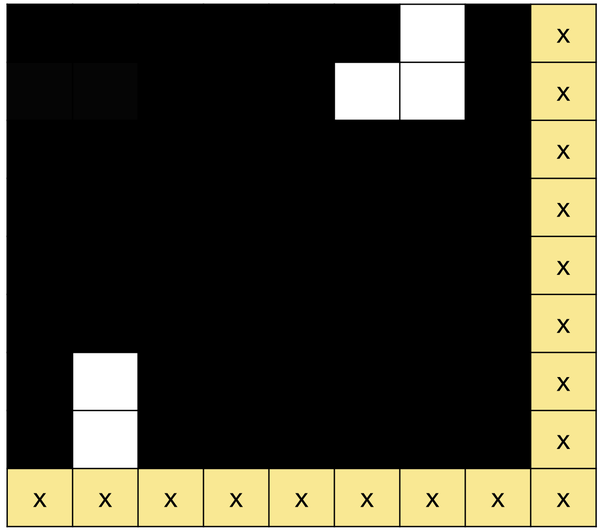
Gradient **Direction**

0	90	0	0	0	0	0	0	x
0	-135	0	0	90	90	45	0	x
0	0	0	45	45	45	0	0	x
0	0	48	45	45	0	0	0	x
0	42	45	45	0	0	0	0	x
0	-45	45	0	0	-90	-90	0	x
0	0	0	0	180	90	45	0	x
0	0	0	0	0	0	0	0	x
x	x	x	x	x	x	x	x	x

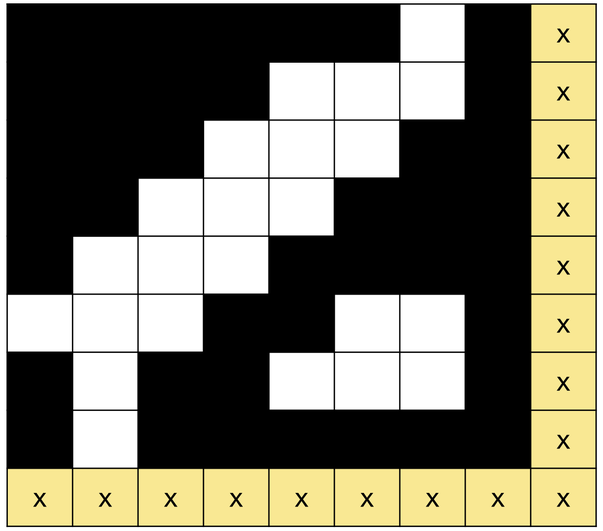
Original Image

0	0	0	0	0	0	0	196	196
0	5	0	0	0	0	0	196	196
0	0	0	0	64	128	196	196	196
0	0	0	64	128	196	196	196	196
0	0	70	128	196	196	196	196	196
0	64	128	196	196	196	196	196	196
0	0	196	196	196	130	130	196	196
0	0	196	196	196	196	196	196	196
0	0	196	196	196	196	196	196	196

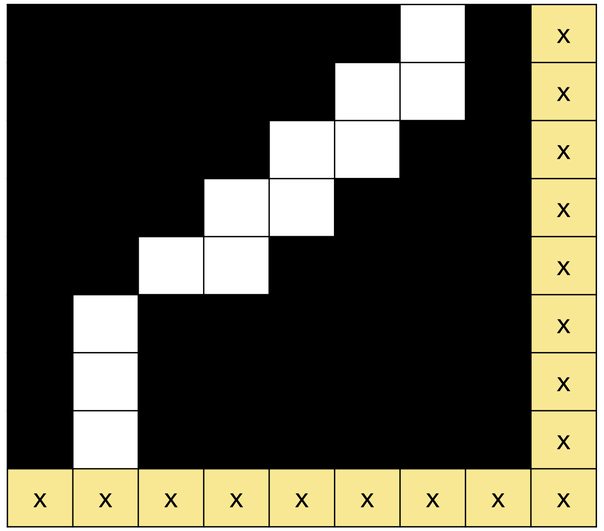
Sobel (threshold = 100)



Sobel (threshold = 50)



Canny Edge Detector



The fact that the edge is shifted can be addressed by better derivative filter (central difference)

How do humans perceive **boundaries**?

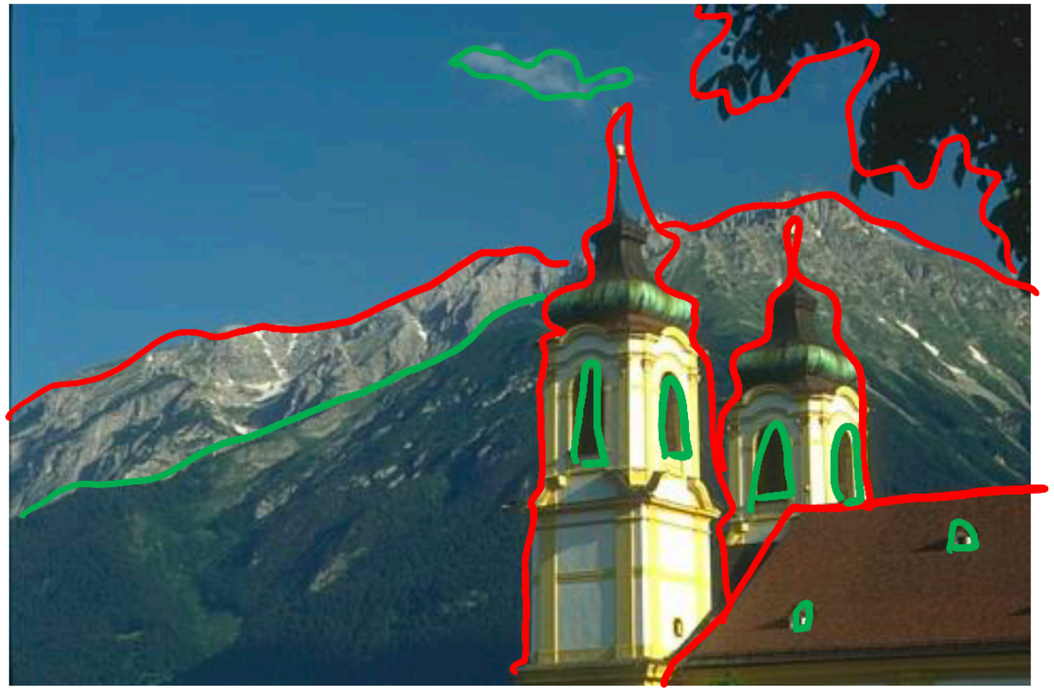
Edges are a property of the 2D image.

It is interesting to ask: How closely do image edges correspond to boundaries that humans perceive to be salient or significant?

Traditional Edge Detection

Generally lacks **semantics** (i.e., too low-level for many task)

How do humans perceive **boundaries**?



"Divide the image into some number of segments, where the segments represent ’things’ or ’parts of things’ in the scene. The number of segments is up to you, as it depends on the image. Something between 2 and 30 is likely to be appropriate. It is important that all of the segments have approximately equal importance."

(Martin et al. 2004)

How do humans perceive **boundaries**?

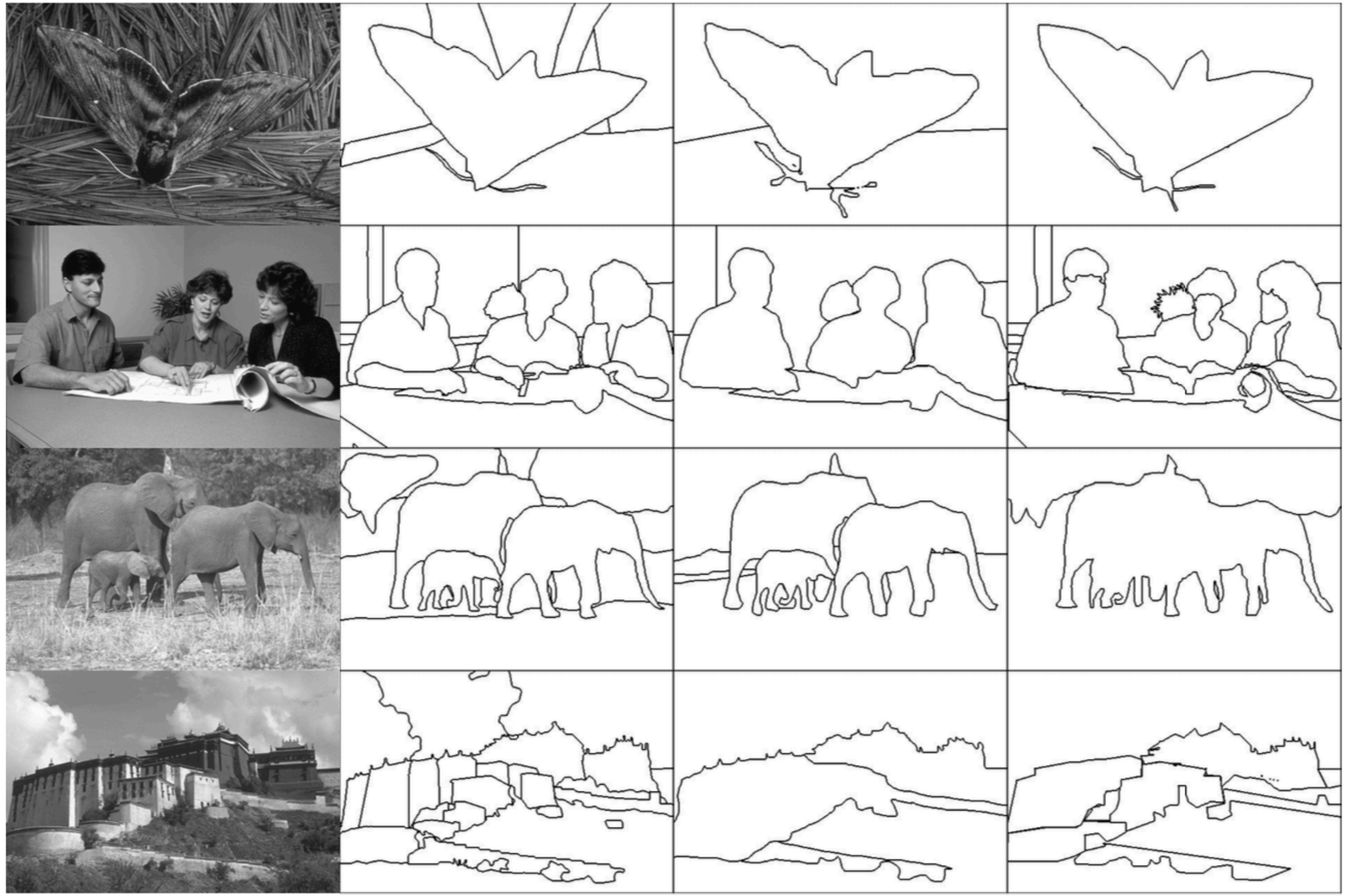


Figure Credit: Martin et al. 2001

How do humans perceive **boundaries**?

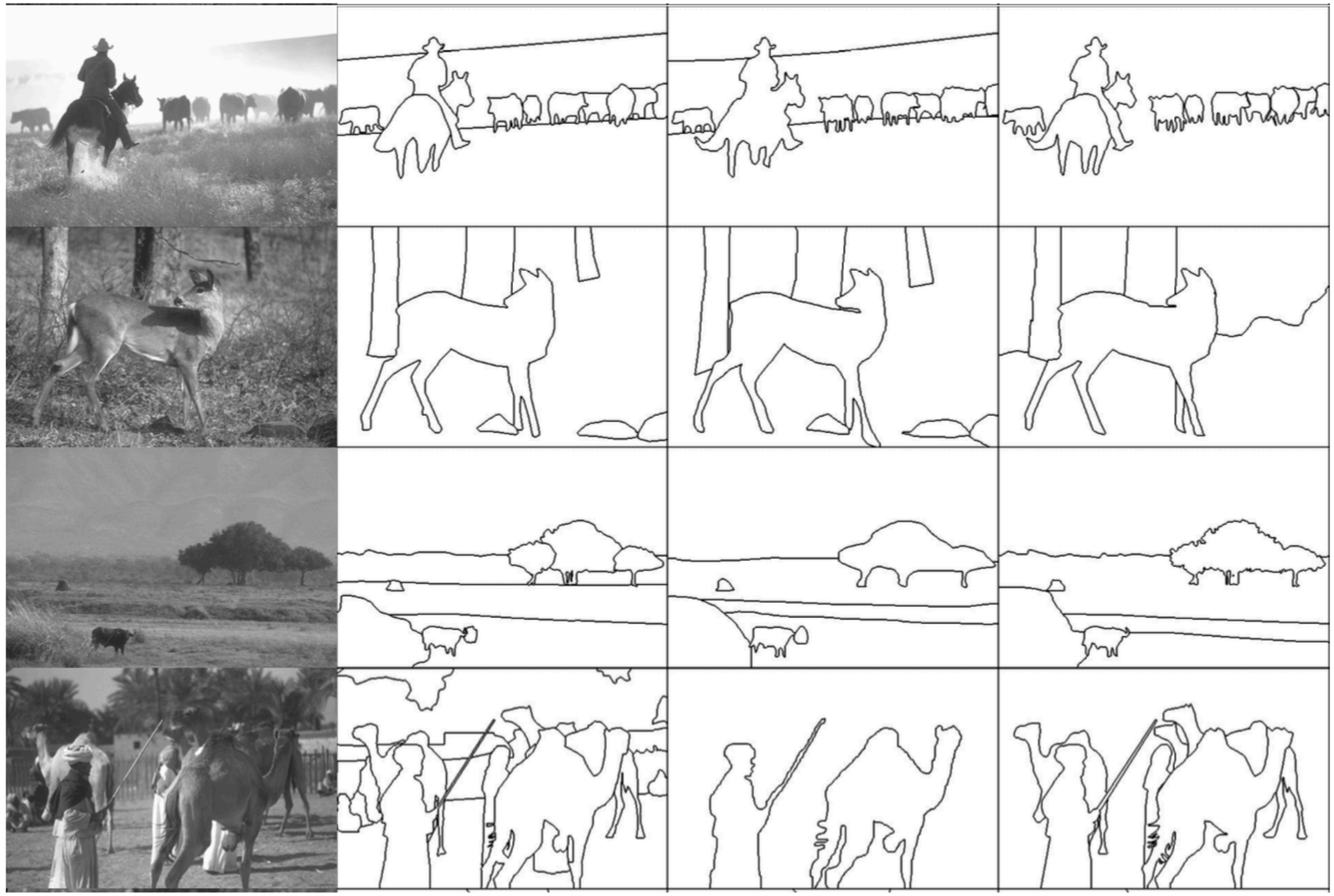
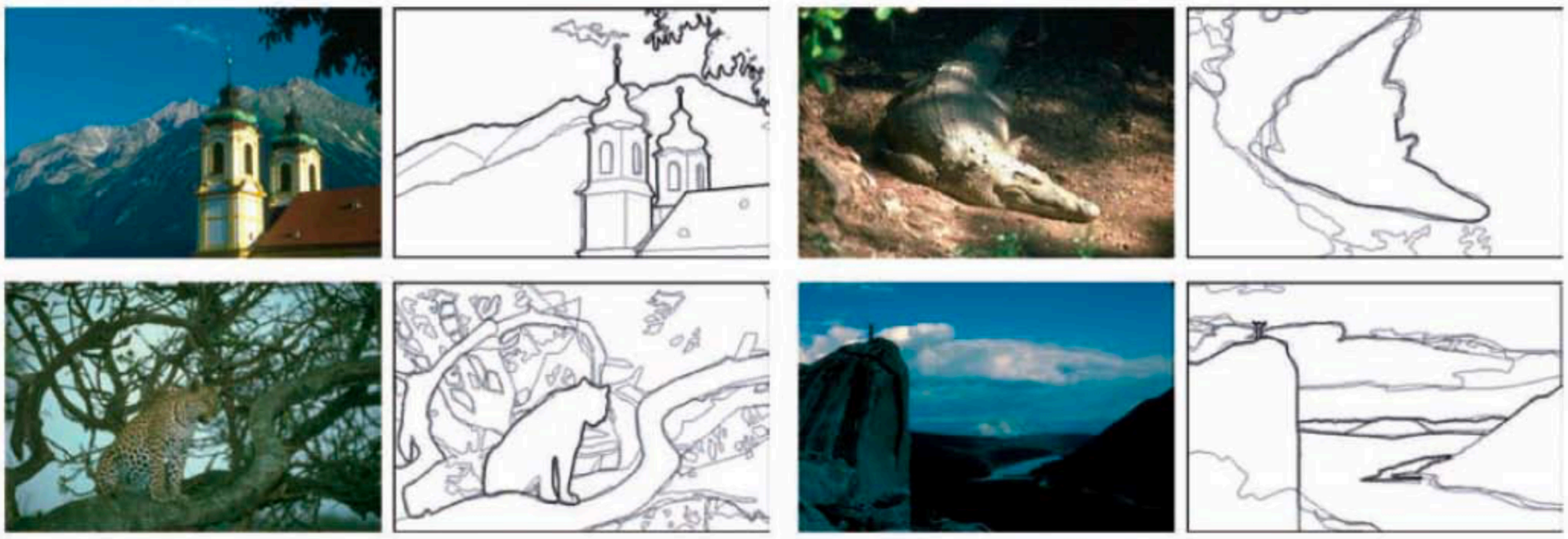


Figure Credit: Martin et al. 2001

How do humans perceive **boundaries**?



Each image shows multiple (4-8) human-marked boundaries. Pixels are darker where more humans marked a boundary.

Figure Credit: Szeliski Fig. 4.31. **Original**: Martin et al. 2004

Boundary Detection

We can formulate **boundary detection** as a high-level recognition task

— Try to learn, from sample human-annotated images, which visual features or cues are predictive of a salient/significant boundary

Many boundary detectors output a **probability or confidence** that a pixel is on a boundary

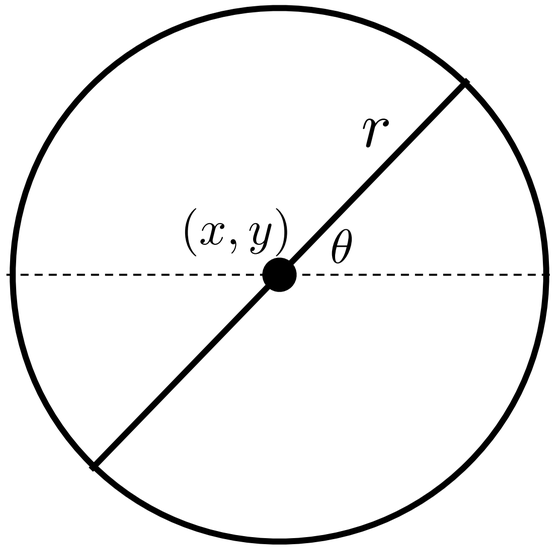
Boundary Detection: Example Approach

— Consider circular windows of radii r at each pixel (x, y) cut in half by an oriented line through the middle

— Compare visual features on both sides of the cut

— If features are very **different** on the two sides, the cut line probably corresponds to a boundary

— Notice this gives us an idea of the orientation of the boundary as well



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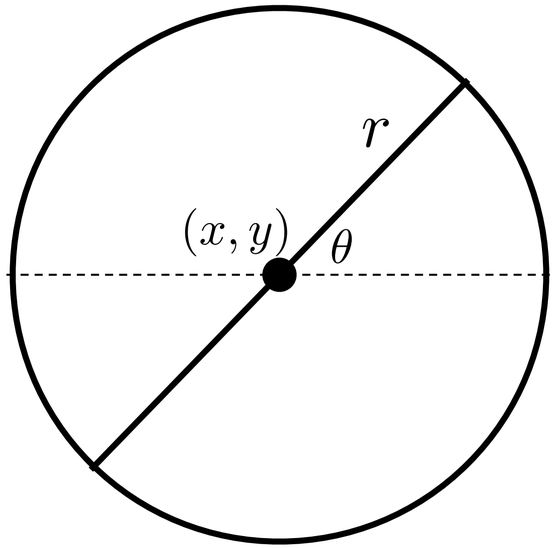


Implementation: consider 8 discrete orientations (θ) and 3 scales (r)

Boundary Detection:

Features:

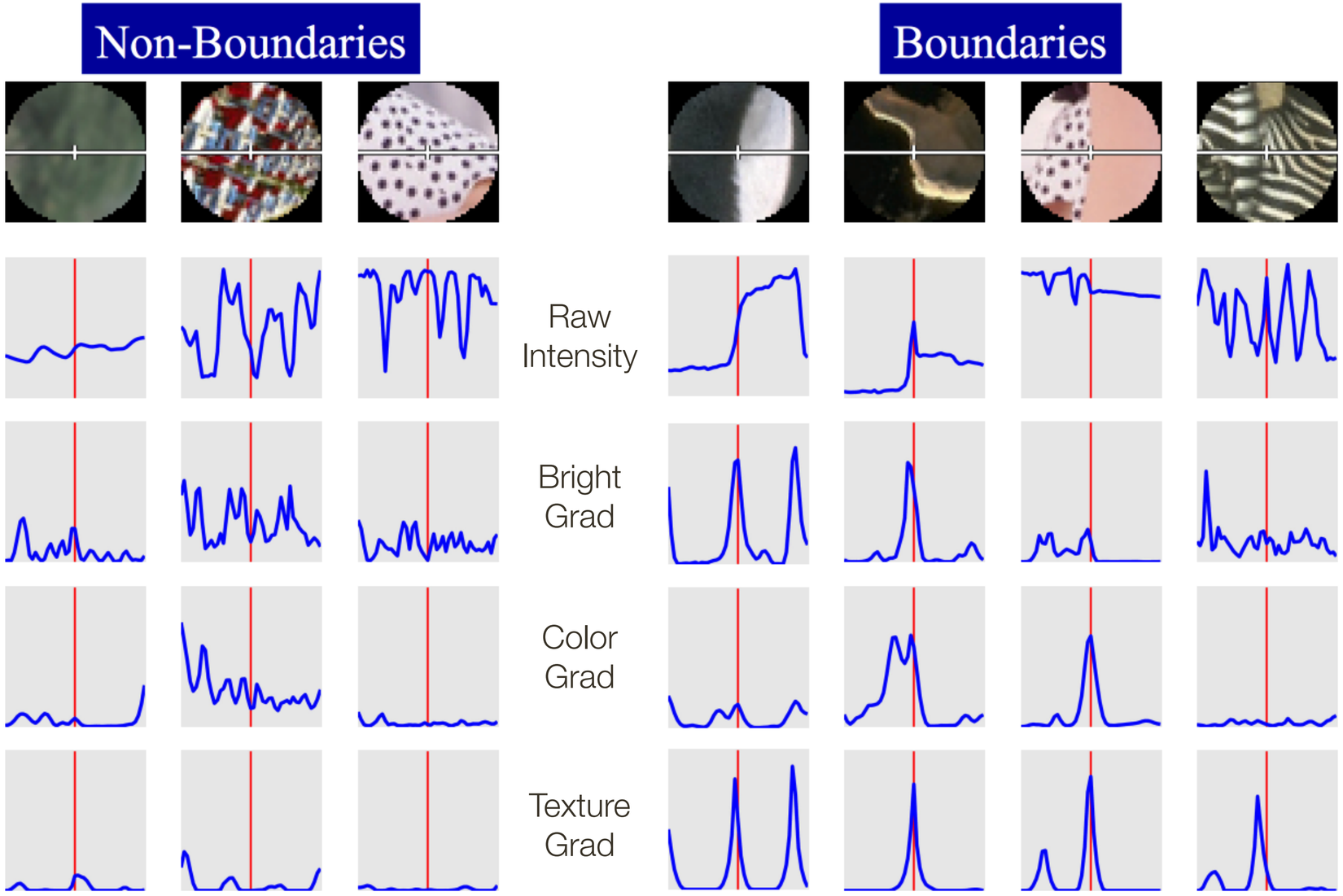
— Raw Intensity

— Orientation Energy

— Brightness Gradient

— Color Gradient

— Texture gradient



Boundary Detection:

For each **feature** type

— Compute non-parametric distribution (histogram) for left side

— Compute non-parametric distribution (histogram) for right side

— Compare two histograms, on left and right side, using statistical test

Use all the histogram similarities as features in a learning based approach that outputs probabilities (Logistic Regression, SVM, etc.)

Boundary Detection: Example Approach

Figure Credit: Szeliski Fig. 4.33. **Original**: Martin et al. 2004

Summary

Physical properties of a 3D scene cause "**edges**" in an image:

— depth discontinuity

— surface orientation discontinuity

— reflectance discontinuity

— illumination boundaries

Two generic approaches to **edge detection**:

— local extrema of a first derivative operator → **Canny**

— zero crossings of a second derivative operator → **Marr/Hildreth**

Many algorithms consider "**boundary detection**" as a high-level recognition task and output a probability or confidence that a pixel is on a human-perceived boundary

THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision

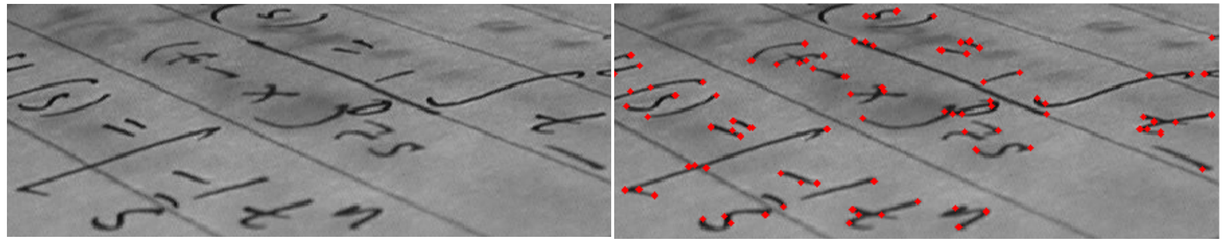


Image Credit: https://en.wikipedia.org/wiki/Corner_detection

Lecture 10: Corner Detection

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Motivation: Template Matching

When might **template matching fail**?

— Different scales 

— Different orientation 

— Lighting conditions 

— Left vs. Right hand 

— Partial Occlusions 

— Different Perspective

— Motion / blur

Motivation: Template Matching in Scaled Representation

When might **template matching** in scaled representation **fail**?

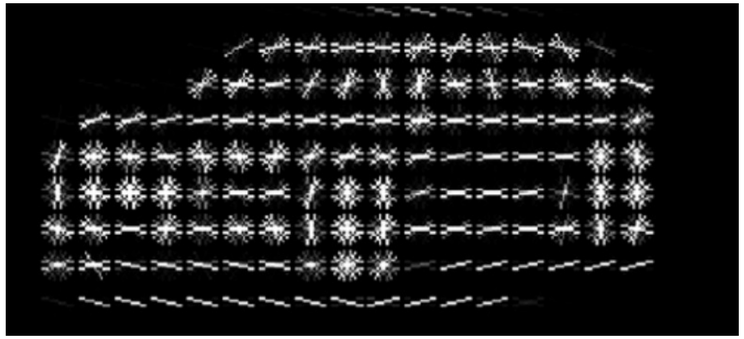
- ~~Different scales~~ 
- Different orientation 
- Lighting conditions 
- Left vs. Right hand 
- Partial Occlusions 
- Different Perspective
- Motion / blur

Motivation: Edge Matching in Scaled Representation

When might **edge matching** in scaled representation **fail**?

- ~~Different scales~~ 
- Different orientation 
- ~~Lighting conditions~~ 
- Left vs. Right hand 
- Partial Occlusions 
- Different Perspective
- Motion / blur

Motivation: Edge Matching in Scaled Representation



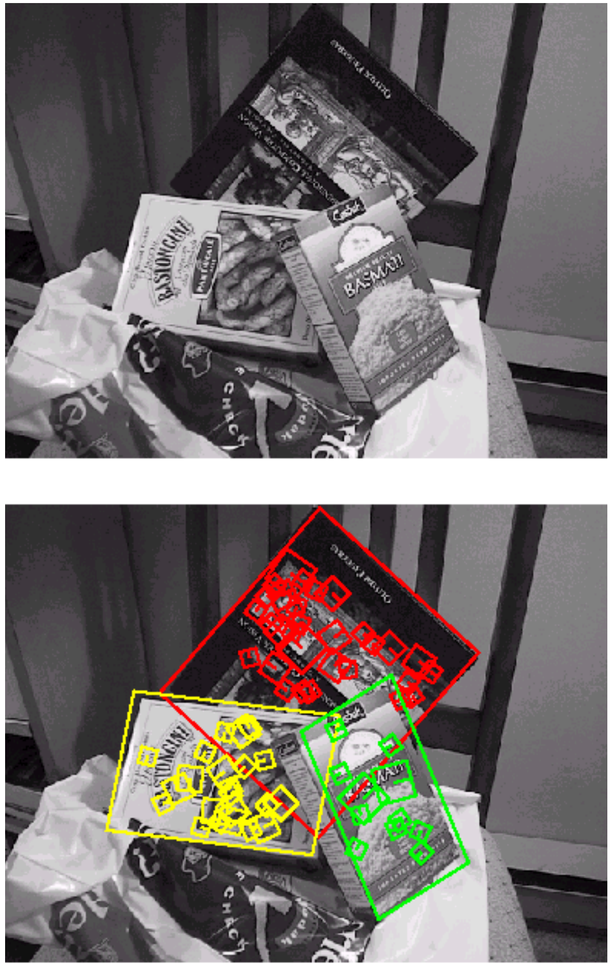
Motivation: Edge Matching in Scaled Representation

- ~~Different scales~~ 
- ~~Different orientation~~ 
- ~~Lighting conditions~~ 
- Left vs. Right hand 
- ~~Partial Occlusions~~ 
- ~~Different Perspective~~
- Motion / blur

Planar Object **Instance Recognition**

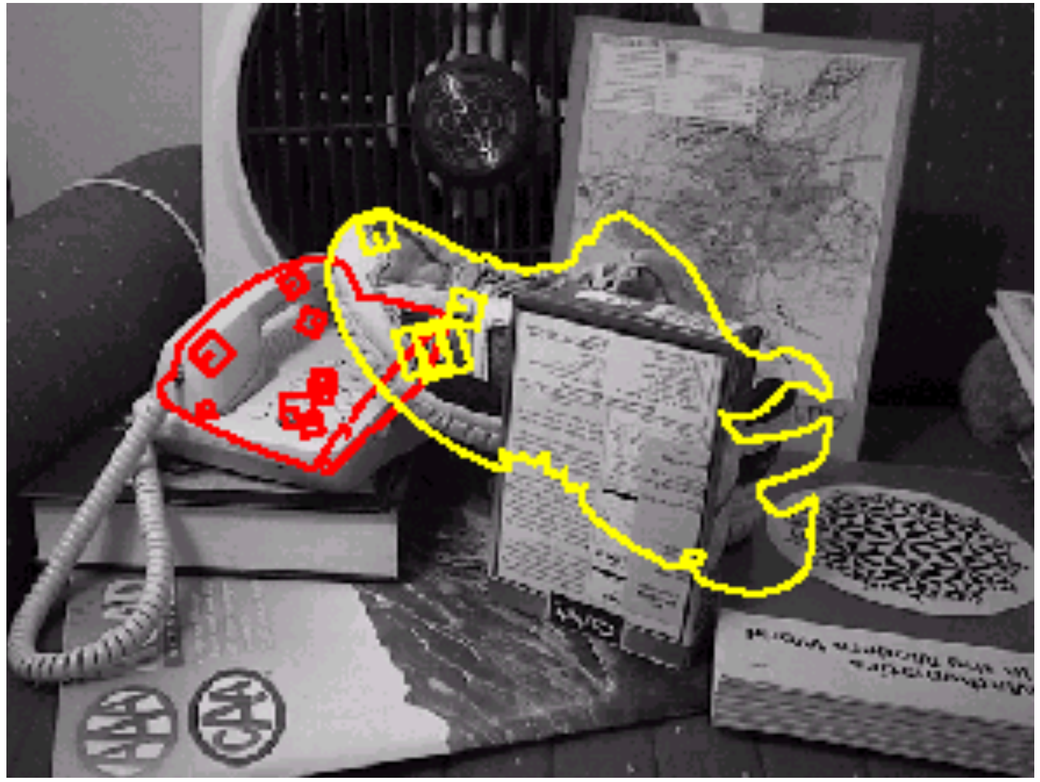
Database of planar objects

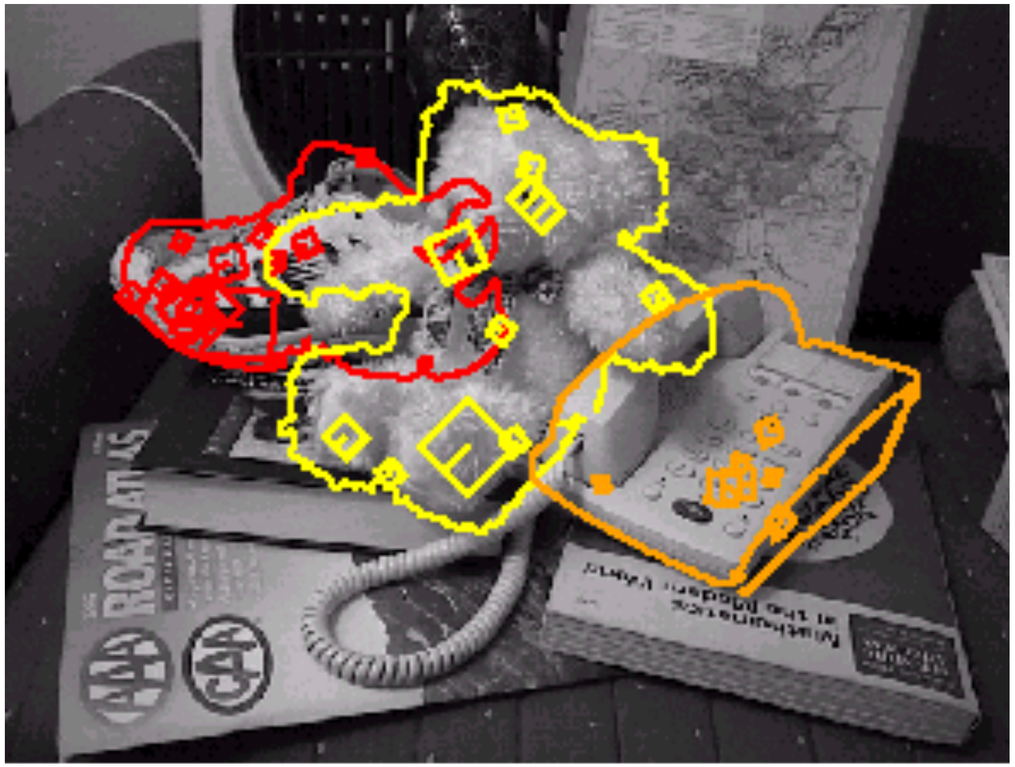
Instance recognition



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Recognition under **Occlusion**





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Image **Matching**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Image **Matching**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What is a **Good Feature**?

Local: features are local, robust to occlusion and clutter

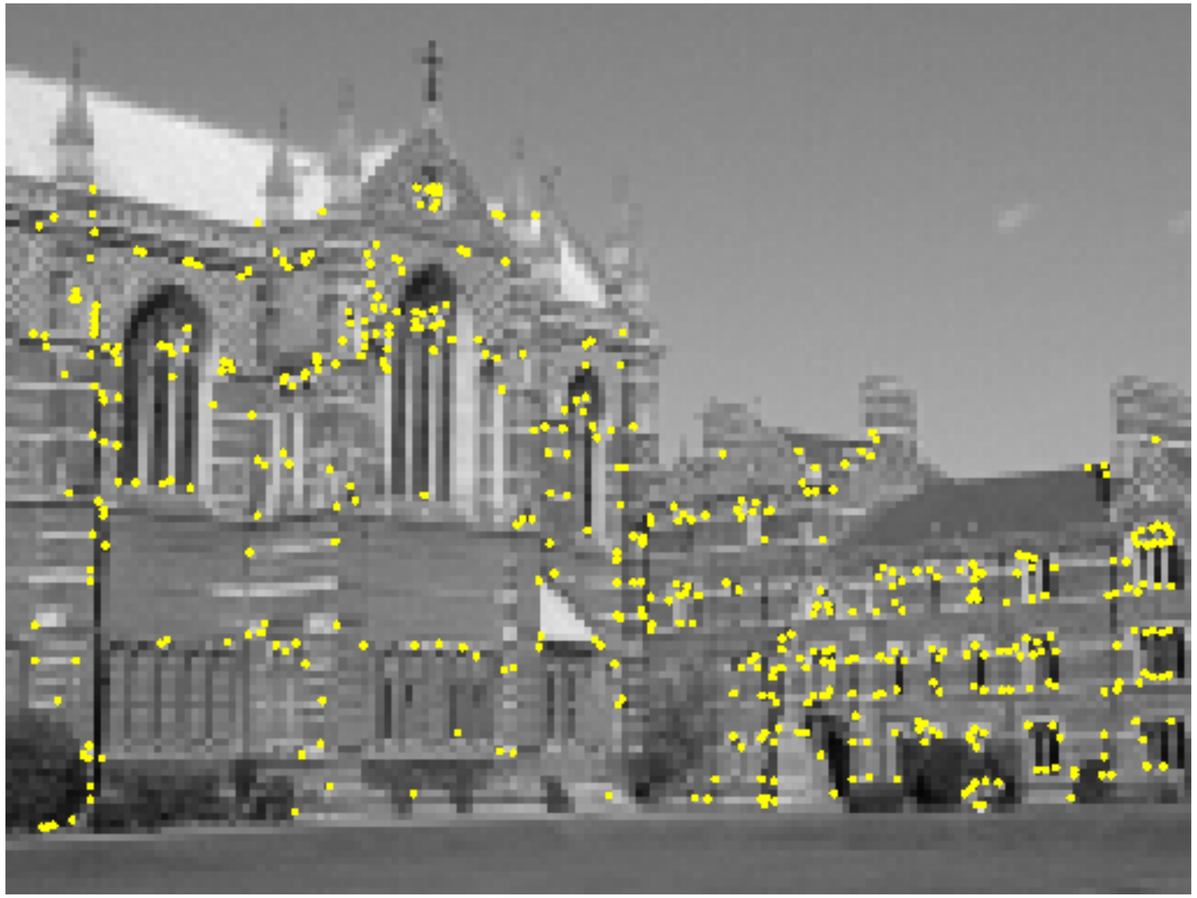
Accurate: precise localization

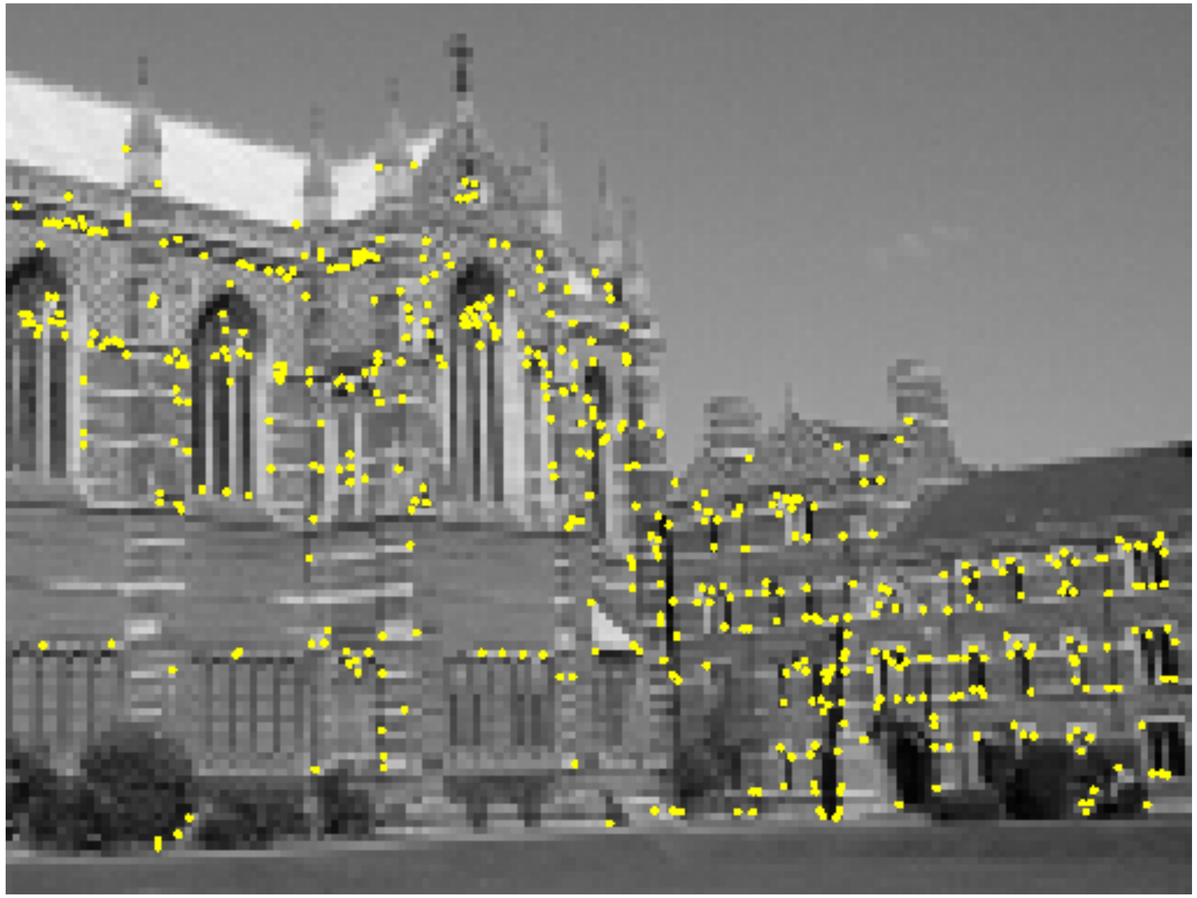
Robust: noise, blur, compression, etc. do not have a big impact on the feature.

Distinctive: individual features can be easily matched

Efficient: close to real-time performance

What is a **Good Feature**?





What is a **corner**?

Image Credit: John Shakespeare, Sydney Morning Herald

We can think of a corner as any **locally distinct** 2D image feature that (hopefully) corresponds to a distinct position on an 3D object of interest in the scene.

What is a **corner**?

Corner

Interest Point

Image Credit: John Shakespeare, Sydney Morning Herald

We can think of a corner as any **locally distinct** 2D image feature that (hopefully) corresponds to a distinct position on an 3D object of interest in the scene.

Why are corners **distinct**?

A corner can be **localized reliably**.

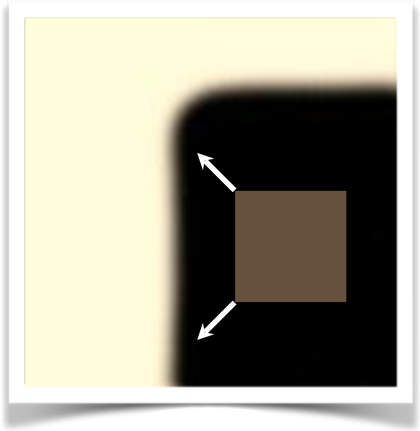
Thought experiment:

Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value.



"**flat**" region:

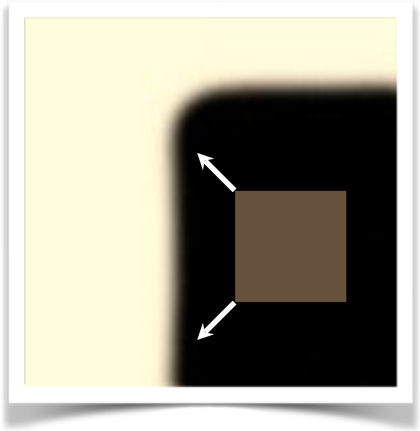
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.



"**flat**" region:
no change in all directions

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

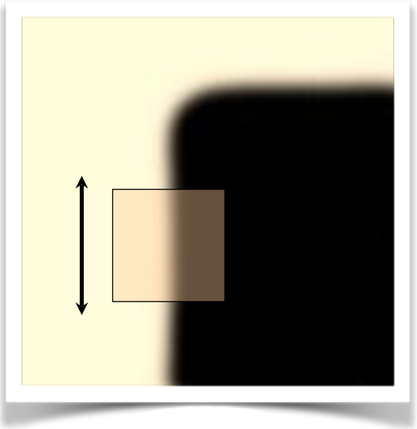
Why are corners **distinct**?

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.

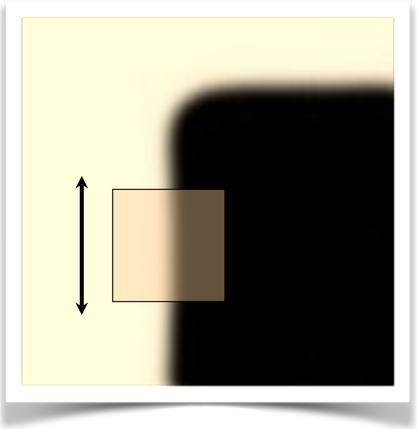
— Place a small window over an edge.



"**edge**":

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?



"**edge**":
no change along
the edge direction

A corner can be **localized reliably**.

Thought experiment:

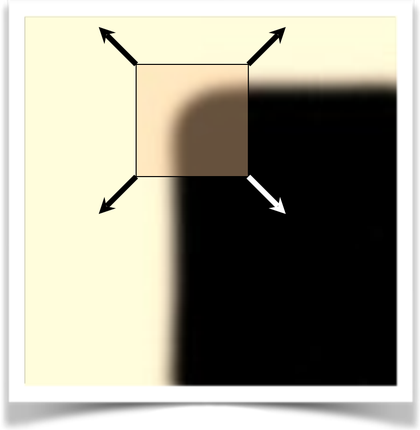
— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.

— Place a small window over an edge. If you slide the window in the direction of the edge, the image in the window will not change

→ Cannot estimate location along an edge (a.k.a., **aperture** problem)

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?



"**corner**":

A corner can be **localized reliably**.

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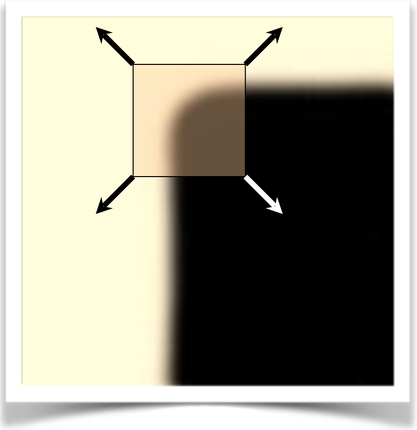
→ Cannot estimate location along an edge (a.k.a., **aperture** problem)

— Place a small window over a corner.

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Why are corners **distinct**?



"**corner**": significant change in all directions

A corner can be **localized reliably**.

Thought experiment:

— Place a small window over a patch of constant image value. If you slide the window in any direction, the image in the window will not change.

— Place a small window over an edge. If you slide the window in the direction of the edge, the image in the window will not change

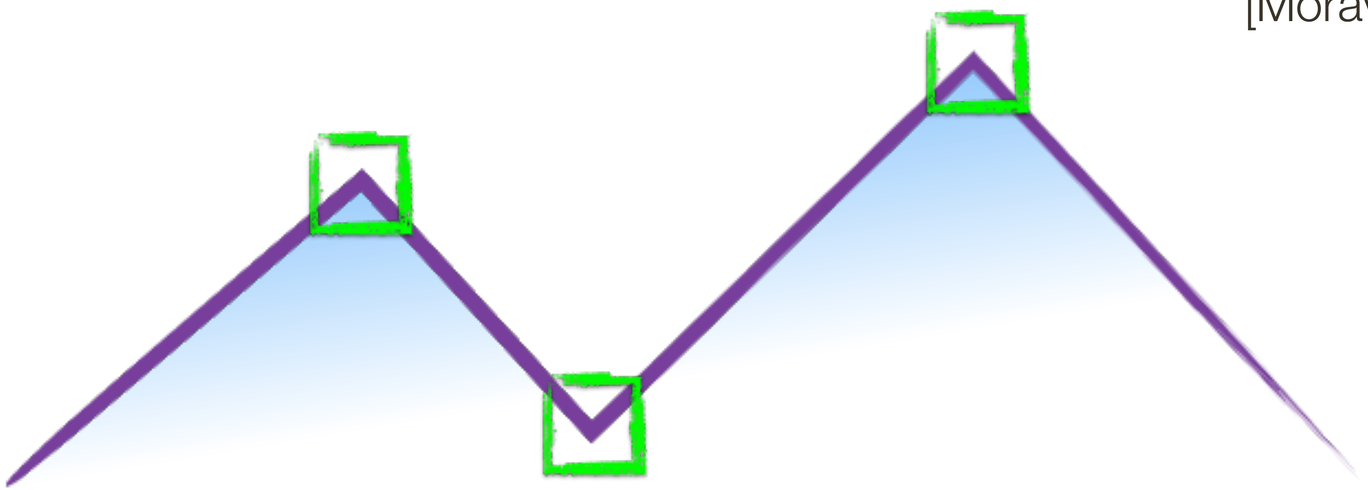
→ Cannot estimate location along an edge (a.k.a., **aperture** problem)

— Place a small window over a corner. If you slide the window in any direction, the image in the window changes.

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

How do you find a **corner**?

[Moravec 1980]



Easily recognized by looking through a small window

Shifting the window should give large change in intensity

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Autocorrelation

Autocorrelation is the correlation of the image with itself.

— Windows centered on an edge point will have autocorrelation that falls off slowly in the direction along the edge and rapidly in the direction across (perpendicular to) the edge.

— Windows centered on a corner point will have autocorrelation that falls of rapidly in all directions.

Autocorrelation

Szeliski, Figure 4.5

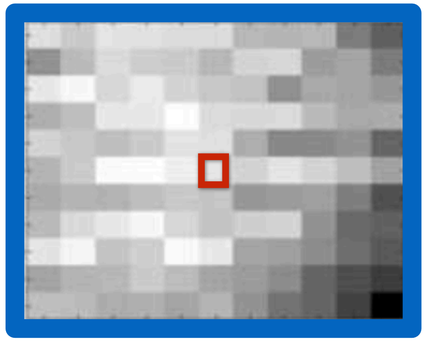
Autocorrelation

Szeliski, Figure 4.5

Autocorrelation

Szeliski, Figure 4.5

Autocorrelation

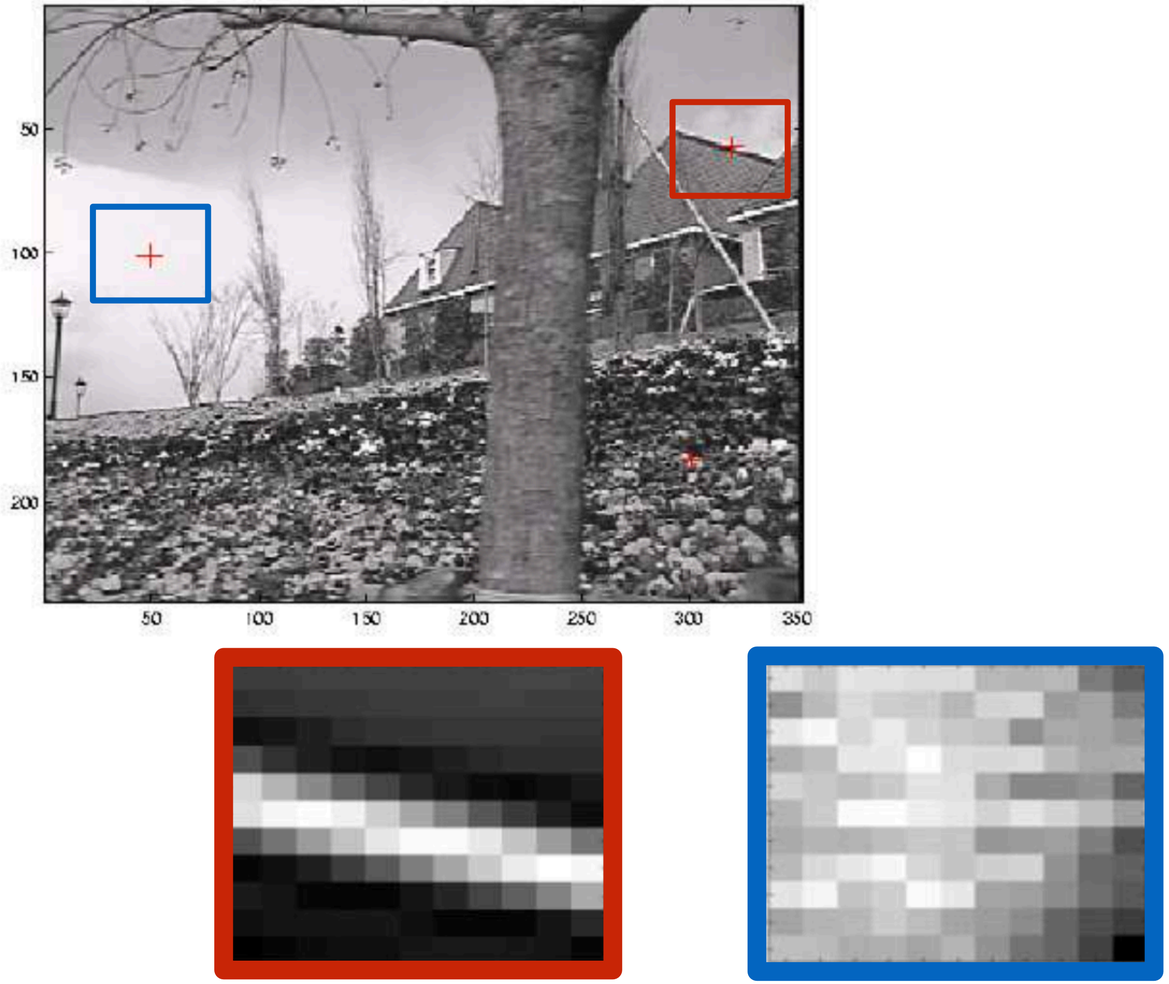


Szeliski, Figure 4.5

Autocorrelation

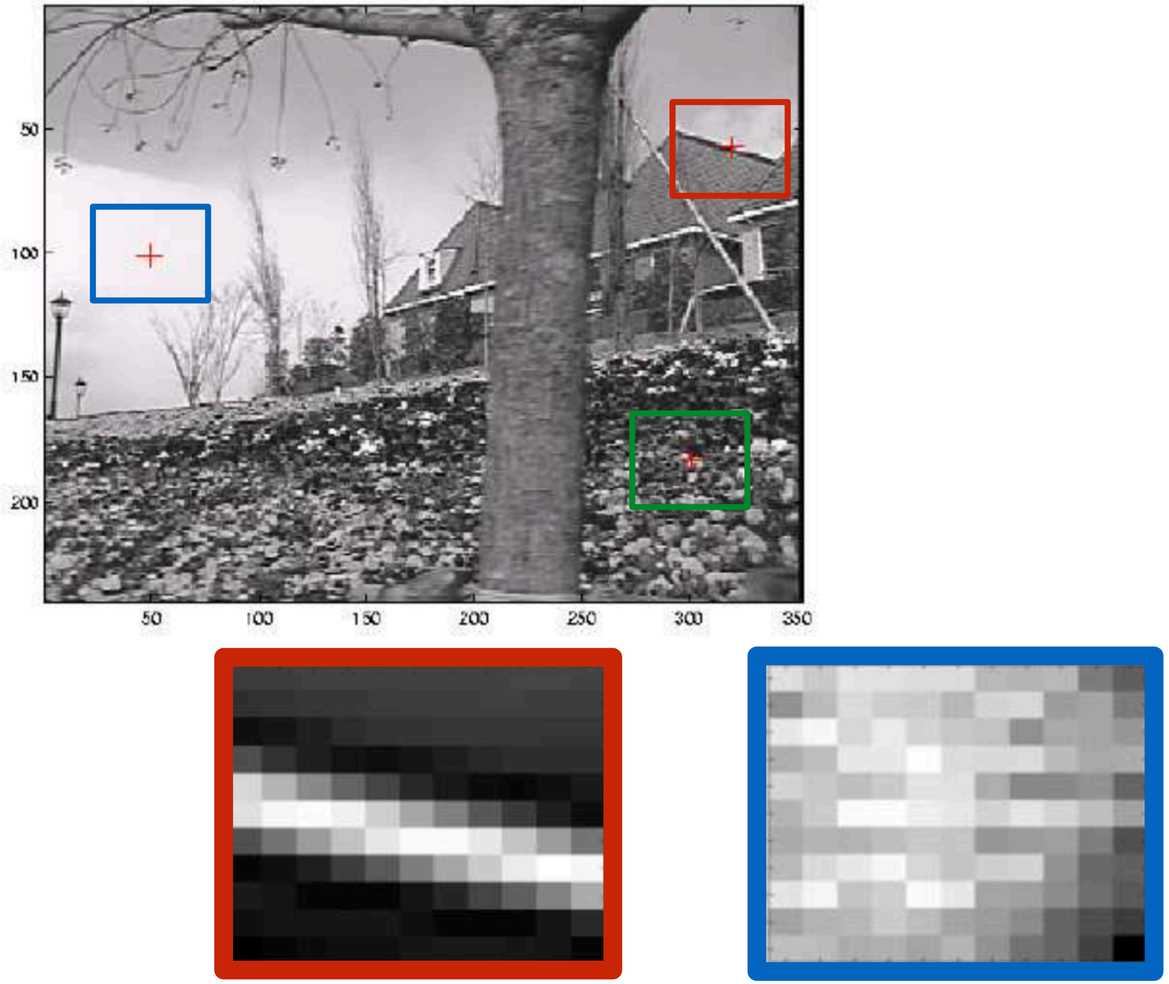
Szeliski, Figure 4.5

Autocorrelation



Szeliski, Figure 4.5

Autocorrelation



Szeliski, Figure 4.5

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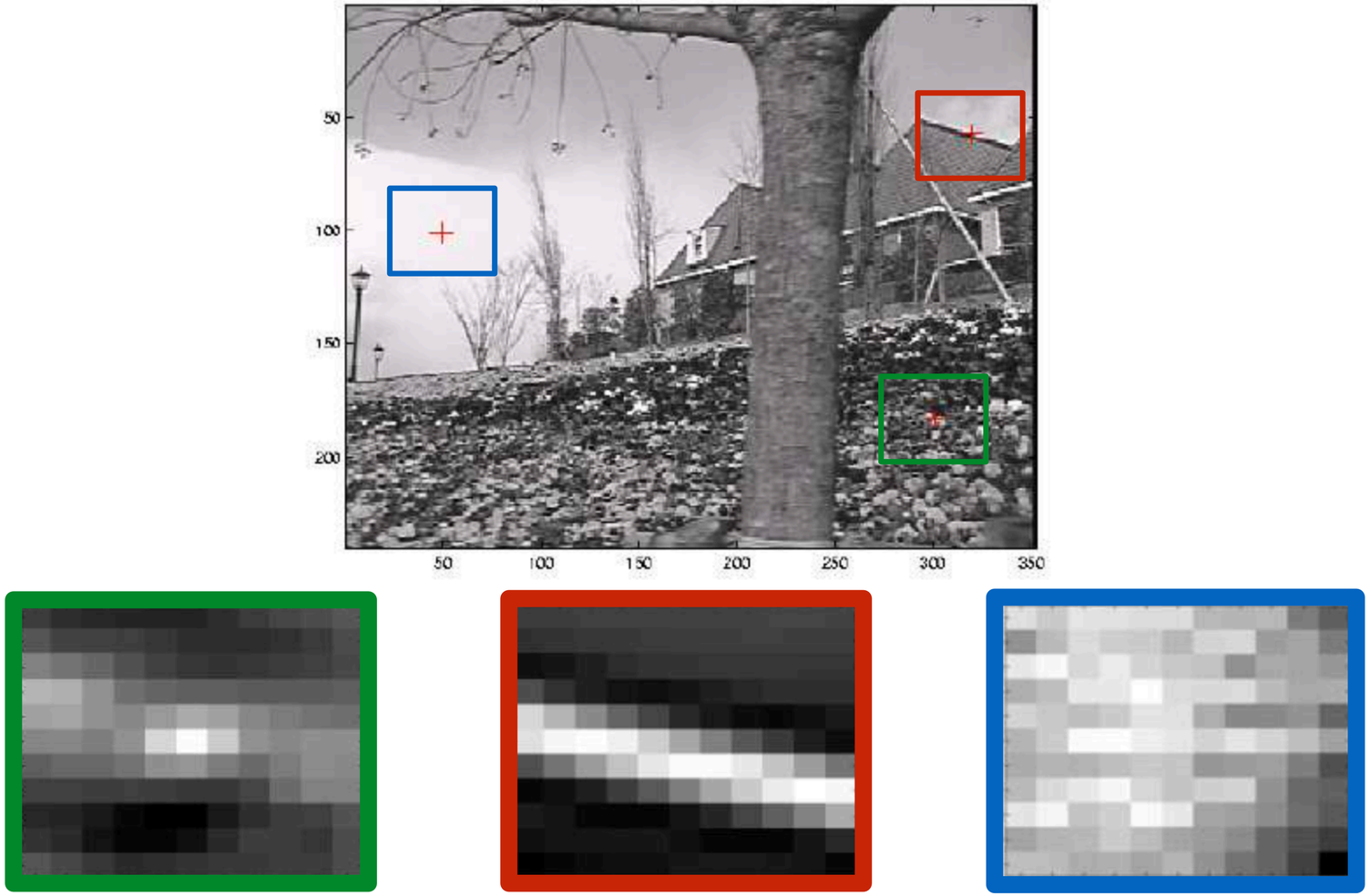

Szeliski, Figure 4.5

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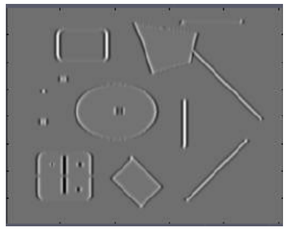
— Windows centered on a corner point will have autocorrelation that falls of rapidly in all directions.

Harris Corner Detection

$$I_x = \frac{\partial I}{\partial x} \qquad I_y = \frac{\partial I}{\partial y}$$



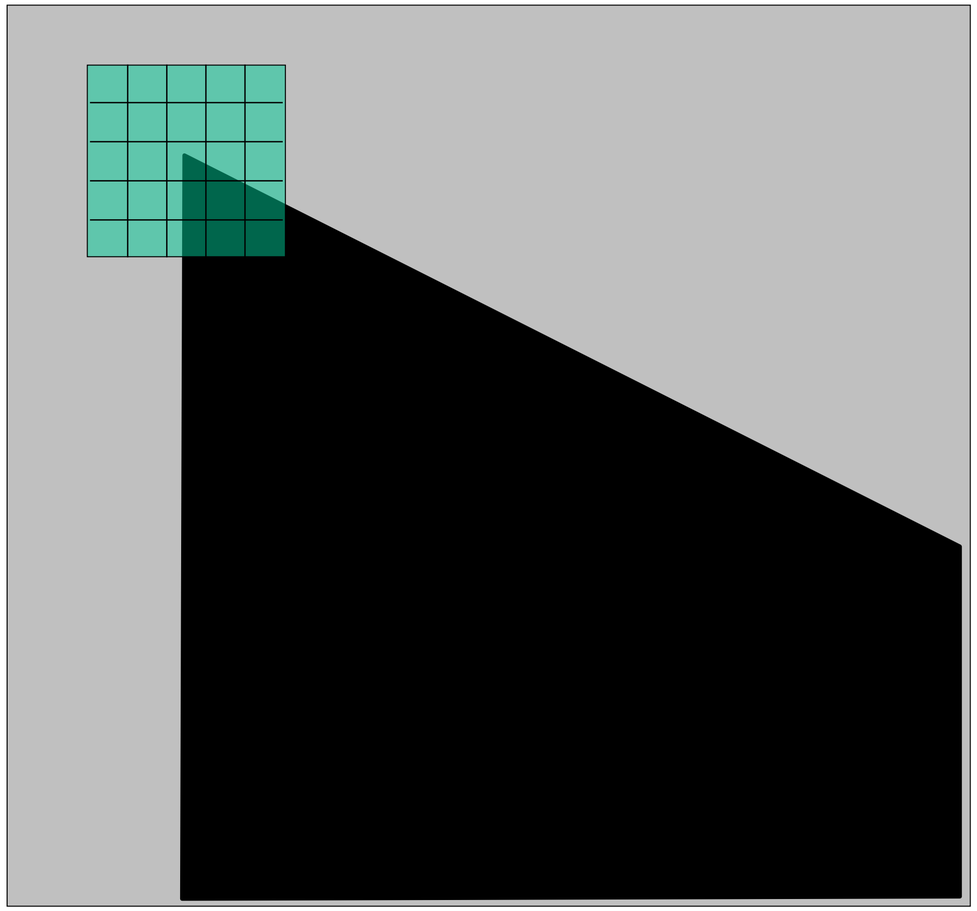
$$\begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

1. Compute image gradients over small region
2. Compute the covariance matrix
3. Compute eigenvectors and eigenvalues
4. Use threshold on eigenvalues to detect corners

Slide Adopted: Ioannis (Yannis) Gkioulekas (CMU)

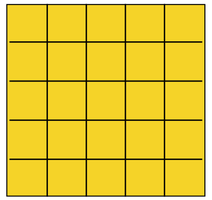
1. Compute **image gradients** over a small region

(not just a single pixel)



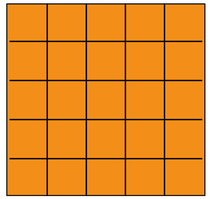
array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$



array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$



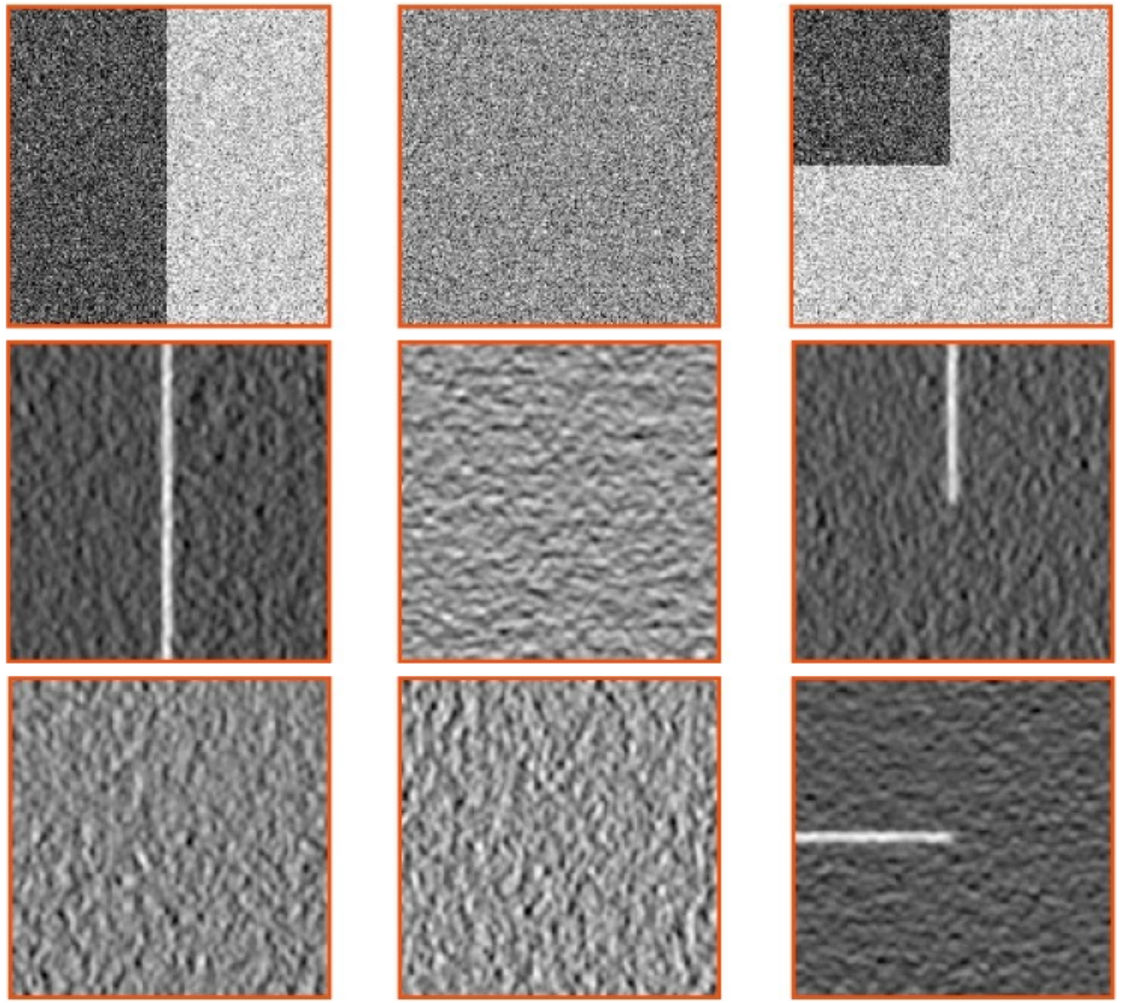
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Visualization of Gradients

image

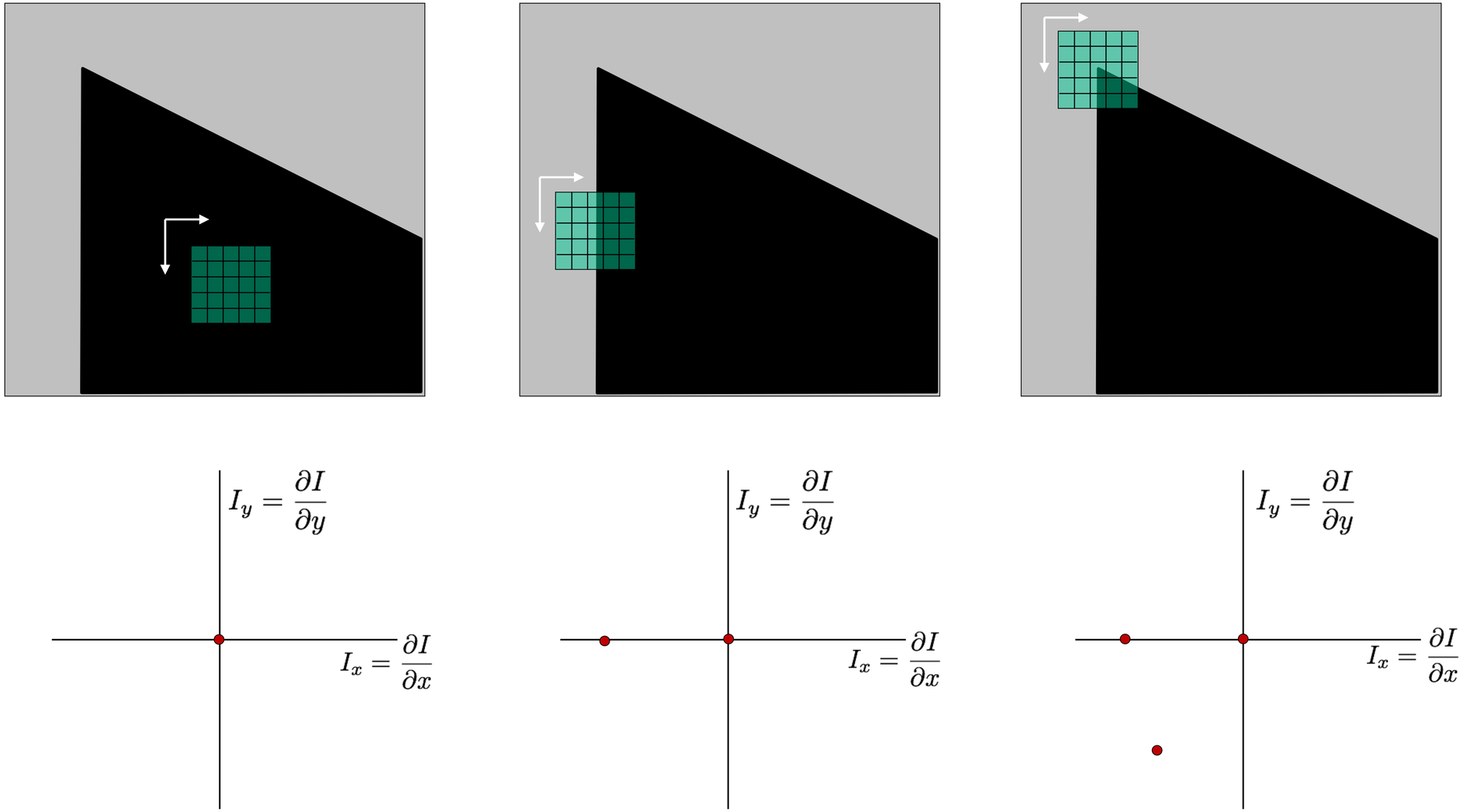
X derivative

Y derivative



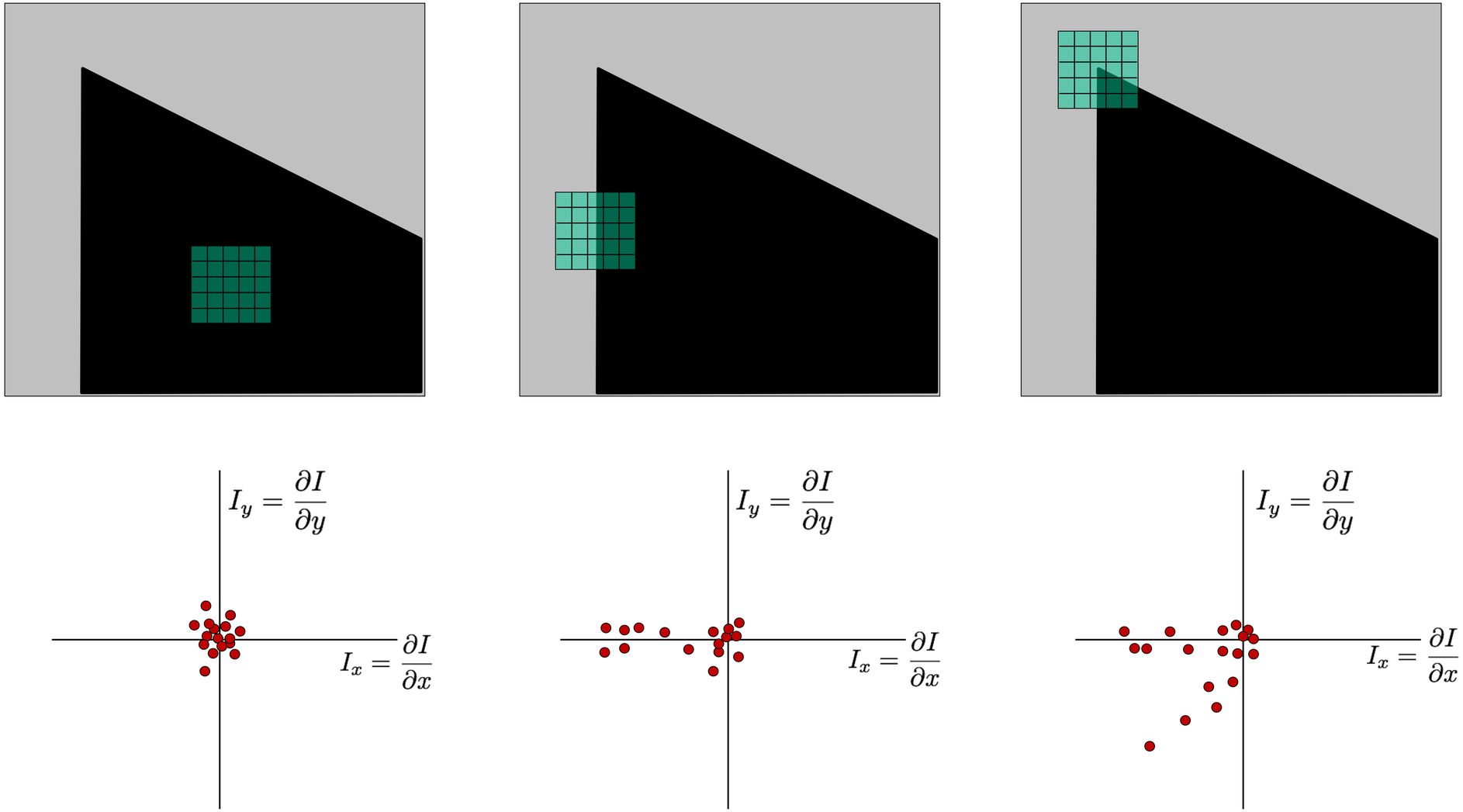
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?



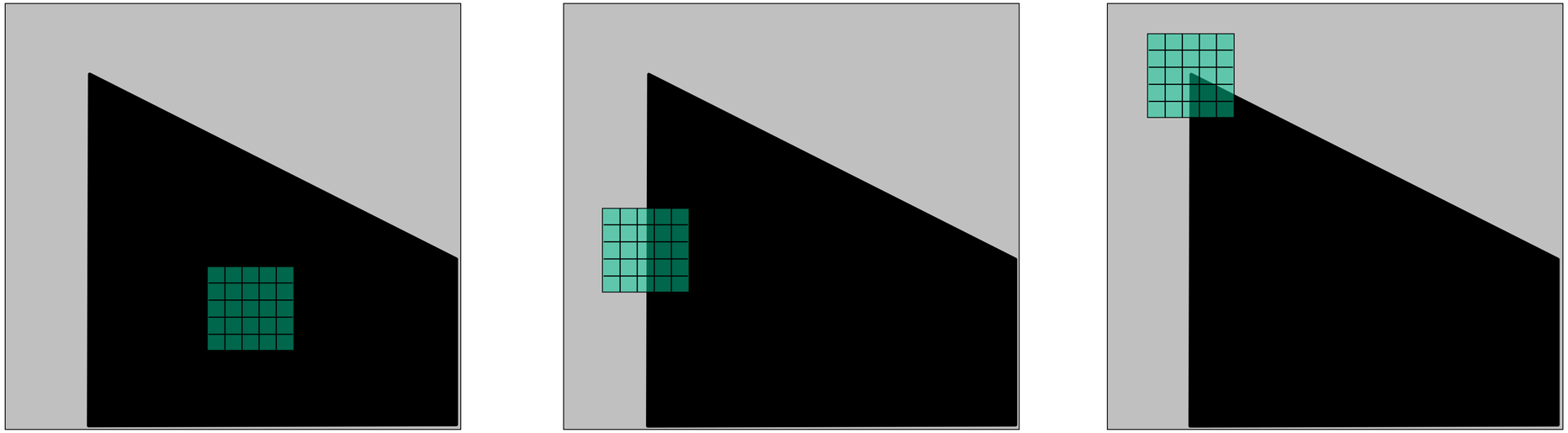
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?

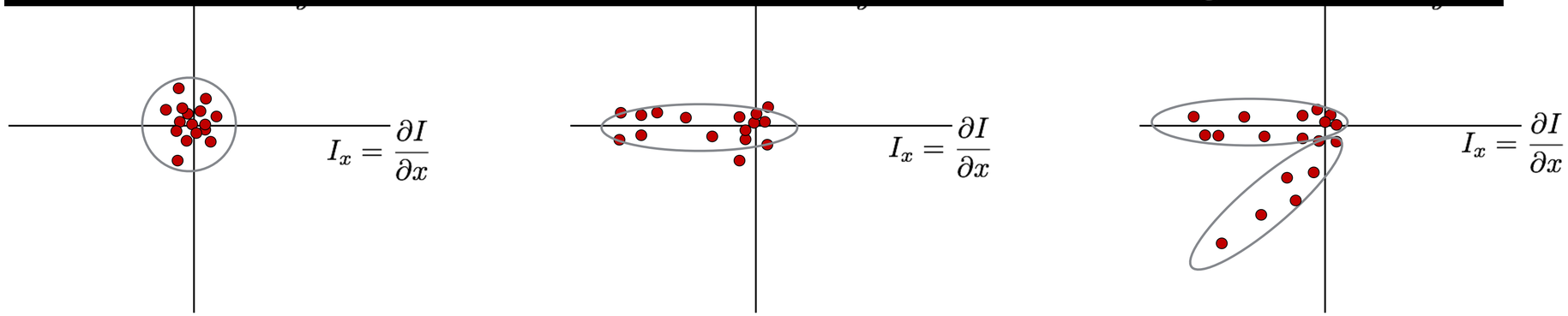


Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?

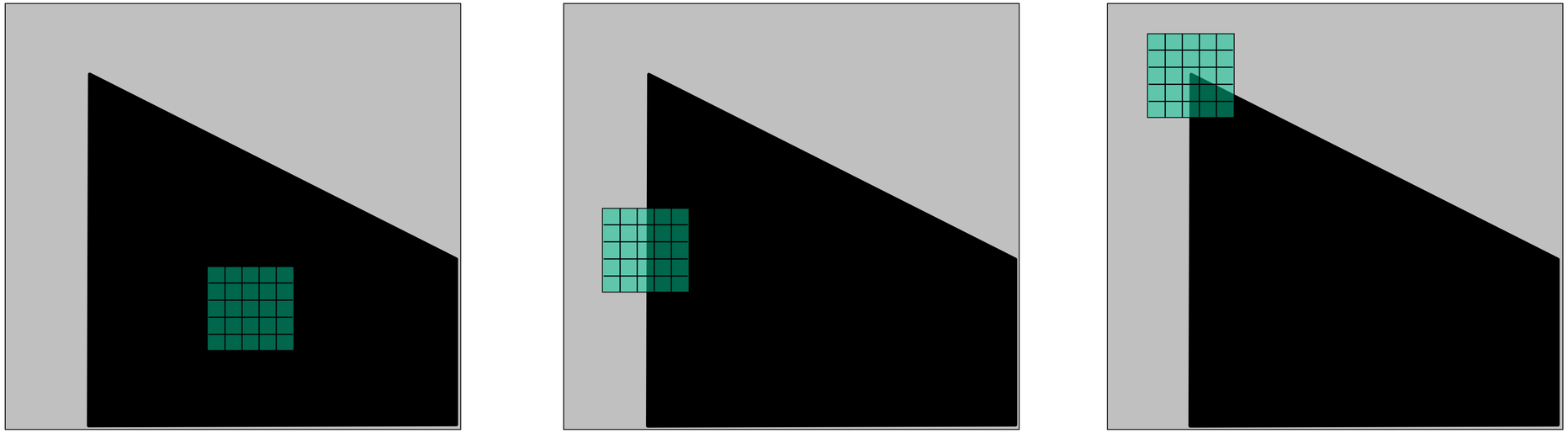


Distribution reveals the **orientation** and **magnitude**

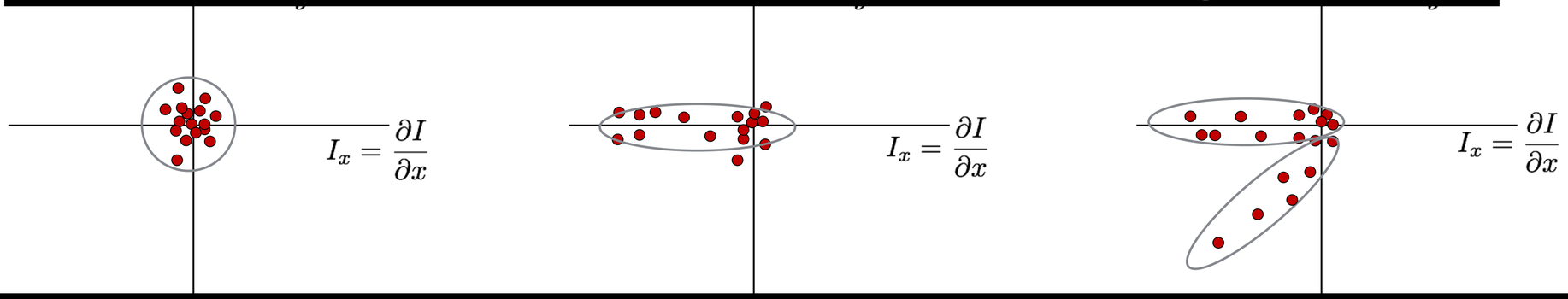


Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

What Does a **Distribution** Tells You About the **Region**?



Distribution reveals the **orientation** and **magnitude**



How do we quantify the **orientation** and **magnitude**?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

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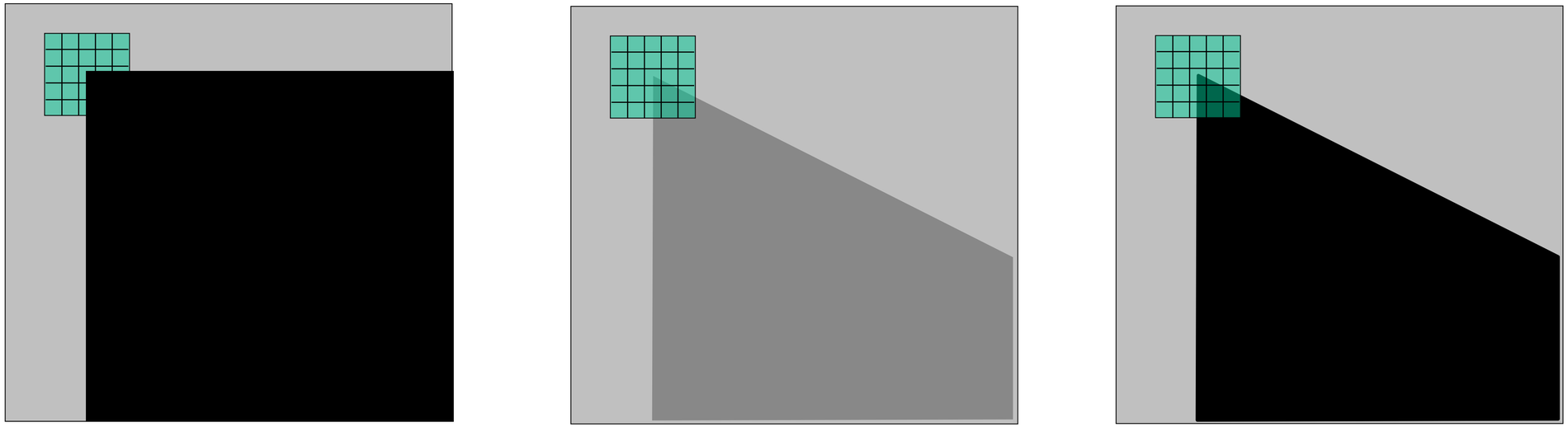
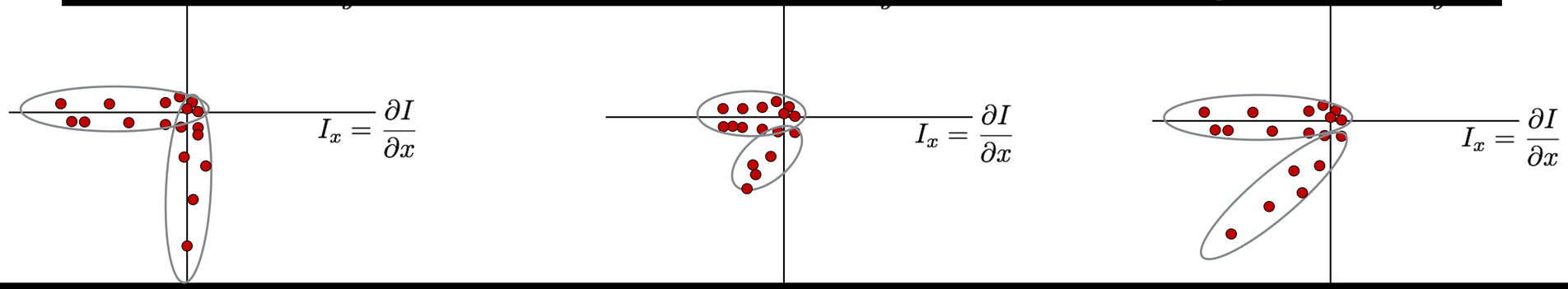


Distribution reveals the **orientation** and **magnitude**



How do we quantify the **orientation** and **magnitude**?

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2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

Sum over small region around the corner

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$$\sum_{p \in P} I_x I_y \quad = \text{sum}(\quad \overset{I_x = \frac{\partial I}{\partial x}}{\underset{\text{array of x gradients}}{\boxed{}}} \quad .* \quad \overset{I_y = \frac{\partial I}{\partial y}}{\underset{\text{array of y gradients}}{\boxed{}}} \quad)$$

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Matrix is **symmetric**

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

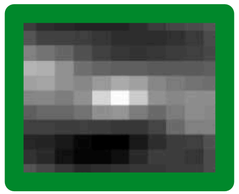
By computing the **gradient covariance matrix** …

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

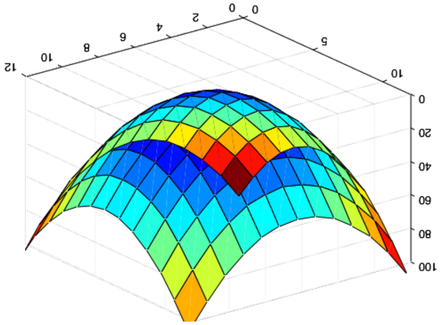
we are fitting a **quadratic** to the gradients over a small image region

2. Compute the **covariance matrix** (a.k.a. 2nd moment matrix)

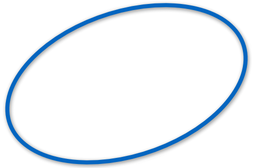
By computing the **gradient covariance matrix** …



$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$



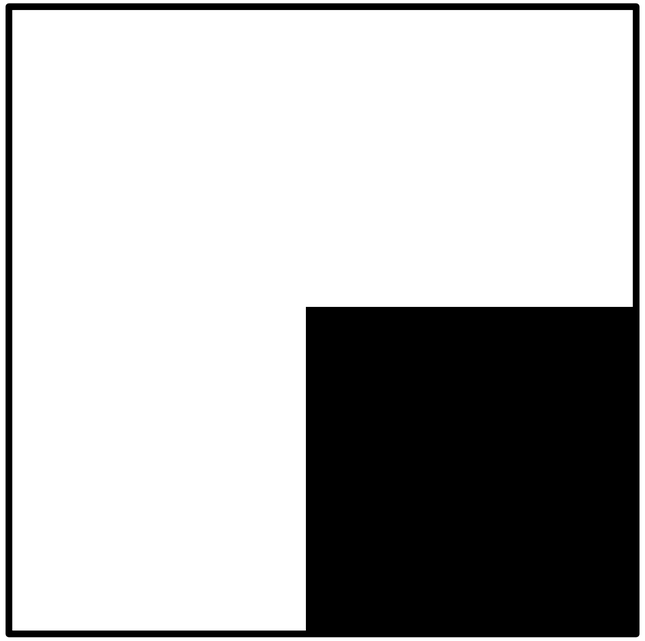
Autocorrelation



Covariance matrix

we are fitting a **quadratic** to the gradients over a small image region

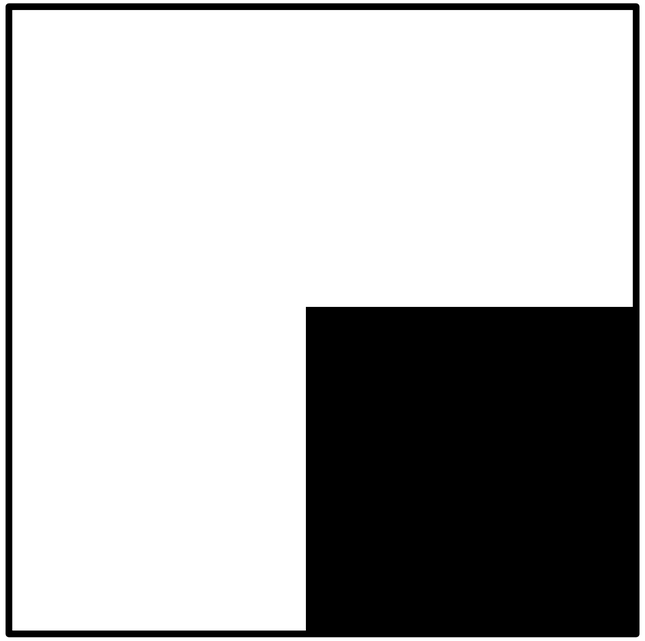
Simple Case



Local Image Patch

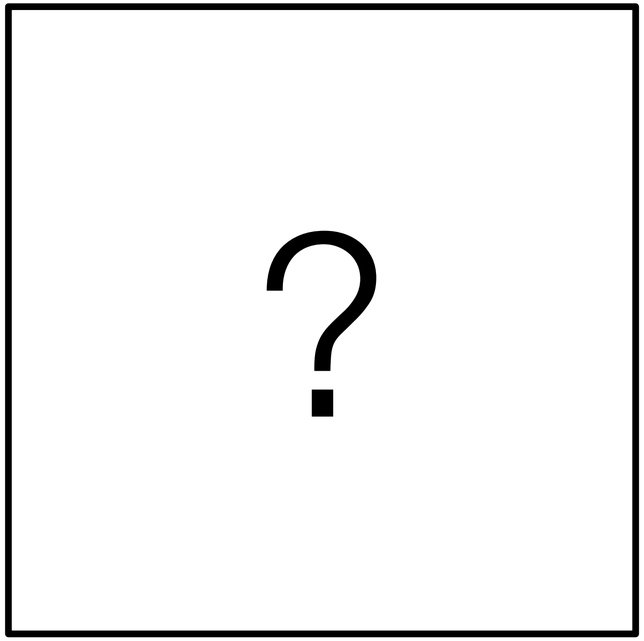
$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix} = ?$$

Simple Case

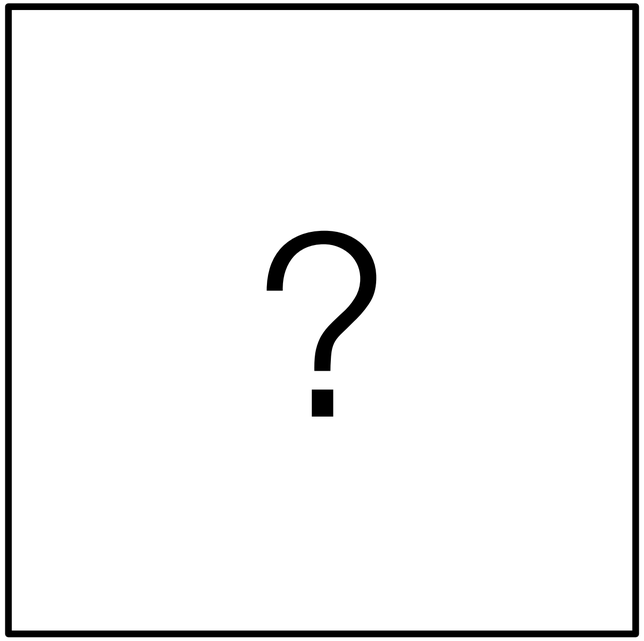


Local Image Patch

I_x



I_y



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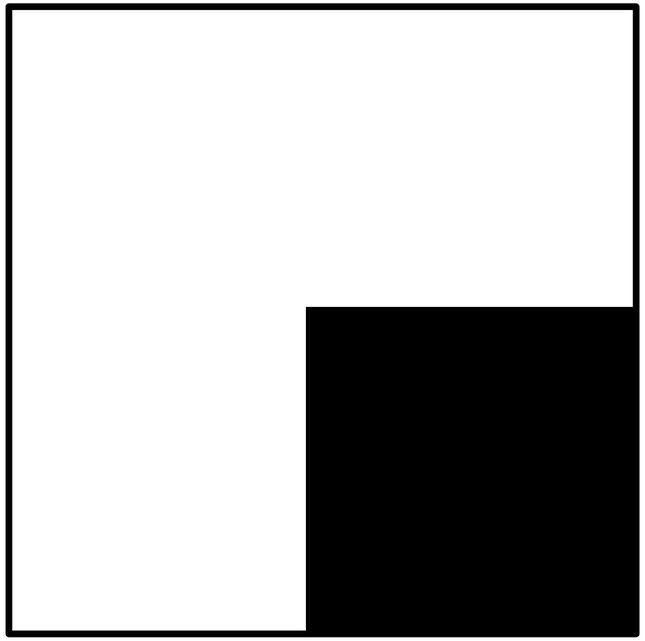
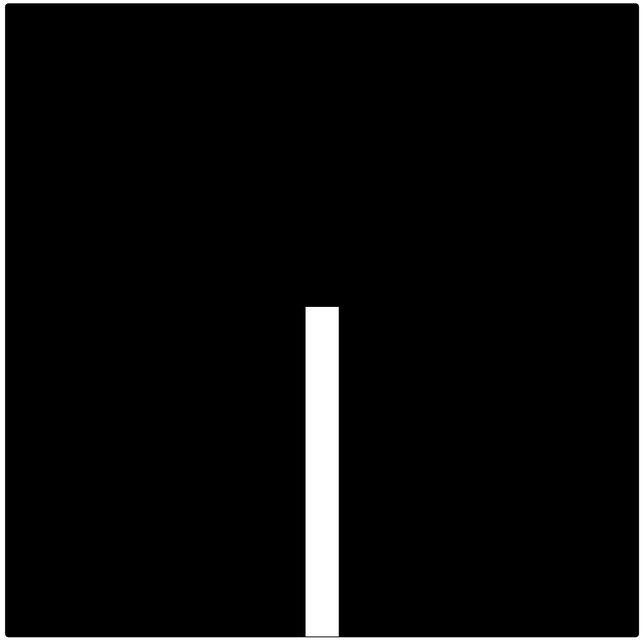
Simple Case



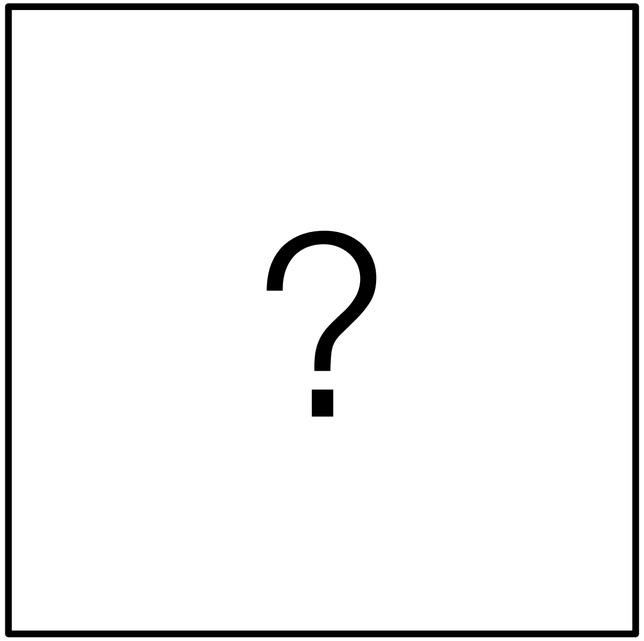
Local Image Patch

I_x



high value along vertical
strip of pixels and 0 elsewhere

I_y



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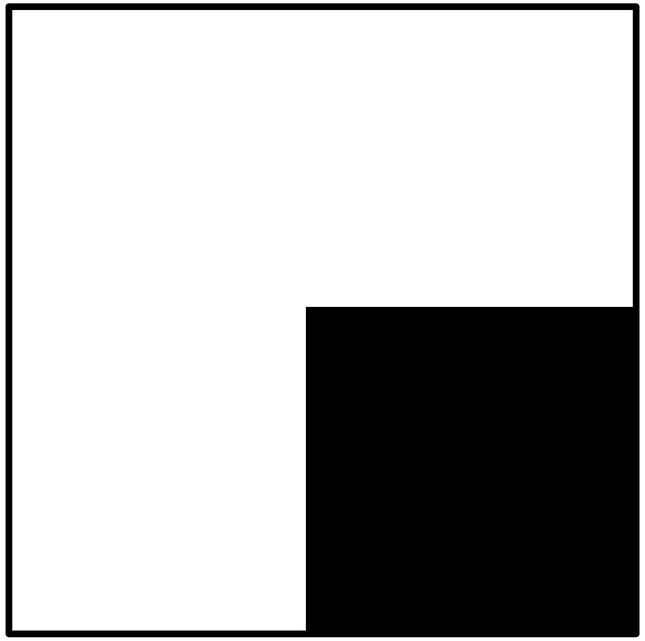
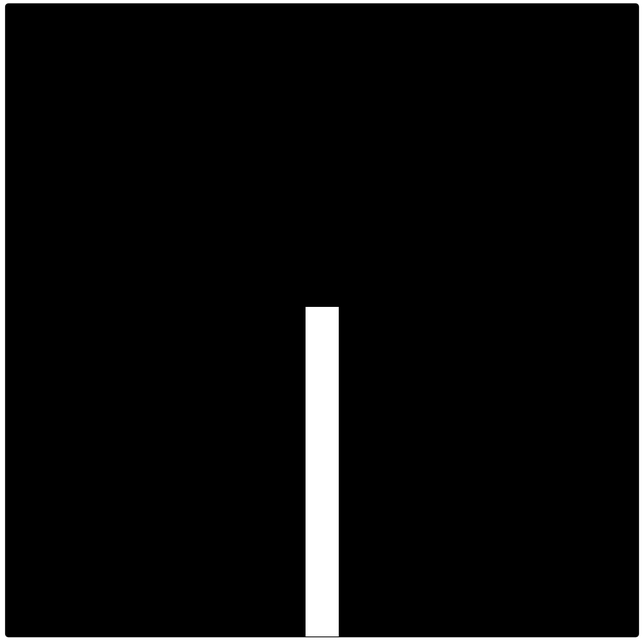


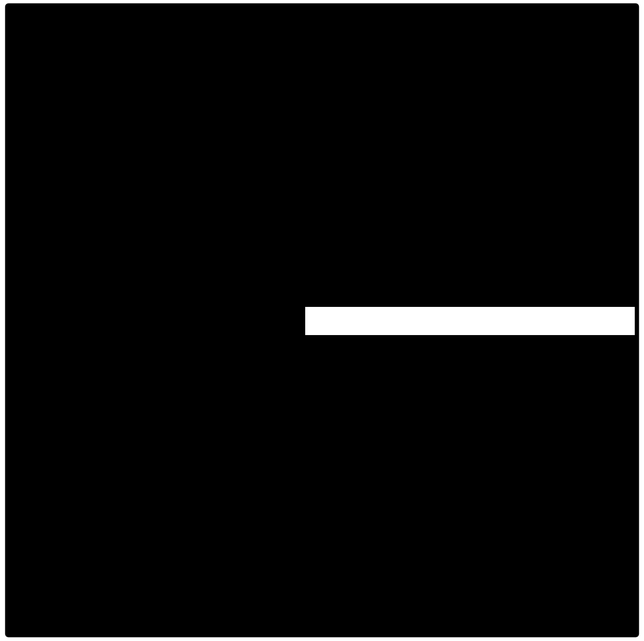
Local Image Patch

I_x



high value along vertical strip of pixels and 0 elsewhere

I_y



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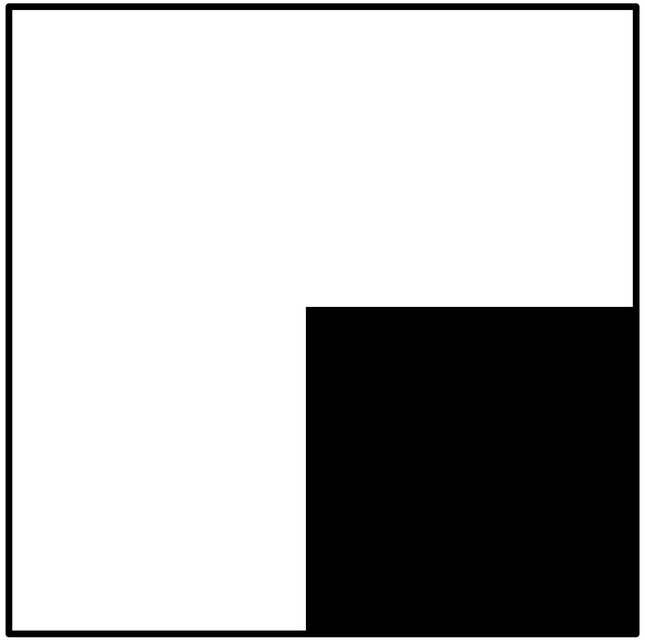

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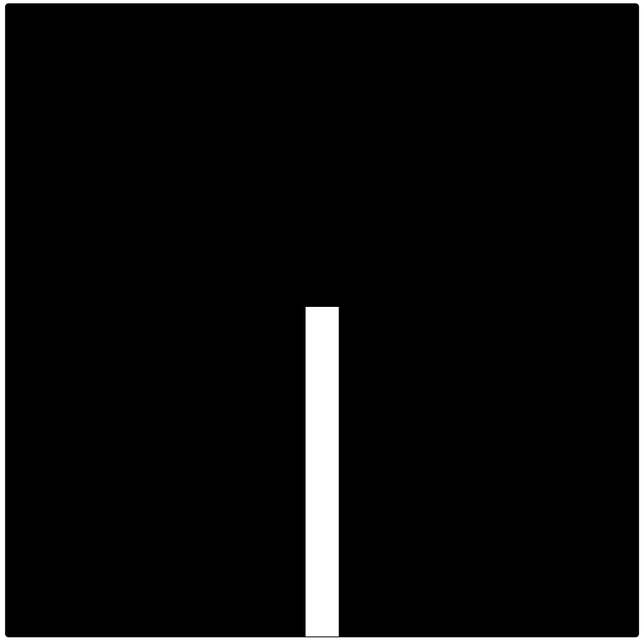

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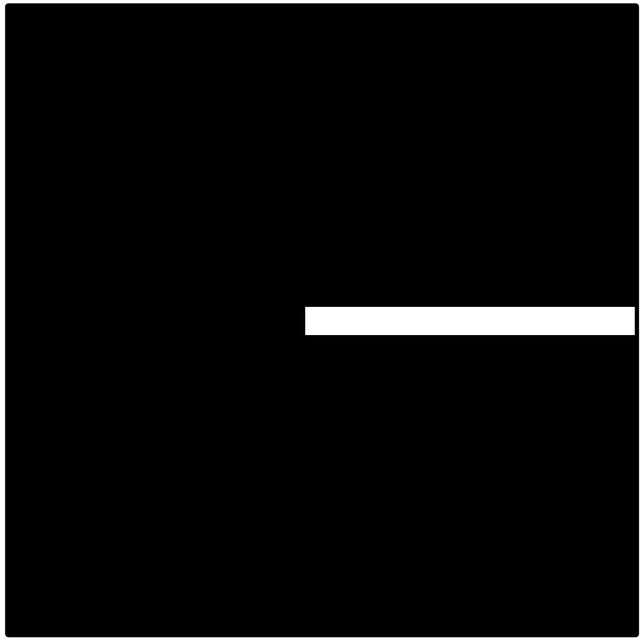

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General Case

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

General Case

It can be shown that since every C is symmetric:

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

… so general case is like a **rotated** version of the simple one

3. Computing **Eigenvalues** and **Eigenvectors**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Quick **Eigenvalue/Eigenvector** Review

Given a square matrix $\mathbf{A}$, a scalar λ is called an **eigenvalue** of $\mathbf{A}$ if there exists a nonzero vector $\mathbf{v}$ that satisfies

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$$

The vector $\mathbf{v}$ is called an **eigenvector** for $\mathbf{A}$ corresponding to the eigenvalue λ.

The eigenvalues of $\mathbf{A}$ are obtained by solving

$$\det(\mathbf{A} - \lambda I) = 0$$

3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

3. Computing **Eigenvalues** and **Eigenvectors**

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$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial) $C - \lambda I$

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3. Computing **Eigenvalues** and **Eigenvectors**

eigenvalue

$$Ce = \lambda e$$

eigenvector

$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial) $C - \lambda I$

2. Find the roots of polynomial (returns eigenvalues) $\det(C - \lambda I) = 0$

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3. Computing **Eigenvalues** and **Eigenvectors**

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2. Find the roots of polynomial (returns eigenvalues) $\det(C - \lambda I) = 0$

3. For each eigenvalue, solve (returns eigenvectors) $(C - \lambda I)e = 0$

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Example

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

1. Compute the determinant of (returns a polynomial)	$C - \lambda I$
2. Find the roots of polynomial (returns eigenvalues)	$\det(C - \lambda I) = 0$
3. For each eigenvalue, solve (returns eigenvectors)	$(C - \lambda I)e = 0$

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Example

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\right)$$

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$$(2-\lambda)(2-\lambda) - (1)(1) = 0$$
$$\lambda^2 - 4\lambda + 3 = 0$$
$$(\lambda - 3)(\lambda - 1) = 0$$
$$\lambda_1 = 1, \lambda_2 = 3$$

1. Compute the determinant of (returns a polynomial) $C - \lambda I$

2. Find the roots of polynomial (returns eigenvalues) $\det(C - \lambda I) = 0$

3. For each eigenvalue, solve (returns eigenvectors) $(C - \lambda I)e = 0$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Visualization as **Ellipse**

Since C is symmetric, we have $$C = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

We can visualize C as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

Ellipse equation:

$$f(x, y) = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \text{const}$$

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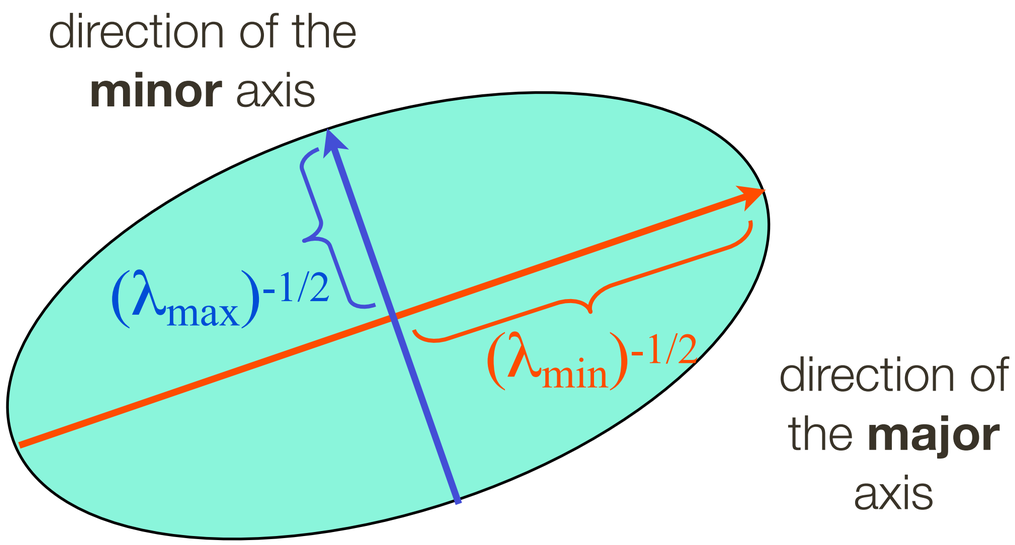

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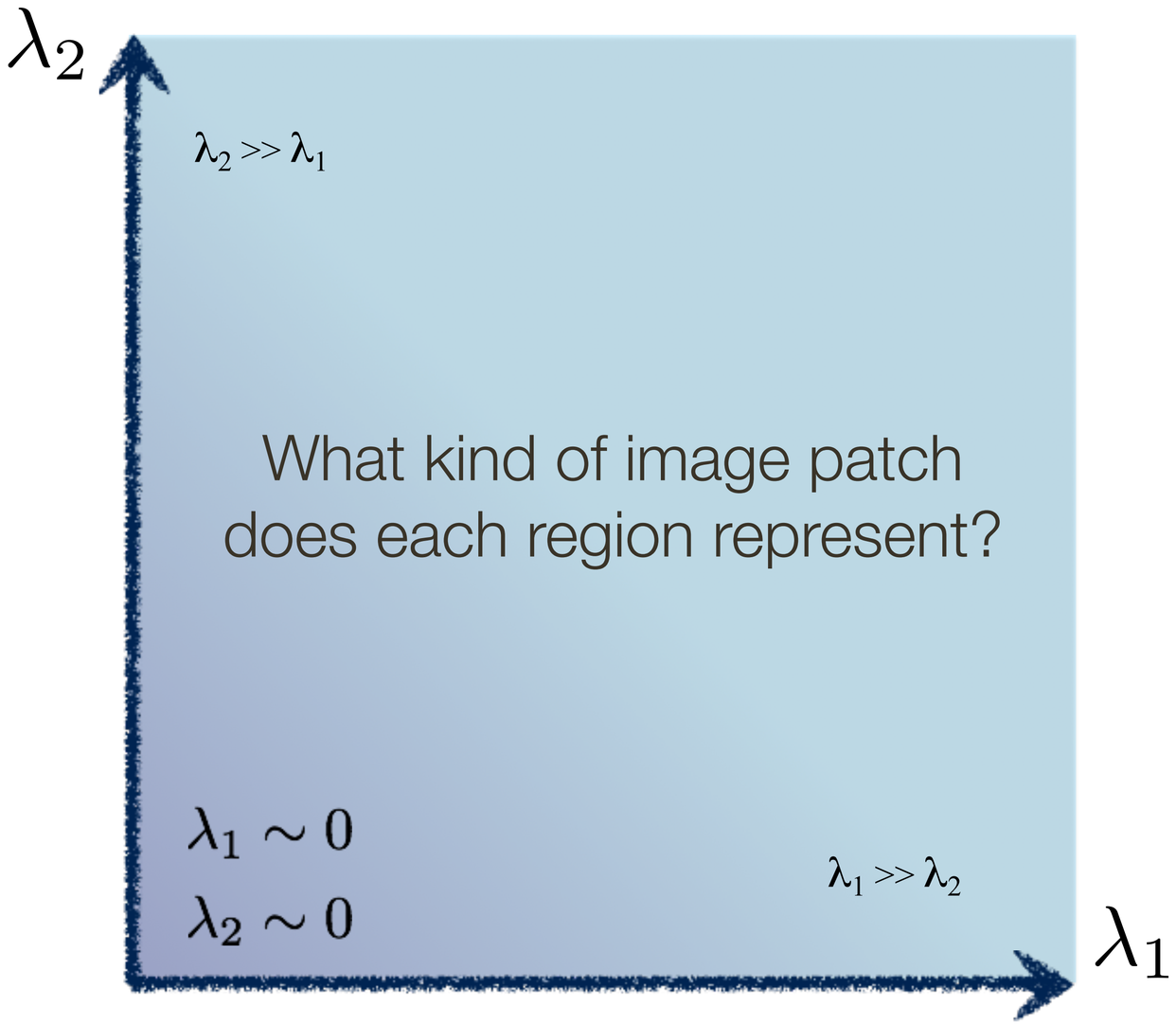
Interpreting **Eigenvalues**



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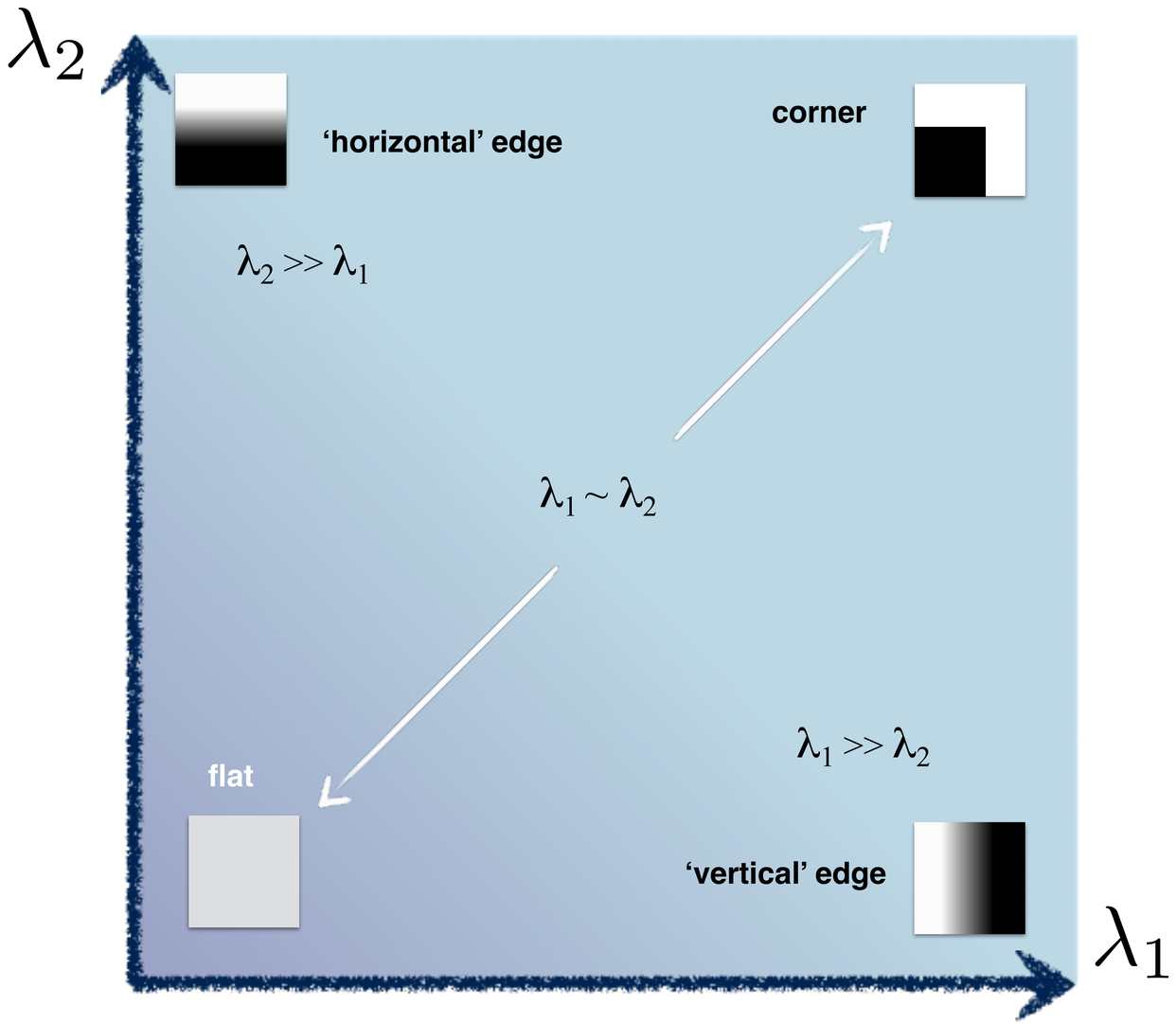

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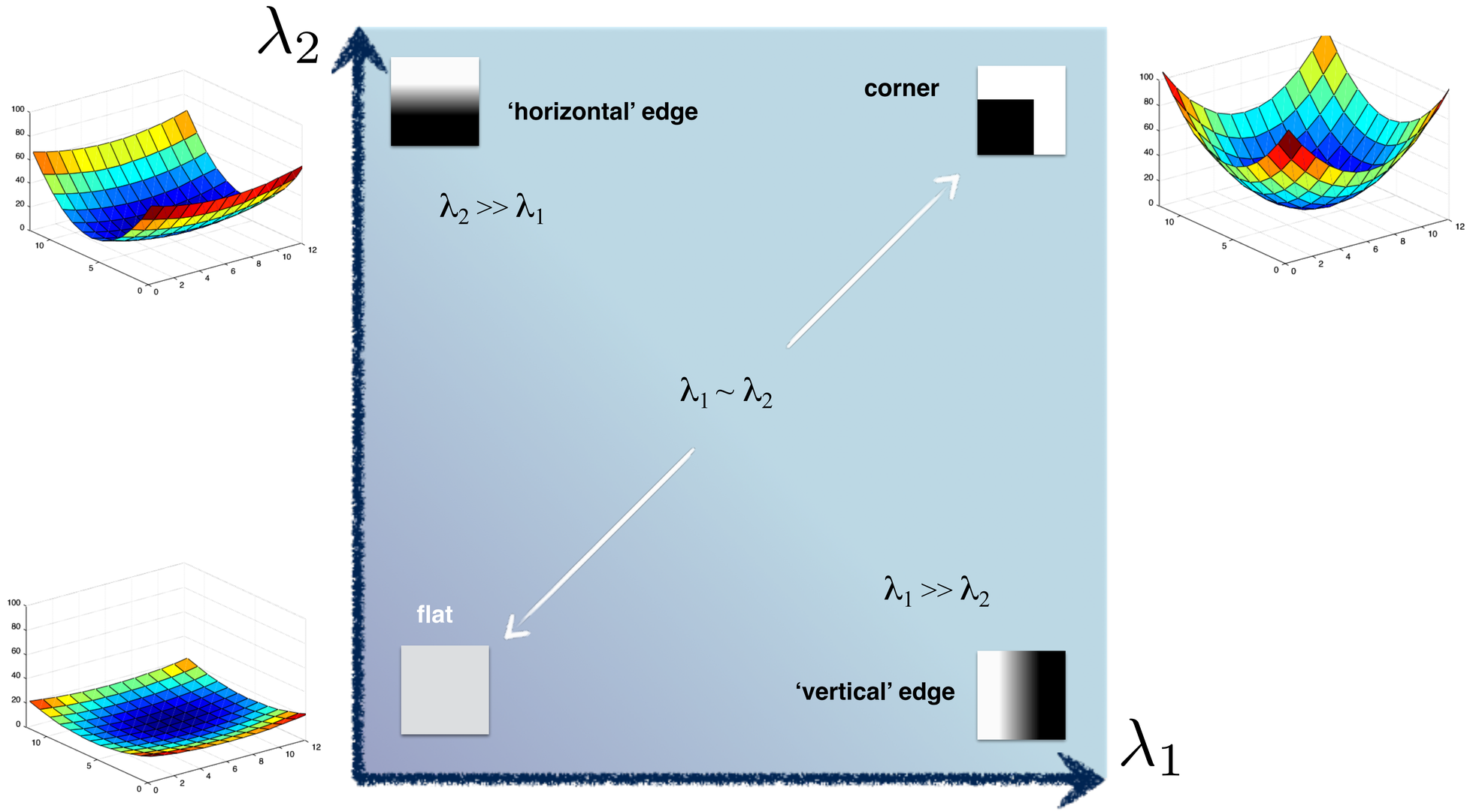

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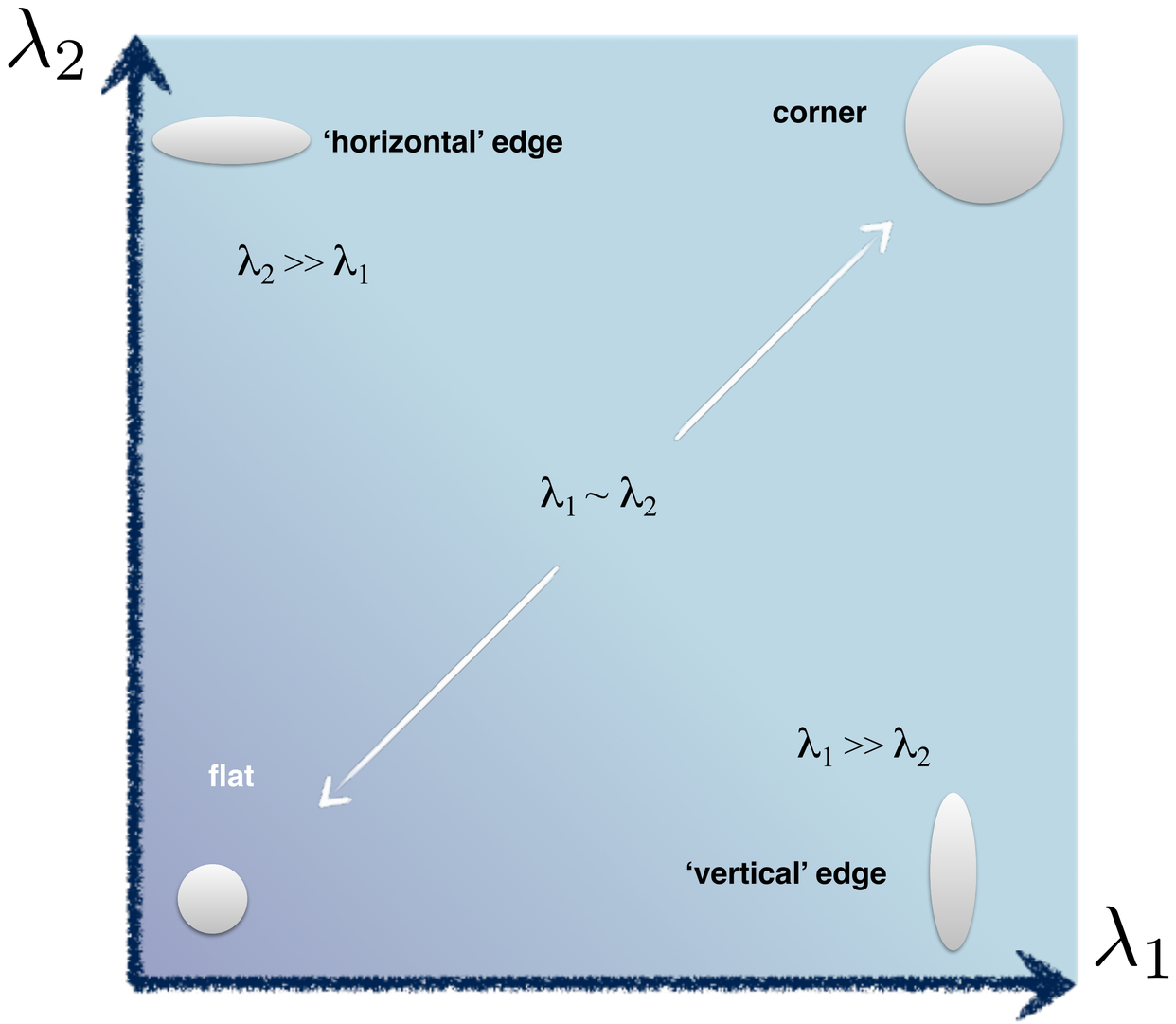

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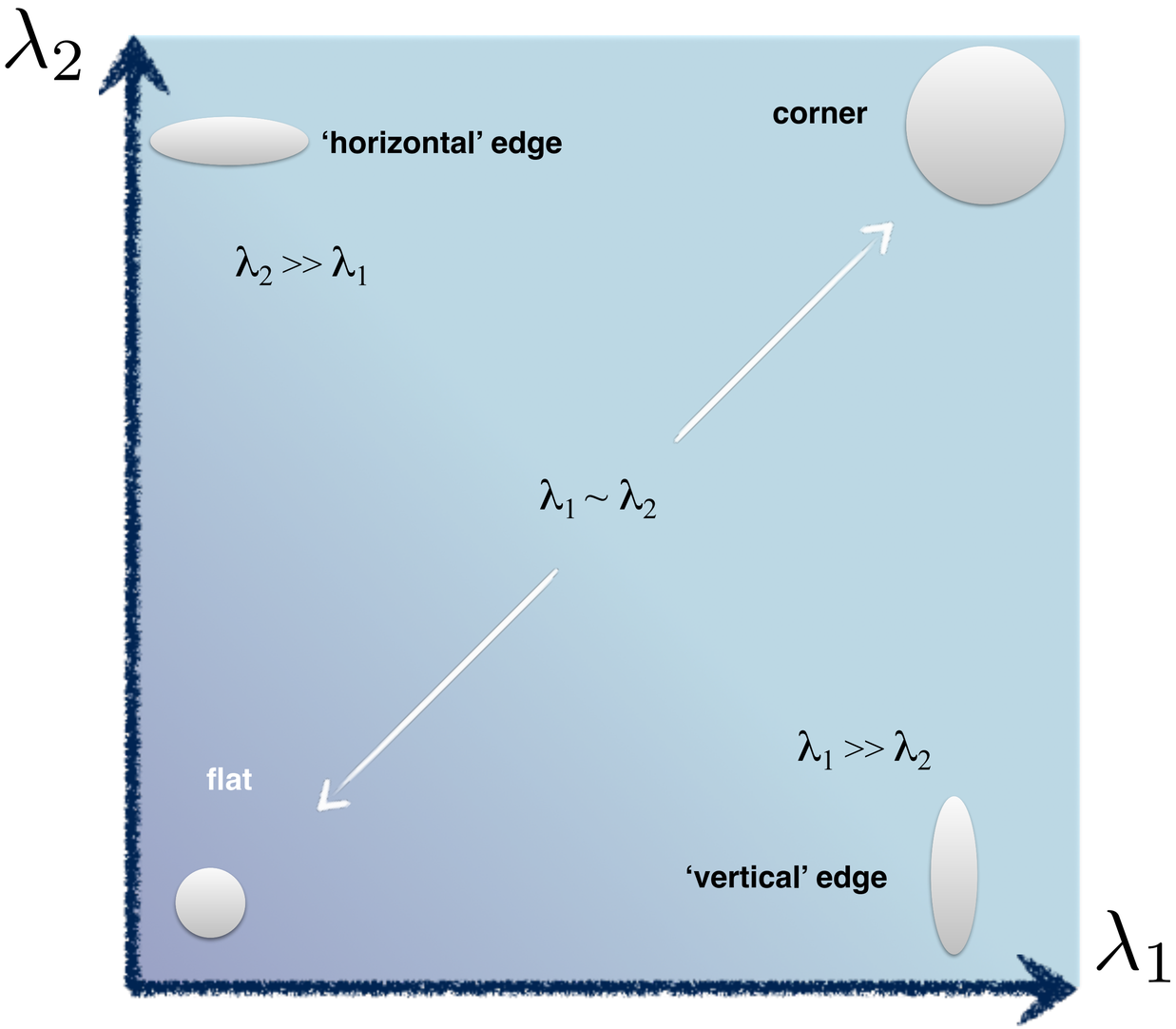


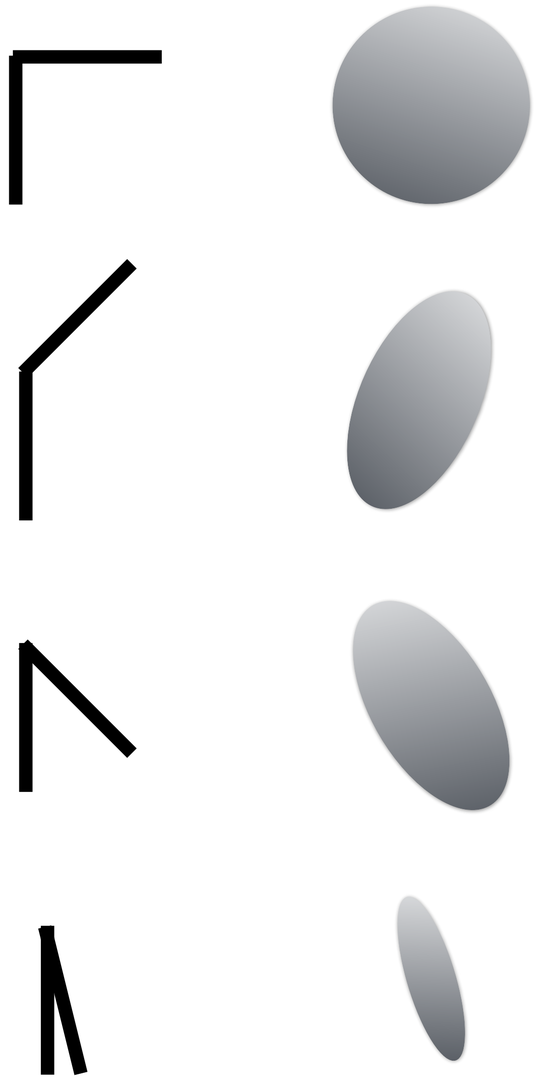


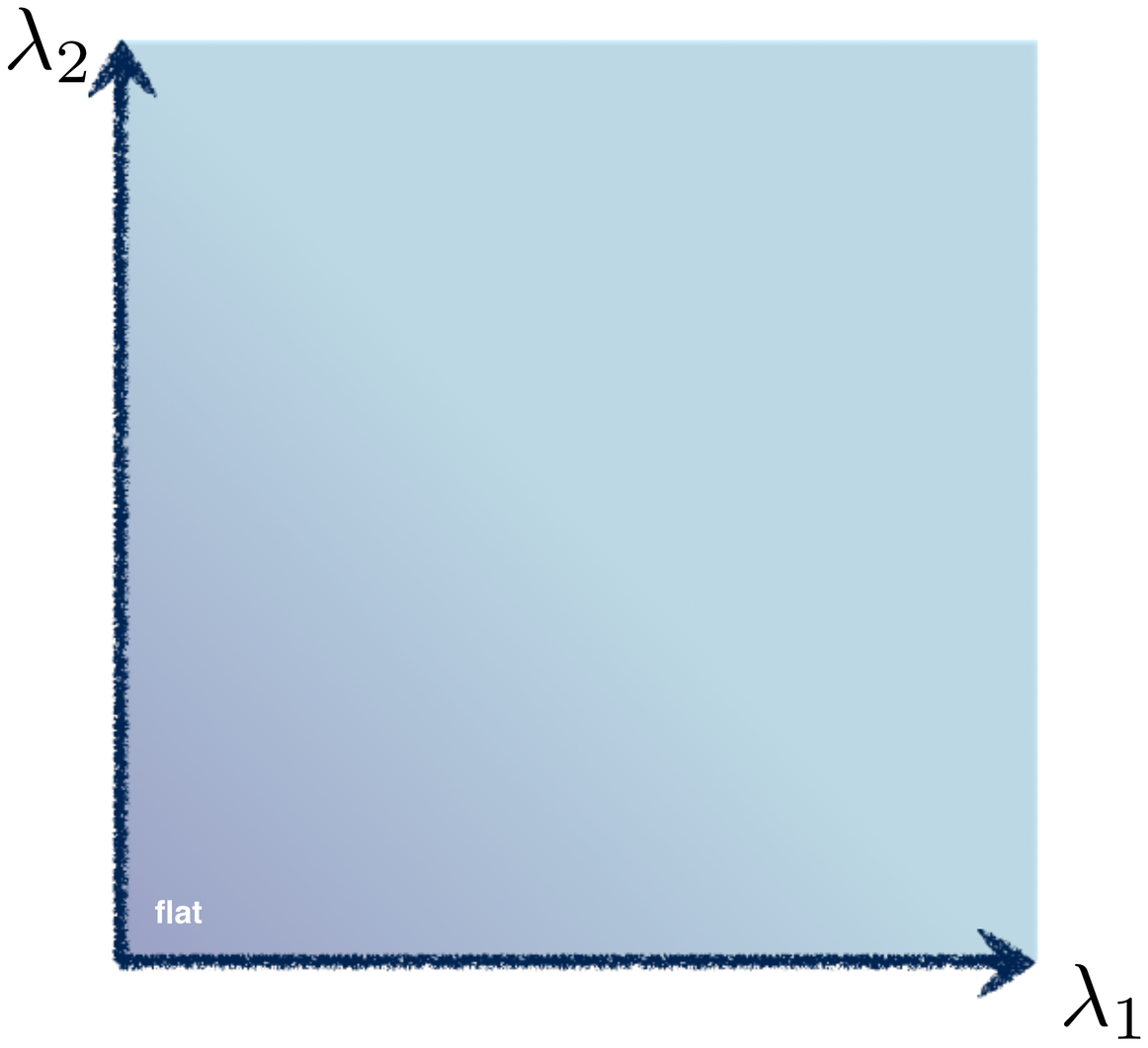
Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

4. **Threshold** on Eigenvalues to **Detect Corners**

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

4. **Threshold** on Eigenvalues to **Detect Corners**

(a function of)



Think of a function to score ‘cornerness’

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

4. **Threshold** on Eigenvalues to **Detect Corners**

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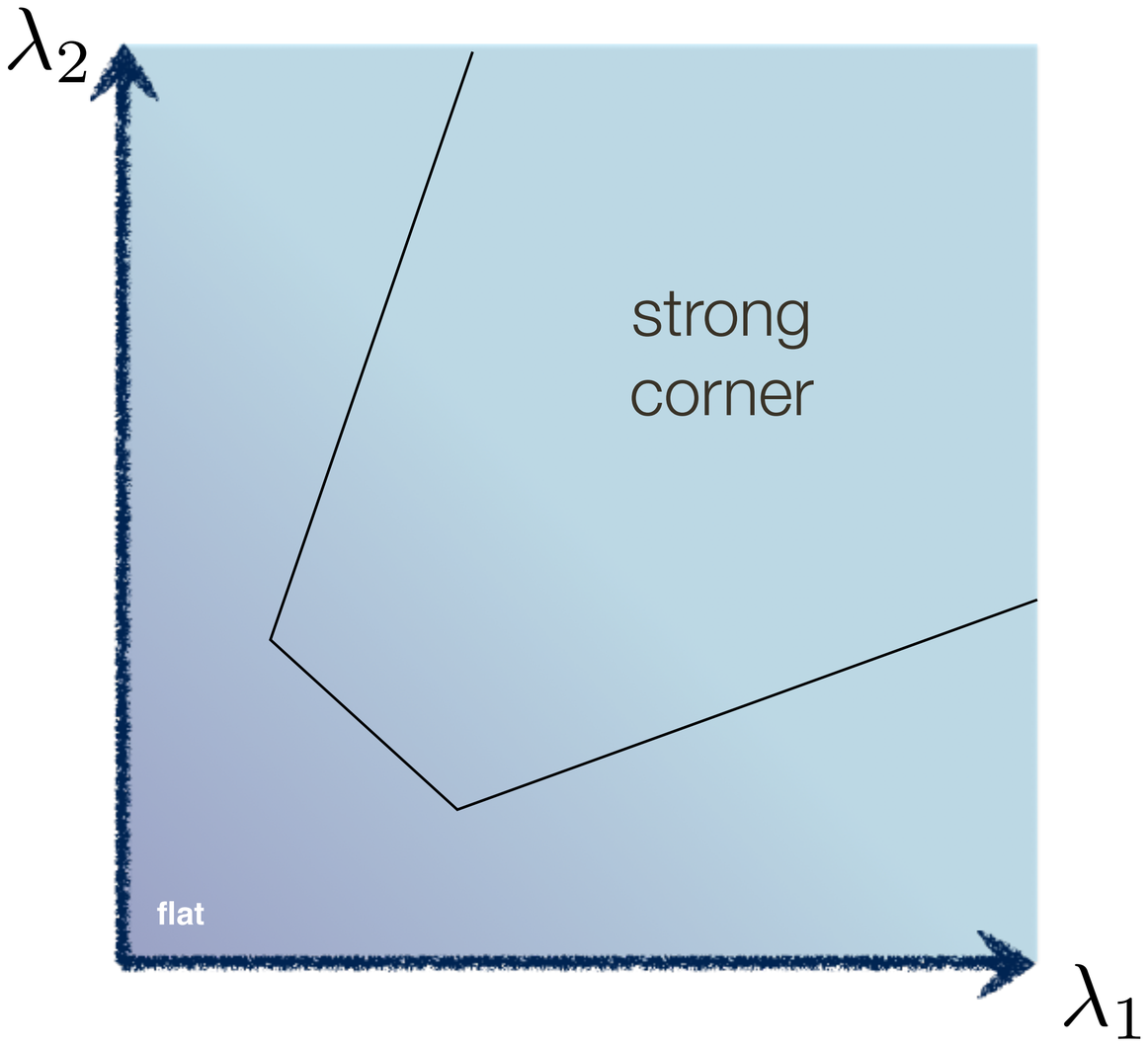

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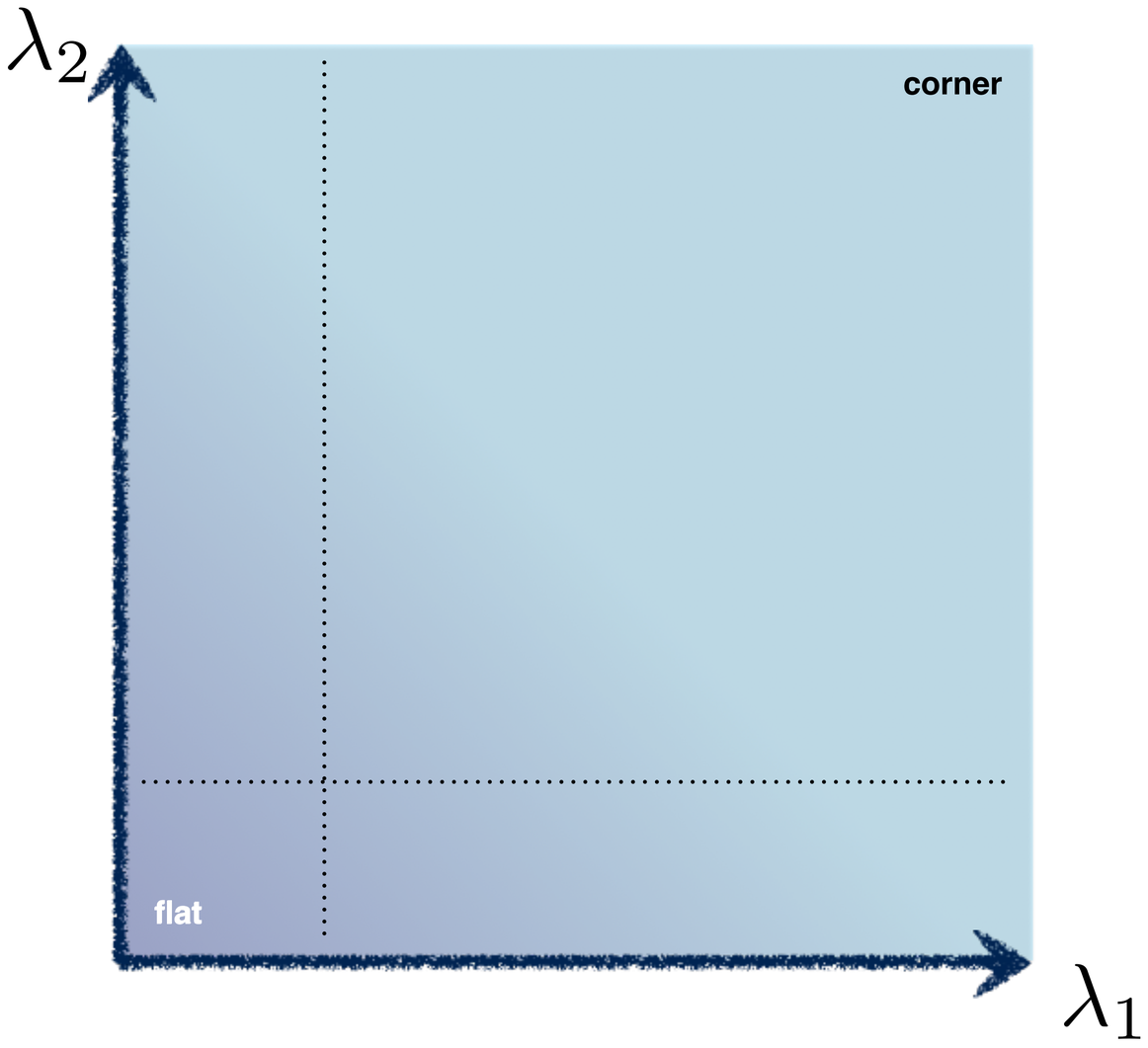

Use the **smallest eigenvalue** as the response function

$$\min(\lambda_1, \lambda_2)$$

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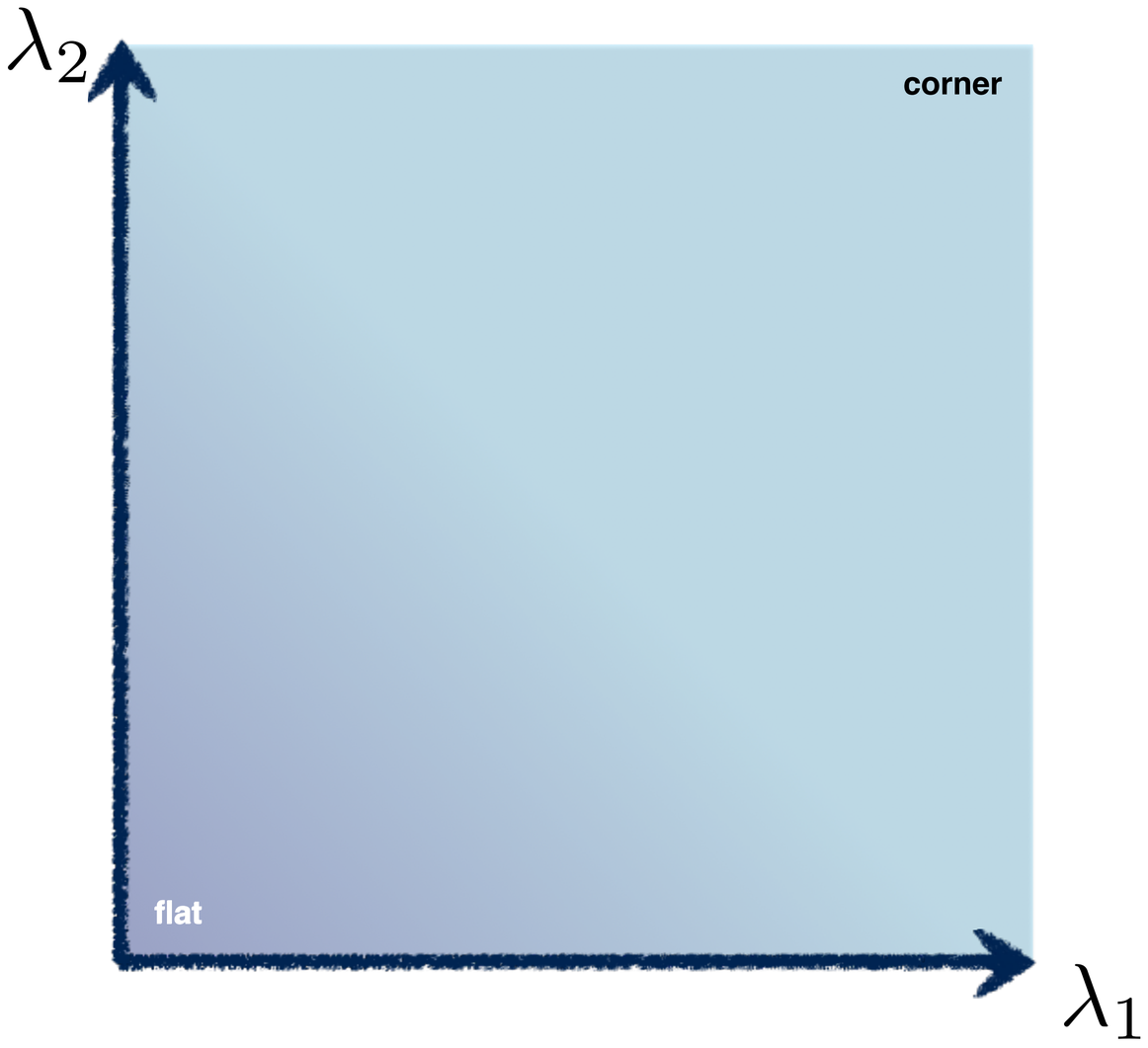
4. **Threshold** on Eigenvalues to **Detect Corners**

(a function of)



$$\lambda_1\lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

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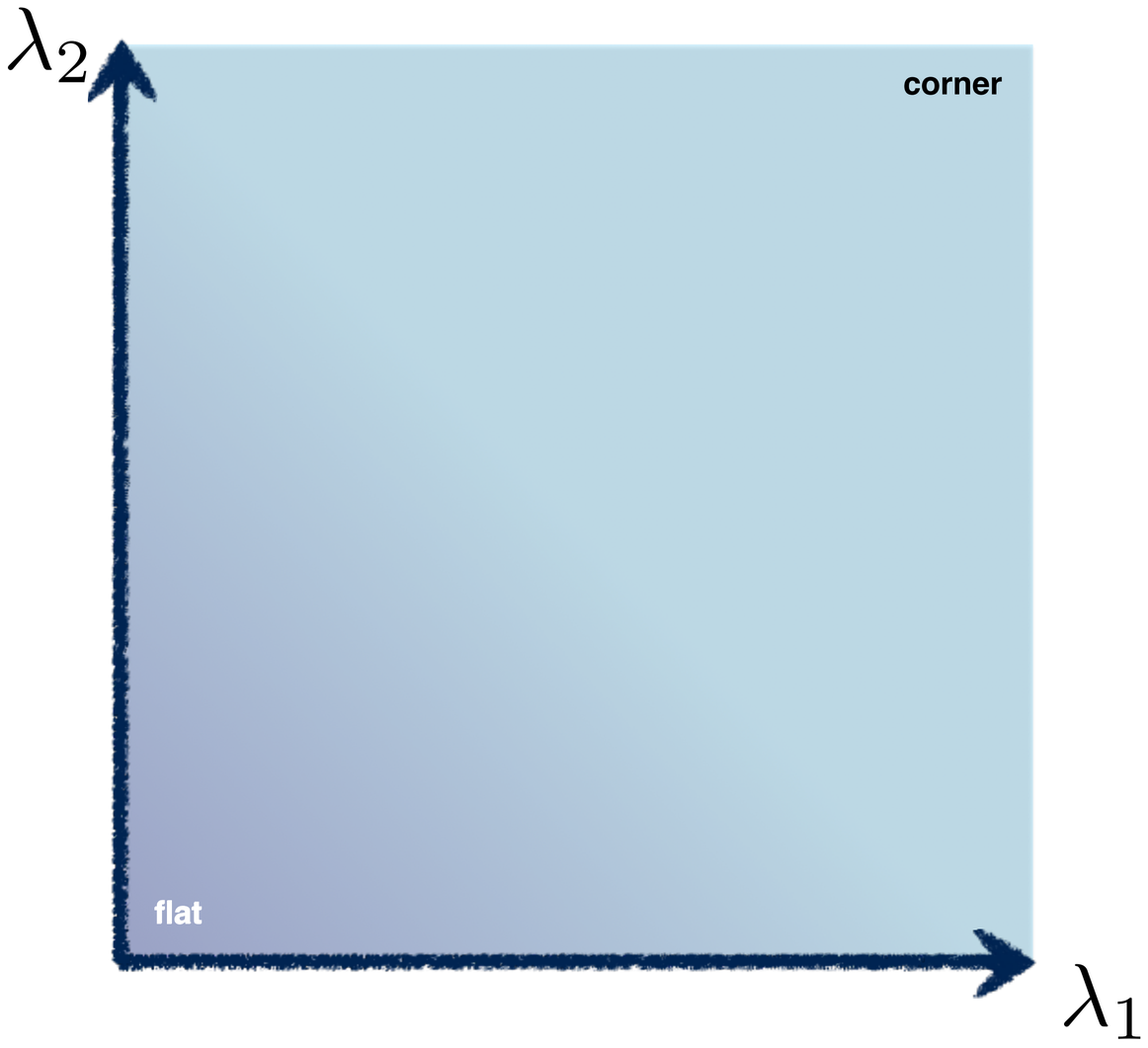
4. **Threshold** on Eigenvalues to **Detect Corners**

(a function of)



$$\lambda_1\lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

$$=$$

$$\det(C) - \kappa \mathrm{trace}^2(C)$$

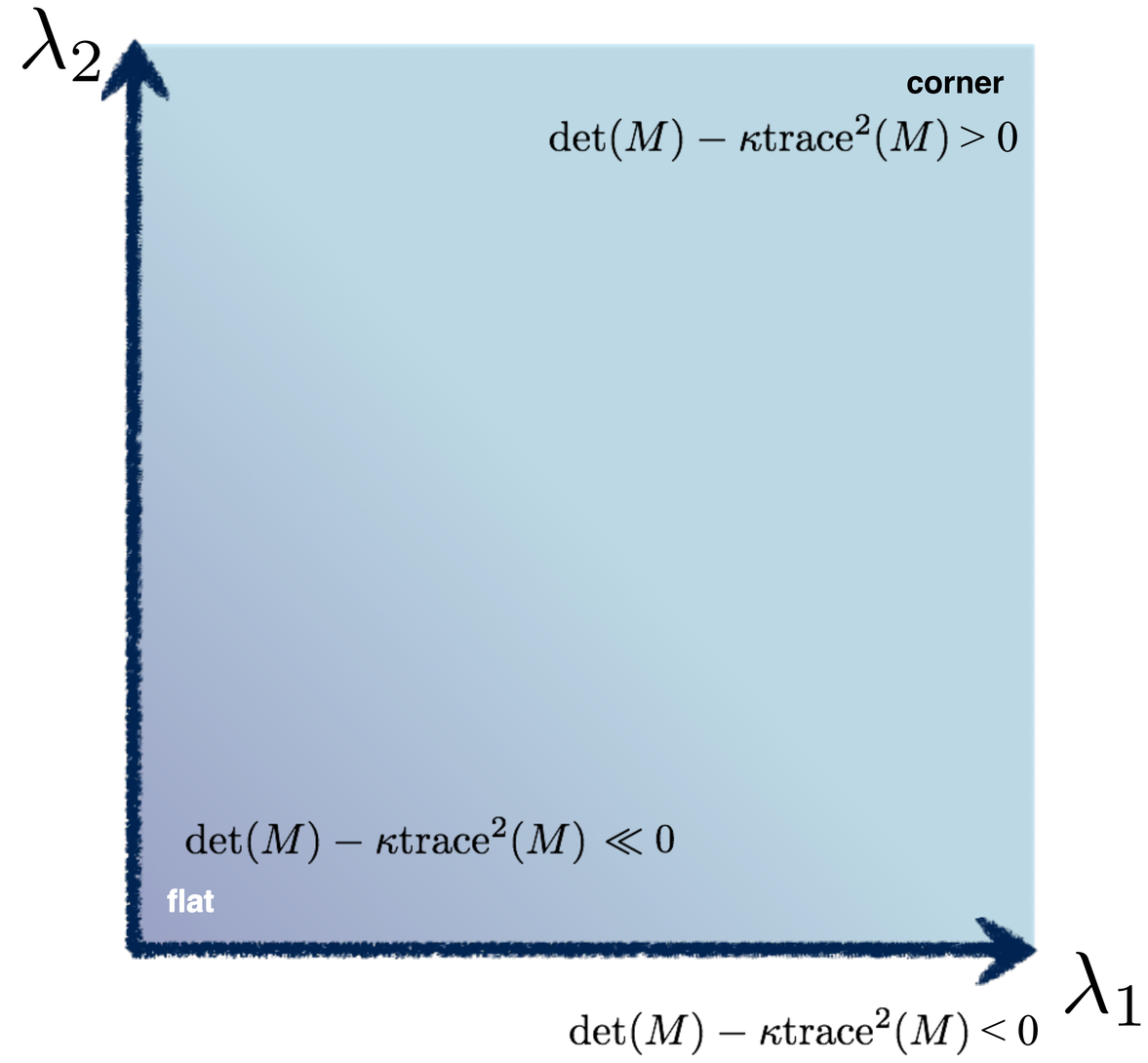
(more efficient)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

4. **Threshold** on Eigenvalues to **Detect Corners**

(a function of)

$\det(M) - \kappa \text{trace}^2(M) < 0$



$$\lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2$$

$$=$$

$$\det(C) - \kappa \text{trace}^2(C)$$

(more efficient)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

4. **Threshold** on Eigenvalues to **Detect Corners**

(a function of)

Harris & Stephens (1988)

$$\det(C) - \kappa \mathrm{trace}^2(C)$$

Kanade & Tomasi (1994)

$$\min(\lambda_1, \lambda_2)$$

Nobel (1998)

$$\frac{\det(C)}{\mathrm{trace}(C) + \epsilon}$$

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Harris Corner Detection Review

- Filter image with **Gaussian**
- Compute magnitude of the x and y **gradients** at each pixel
- Construct C in a window around each pixel
  - Harris uses a **Gaussian window**
- Solve for product of the λ's
- If λ's both are big (product reaches local maximum above threshold) then we have a corner
  - Harris also checks that ratio of λs is not too high

Compute the **Covariance Matrix**

Sum can be implemented as an (unnormalized) box filter with

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Harris uses a **Gaussian** weighting instead

Compute the **Covariance Matrix**

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Error function — Window function — Shifted intensity — Intensity

Sum can be implemented as an (unnormalized) box filter with

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Harris uses a **Gaussian** weighting instead

(has to do with bilinear Taylor expansion of 2D function that measures change of intensity for small shifts … remember AutoCorrelation)

Harris Corner Detection Review

— Filter image with **Gaussian**

— Compute magnitude of the x and y **gradients** at each pixel

— Construct C in a window around each pixel

  — Harris uses a **Gaussian window**

— Solve for product of the λ's

Harris & Stephens (1988)

$$\det(C) - \kappa \mathrm{trace}^2(C)$$

— If λ's both are big (product reaches local maximum above threshold) then we have a corner

  — Harris also checks that ratio of λs is not too high

Example: Harris Corner Detection

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\sum \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = 3$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix}$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0
-1	1	0	0	-1	1
-1	0	0	0	1	0
-1	0	0	0	1	0
0	-1	0	0	1	0
0	-1	0	0	1	0
0	-1	0	0	1	0
0	-1	0	0	1	0

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 2 \\ 2 & 4 \end{bmatrix} => \lambda_1 = 1.4384; \lambda_2 = 5.5616$$

$$\det(\mathbf{C}) - 0.04\mathrm{trace}^2(\mathbf{C}) = 6.04$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 0$$

$$\det(\mathbf{C}) - 0.04\mathrm{trace}^2(\mathbf{C}) = -0.36$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Example: Harris Corner Detection

Lets compute a measure of “corner-ness” for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

$$\mathbf{C} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} => \lambda_1 = 3; \lambda_2 = 2$$

$$\det(\mathbf{C}) - 0.04\mathrm{trace}^2(\mathbf{C}) = 5$$

$$I_x = \frac{\partial I}{\partial x}$$

0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

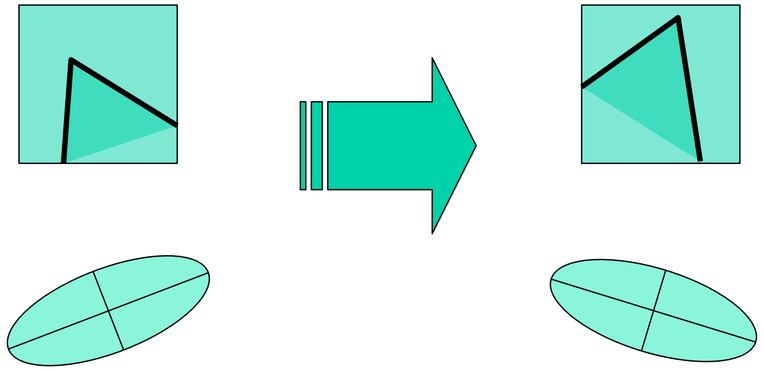
$$I_y = \frac{\partial I}{\partial y}$$

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Harris Corner Detection Review

- Filter image with **Gaussian**
- Compute magnitude of the x and y **gradients** at each pixel
- Construct C in a window around each pixel
  - Harris uses a **Gaussian window**
- Solve for product of the λ's
- If λ's both are big (product reaches local maximum above threshold) then we have a corner
  - Harris also checks that ratio of λs is not too high

Properties: Rotational Invariance



Ellipse rotates but its shape
(**eigenvalues**) remains the same

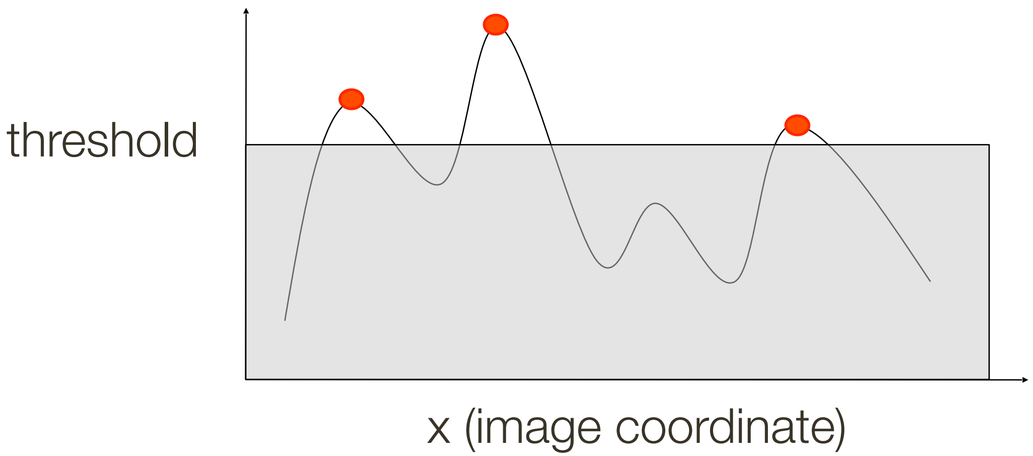
Corner response is **invariant** to image rotation

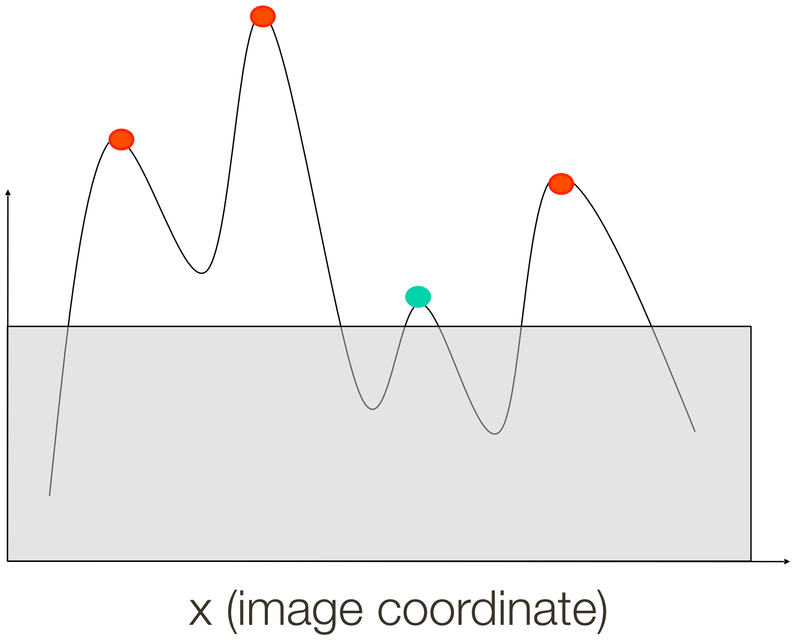
Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Properties: (partial) Invariance to Intensity Shifts and Scaling

Only derivatives are used -> Invariance to intensity shifts

Intensity scale could effect performance

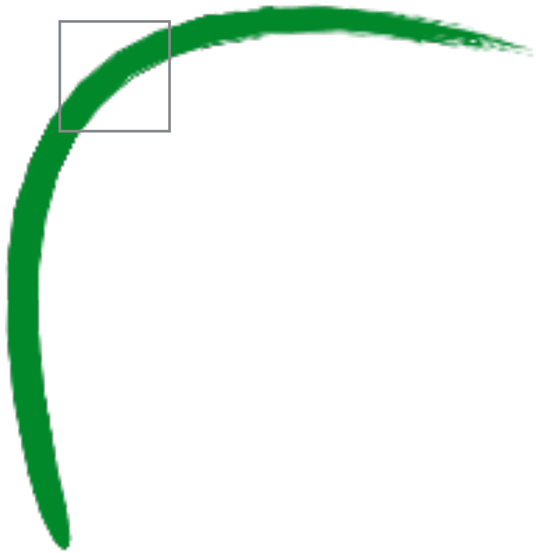




Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Properties: NOT Invariant to Scale Changes

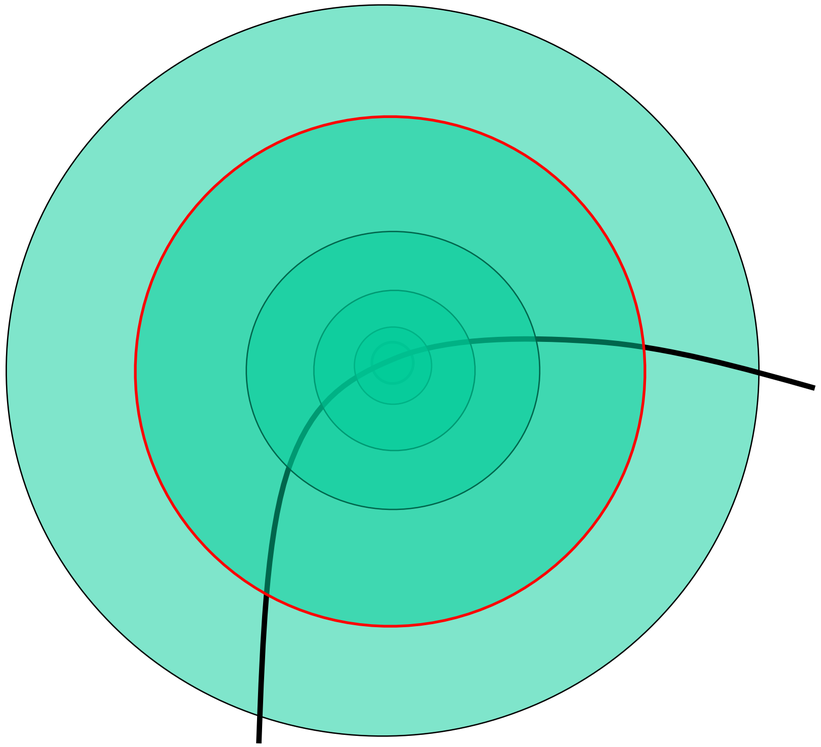
edge!

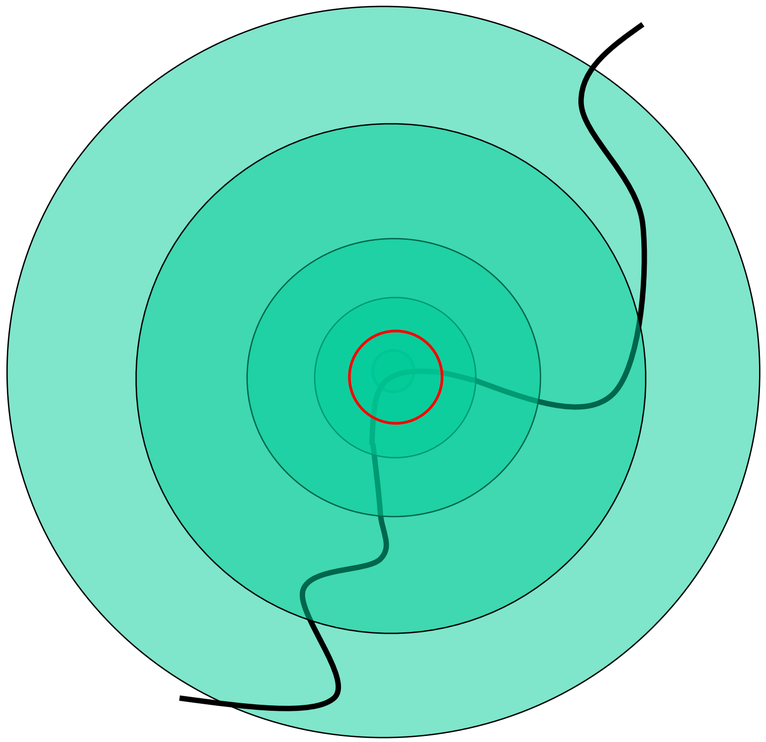


corner!

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Intuitively …

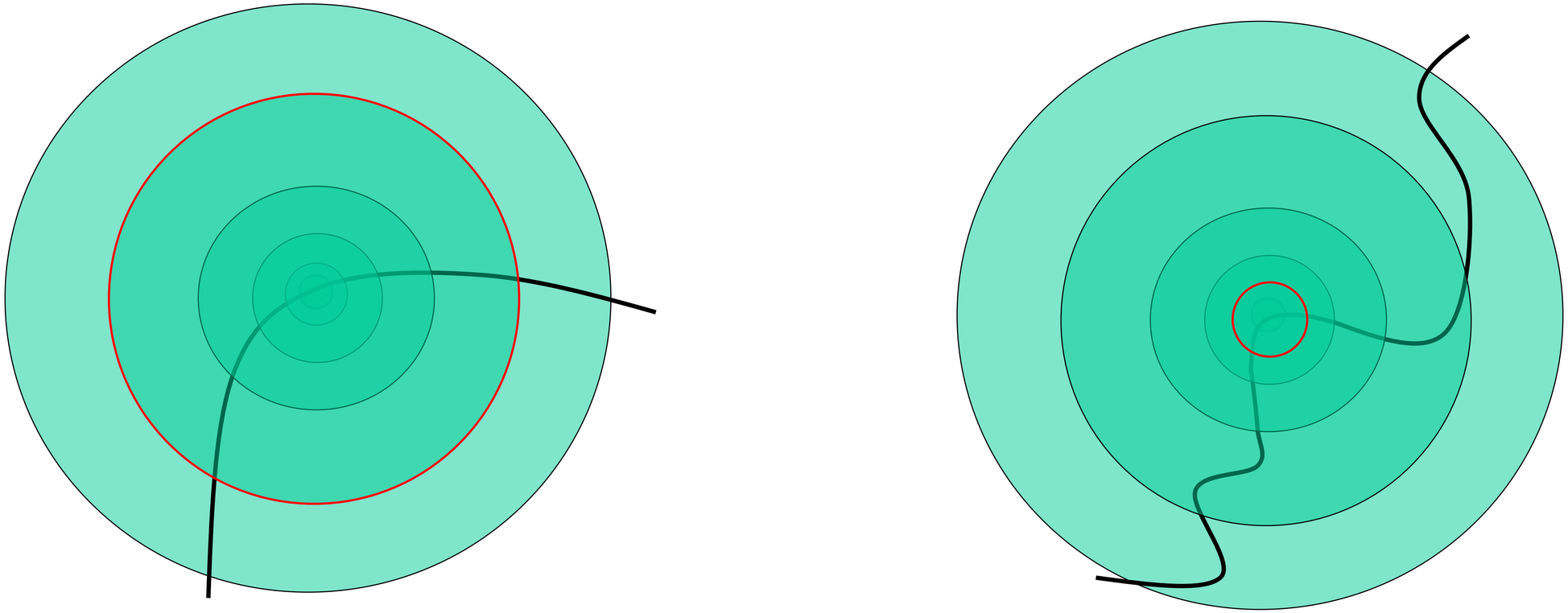




Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

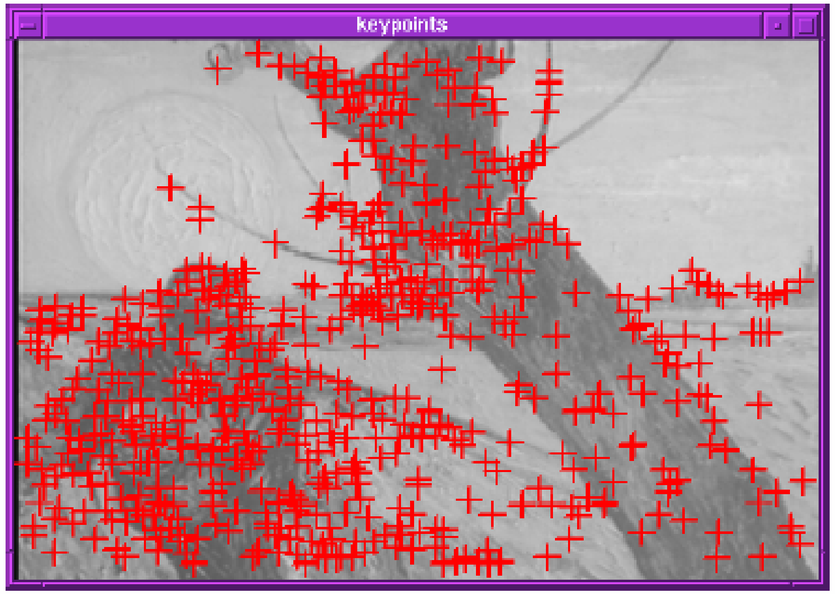
Intuitively …

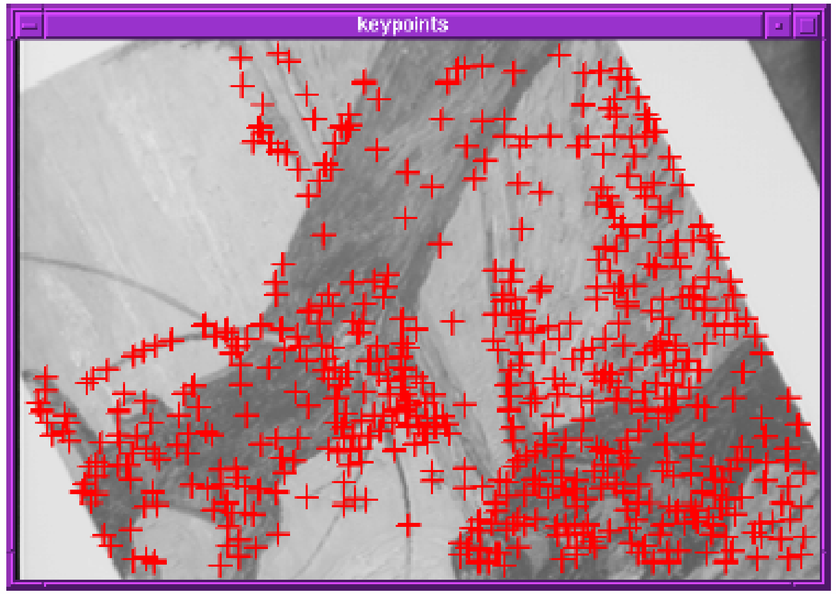
Find local maxima in both **position** and **scale**



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Example 1:





Example 2: Wagon Wheel (Harris Results)

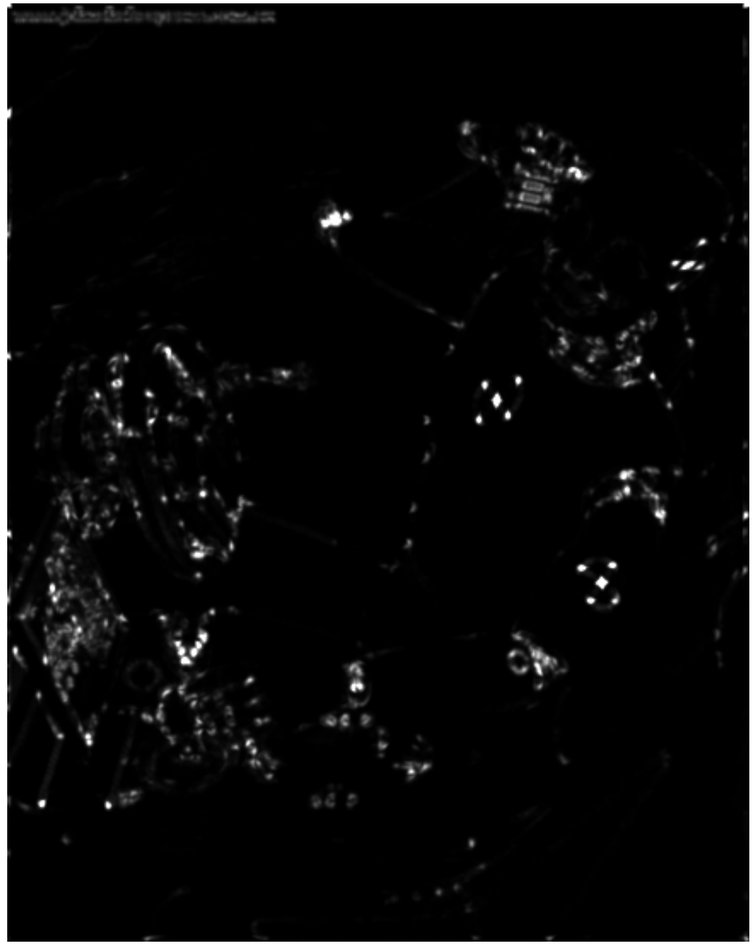
$\sigma = 1$ (219 points)

$\sigma = 2$ (155 points)

$\sigma = 3$ (110 points)

$\sigma = 4$ (87 points)

Example 3: Crash Test Dummy (Harris Result)



corner response image

$\sigma = 1$ (175 points)

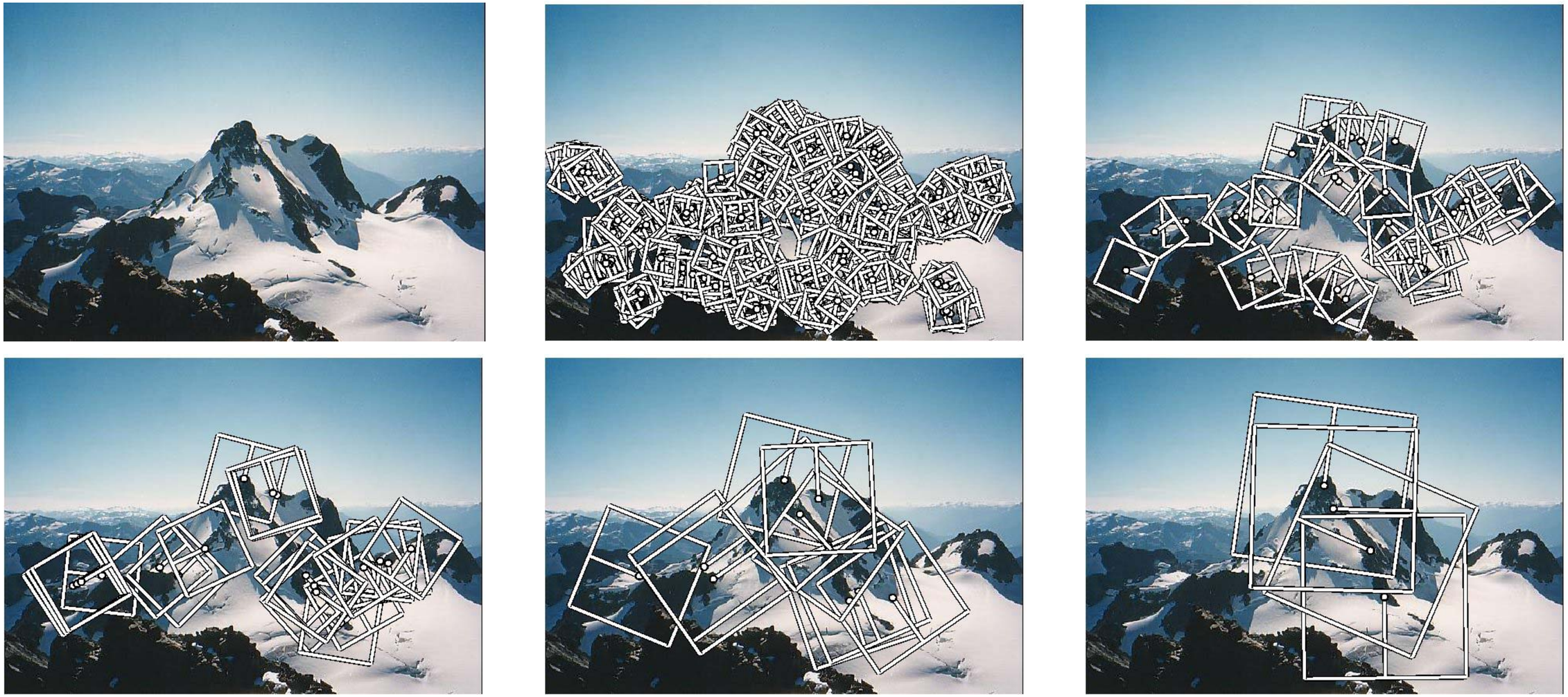
Original Image Credit: John Shakespeare, Sydney Morning Herald

Implementation

```
For each level of the Gaussian pyramid
compute feature response (e.g. Harris, Laplacian)

For each level of the Gaussian pyramid
    if local maximum and cross-scale
    save scale and location of feature
```
(x, y, s)

Multi-Scale Harris Corners



Summary Table

Summary of what we have seen so far:

Representation	Result is. . .	Approach	Technique
intensity	dense	template matching	(normalized) correlation
edge	relatively sparse	derivatives	$\nabla^2 G$, Canny
corner	sparse	locally distinct features	Harris

Summary

Edges are useful image features for many applications, but suffer from the aperture problem

Canny Edge detector combines edge filtering with linking and hysteresis steps

Corners / Interest Points have 2D structure and are useful for correspondence

Harris corners are minima of a local SSD function

DoG maxima can be reliably located in scale-space and are useful as interest points