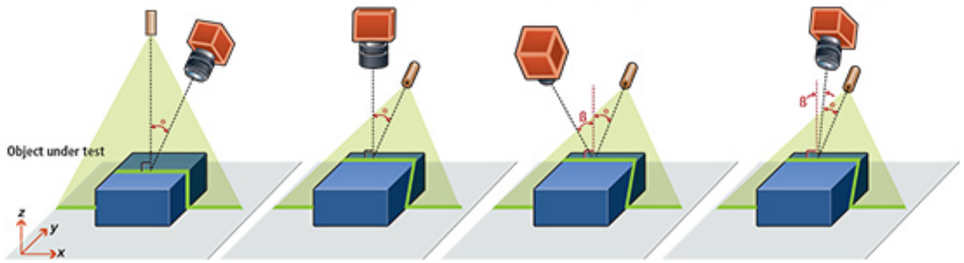
THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 2: Image Formation

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (**January 8, 2019**)

Updates:

— **Canvas** should now be working now

— TA office hours are out

— **Assignment 0** is out (optional)

— Registration

Reminders:

— **WWW**: assignments, lecture notes, readings

— **Piazza**: discussion (lecture notes and assignment will also be posted here)

— **Canvas**: assignment hand in and grading

Menu for Today (**January 8, 2019**)

Topics:

- Image Formation
- Cameras and Lenses
- Projection
- Human eye (as camera)

Redings:

- **Today's** Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3
- **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

- Complete **Assignment 0** (ungraded) by Friday, **January 11**
- **WWW**: http://www.cs.ubc.ca/~lsigal/teaching.html
- **Piazza**: piazza.com/ubc.ca/winterterm12018/cpsc425

Today's "fun" Example

Photo credit: reddit user Liammm

Today's "fun" Example: **Eye Sink Illusion**

Pareidolia

Photo credit: reddit user Liammm

Today's "fun" Example: **Eye Sink Illusion**

"Tried taking a picture of a sink draining, wound up with a picture of an eye instead"

Photo credit: reddit user Liammm

Lecture 1: Re-cap

Types of computer vision **problems**:

— Computing properties of the 3D world from visual data (***measurement***)
— Recognition of objects and scenes (***perception and interpretation***)
— Search and interact with visual data (***search and organization***)
— Manipulation or creation of image or video content (***visual imagination***)

Computer vision **challenges**:

— Fundamentally **ill-posed**
— Enormous **computation** and **scale**
— Lack of fundamental understanding of how **human perception** works

Lecture 1: Re-cap

Computer vision technologies have moved **from research labs into commercial products and services**. Examples cited include:

— broadcast television sports
— electronic games (Microsoft Kinect)
— biometrics
— image search
— visual special effects
— medical imaging
— robotics

… many others

Lecture 2: Goal

To understand how images are formed

Overview: Image Formation, Cameras and Lenses

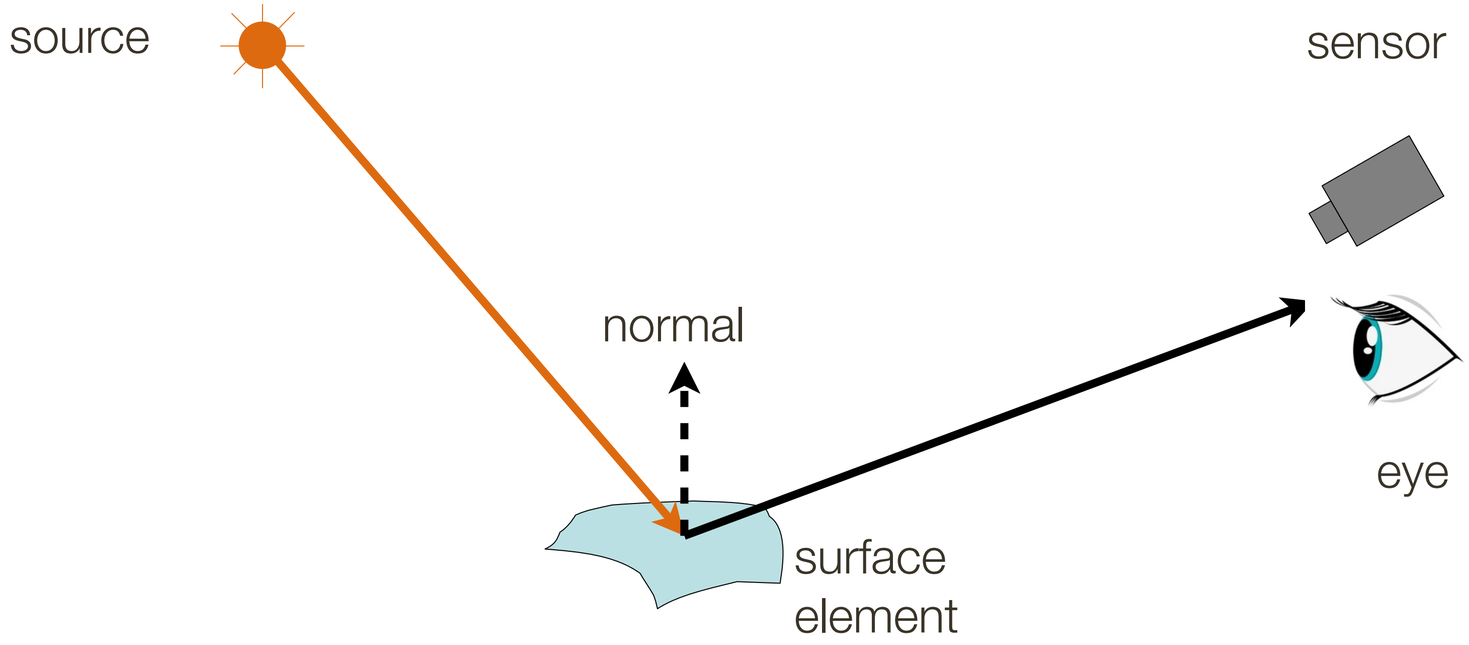
The **image formation process** that produces a particular image depends on

— **Lightening** condition

— Scene **geometry**

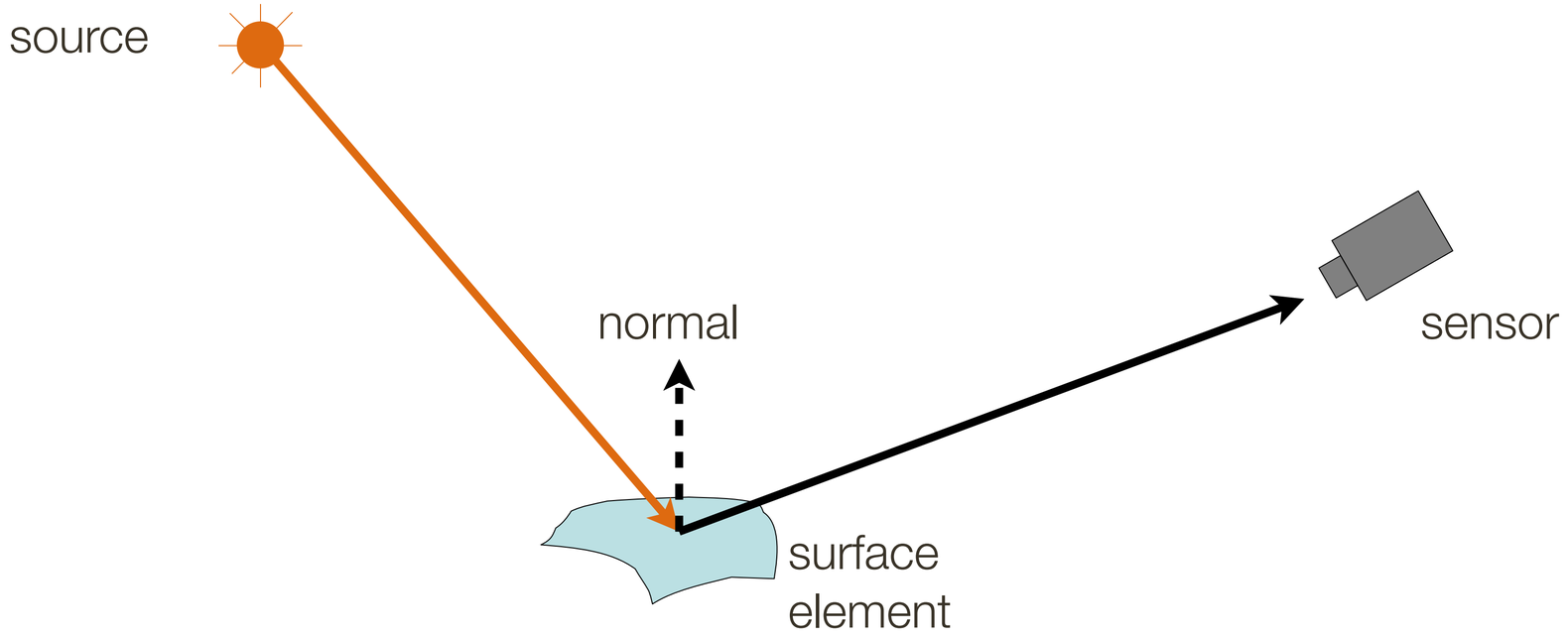
— **Surface** properties

— Camera **optics**



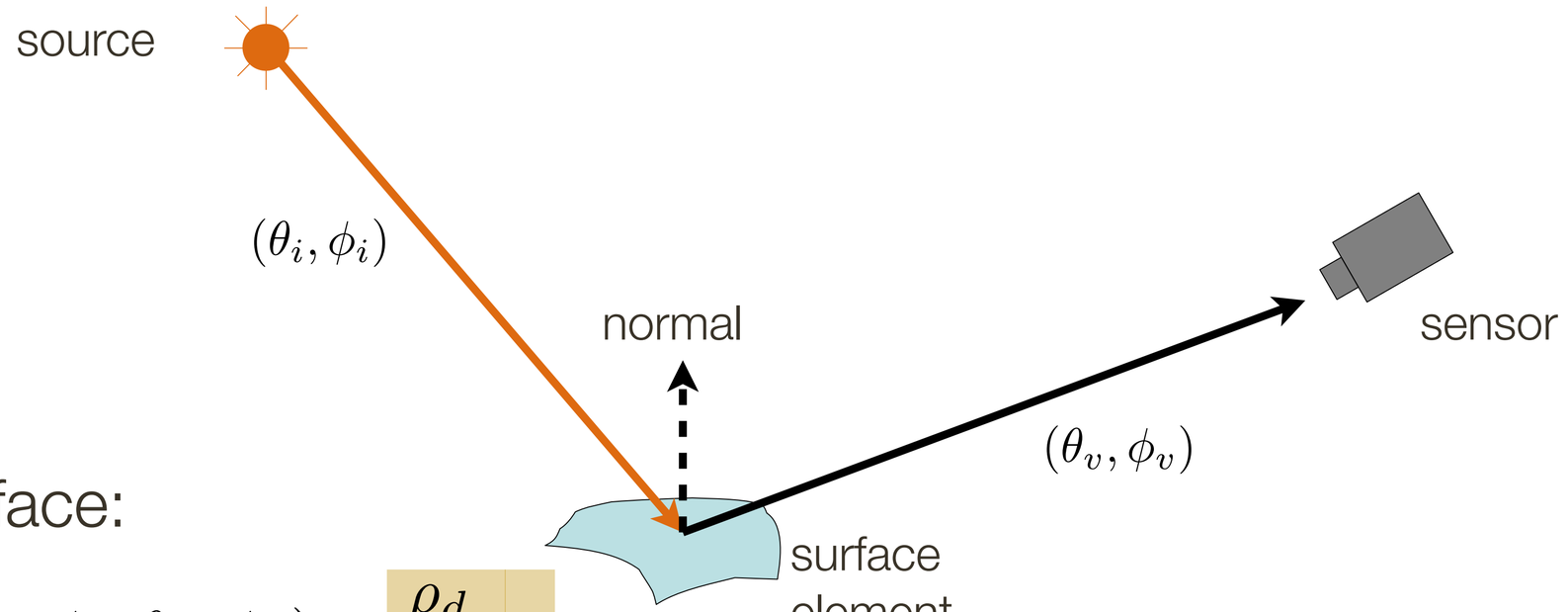
Sensor (or eye) **captures amount of light** reflected from the object

(small) **Graphics** Review



(small) **Graphics** Review

Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: $\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)$



Lambertian surface:

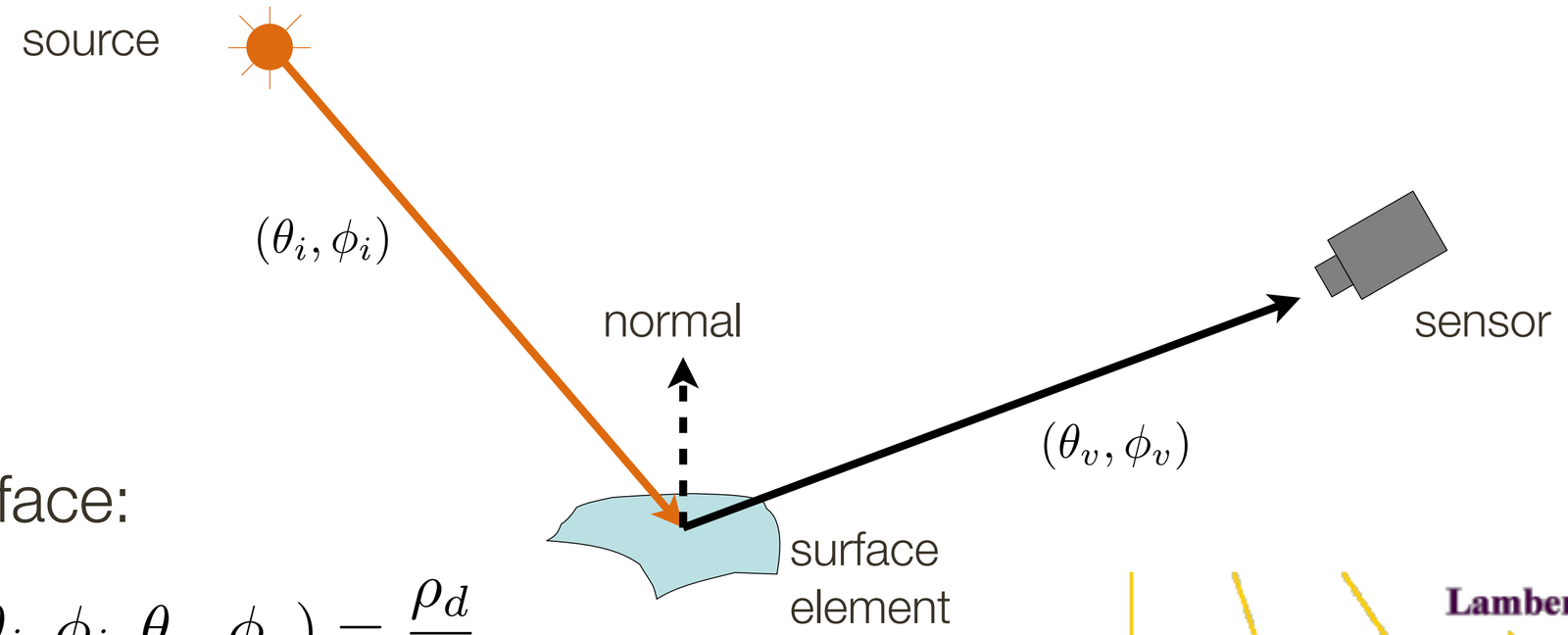
$$\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

constant, called **albedo**

Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)

(small) **Graphics** Review

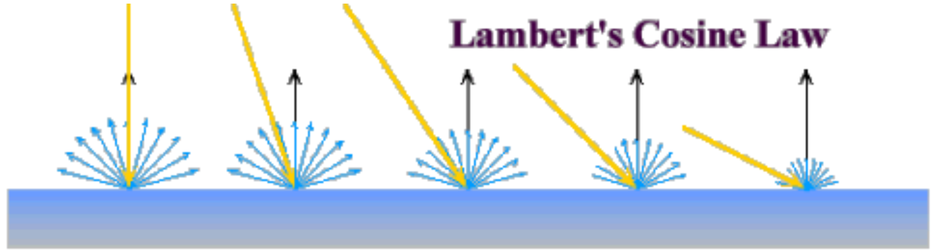
Surface reflection depends on both the **viewing** (θ_v, ϕ_v) and **illumination** (θ_i, ϕ_i) direction, with Bidirectional Reflection Distribution Function: $\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)$



Lambertian surface:

$$\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

$$L = \frac{\rho_d}{\pi} I(\vec{i} \cdot \vec{n})$$



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(small) **Graphics** Review

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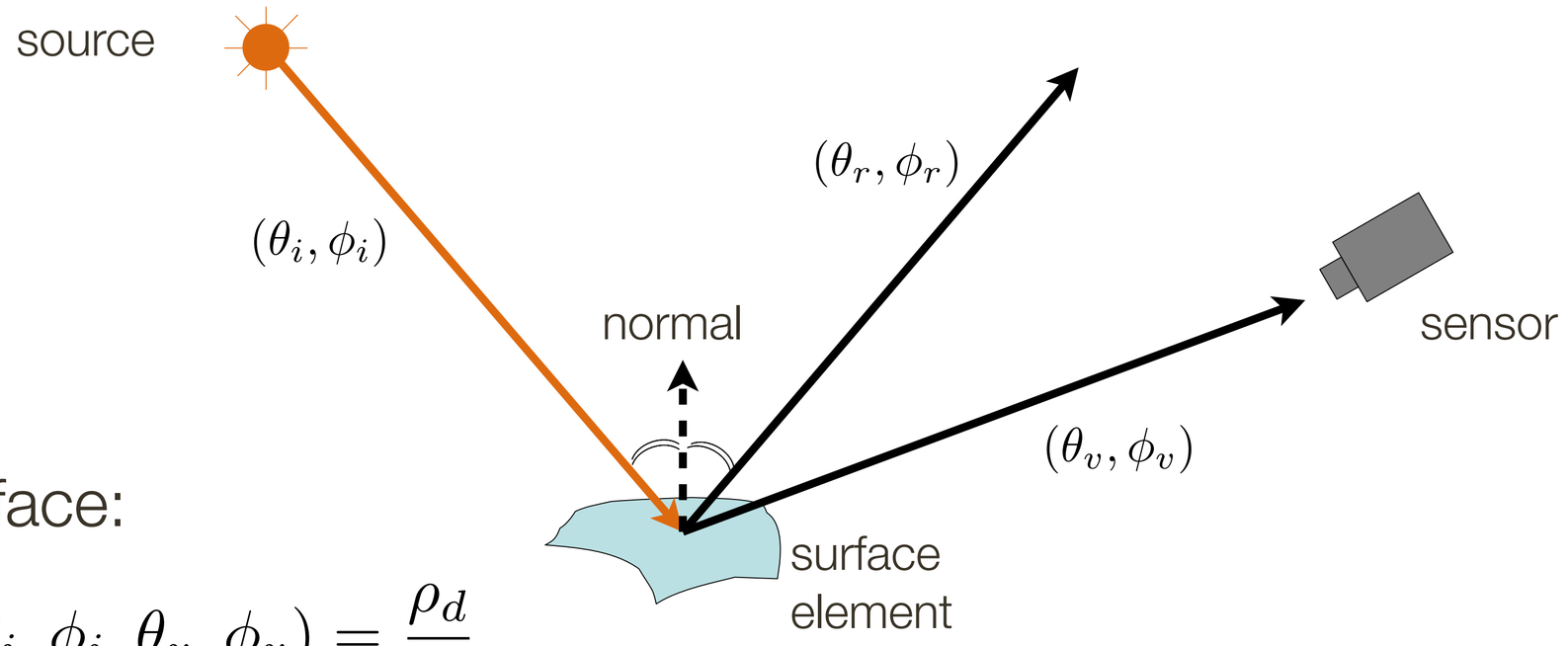


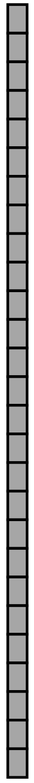
Lambertian surface:

$$\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

Mirror surface: all incident light reflected in one directions $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

Slide adopted from: Ioannis (Yannis) Gkioulekas (CMU)

Let’s say we have a **sensor …**



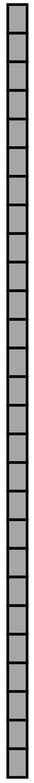
digital sensor (CCD or CMOS)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

… and the **object** we would like to photograph

What would an image taken like this look like?

real-world object



digital sensor (CCD or CMOS)

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Bare-sensor imaging

real-world object

digital sensor (CCD or CMOS)

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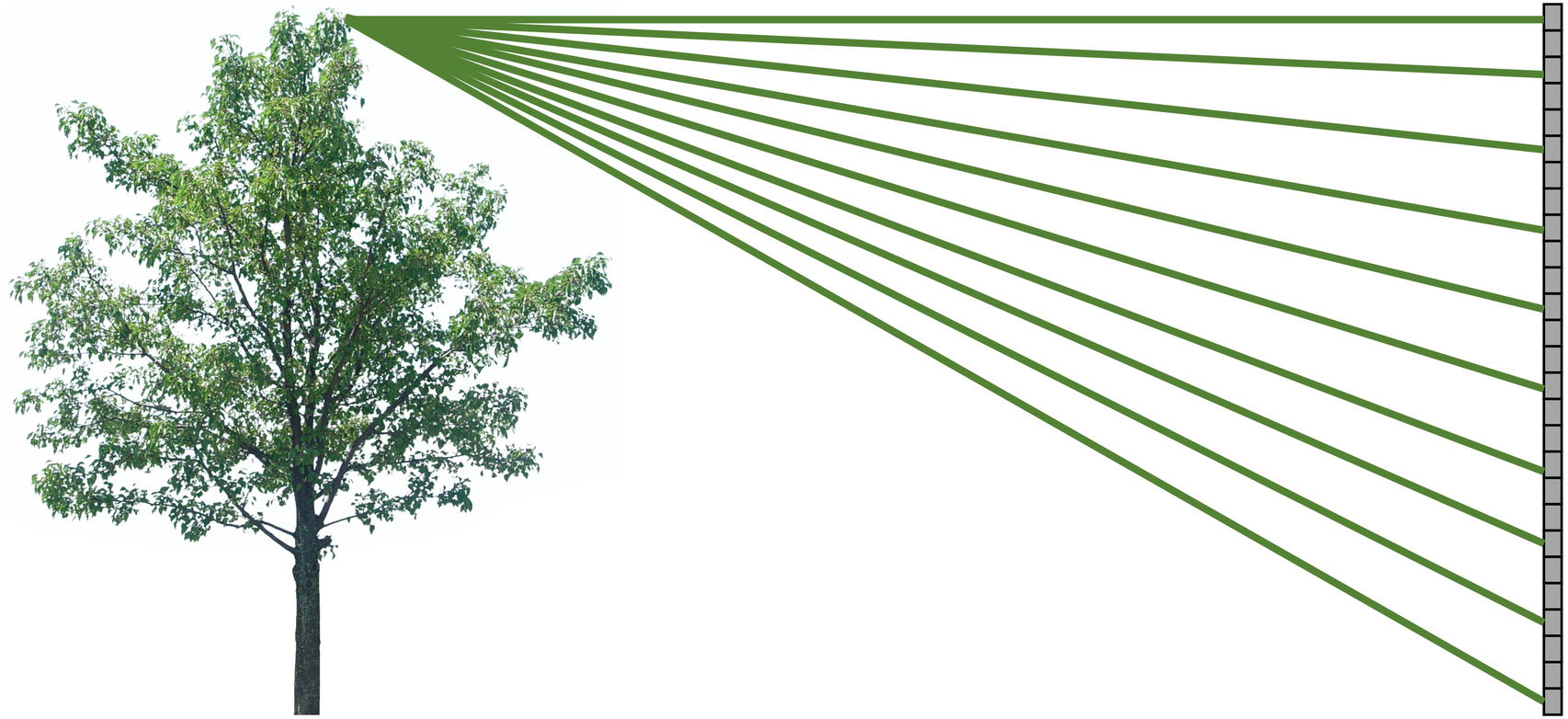

digital sensor (CCD or CMOS)

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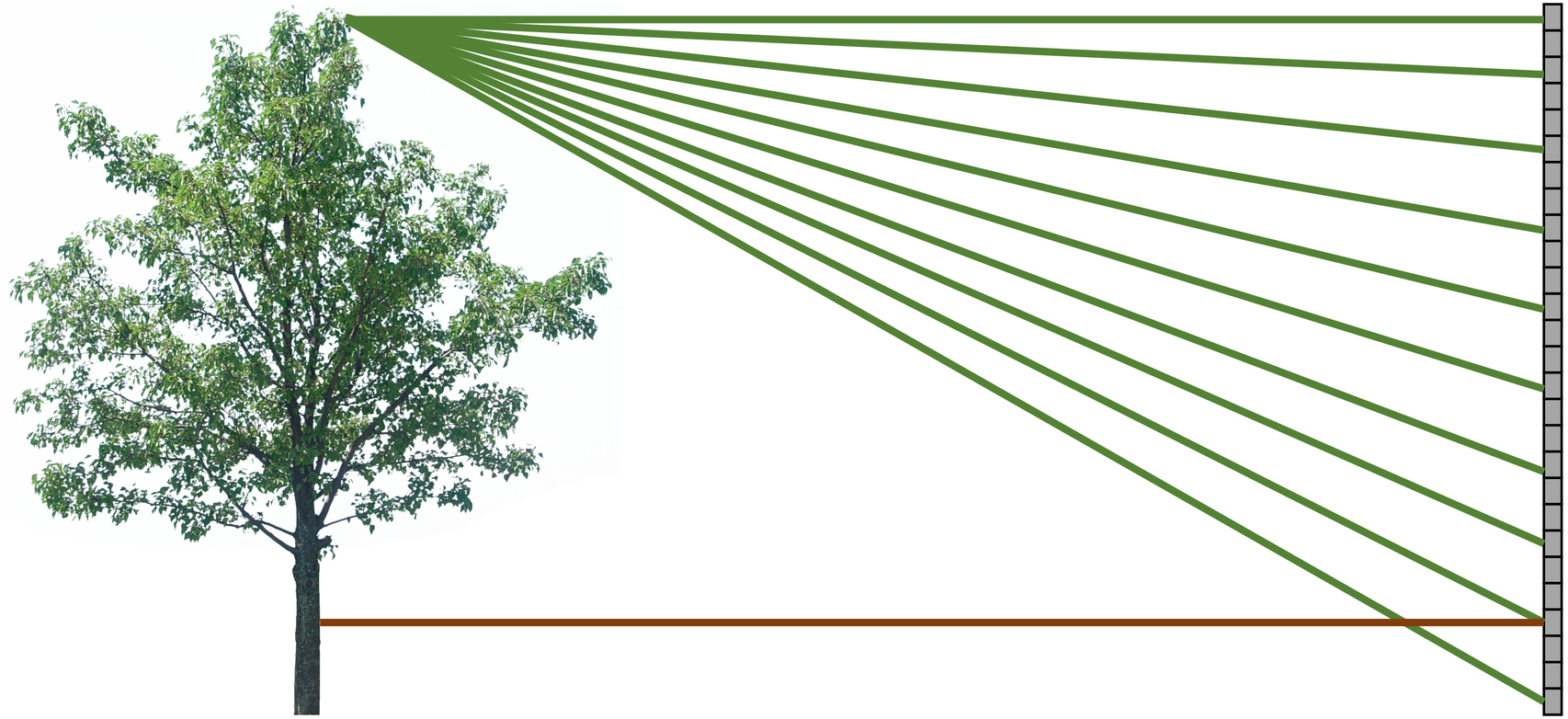
Bare-sensor imaging

real-world object



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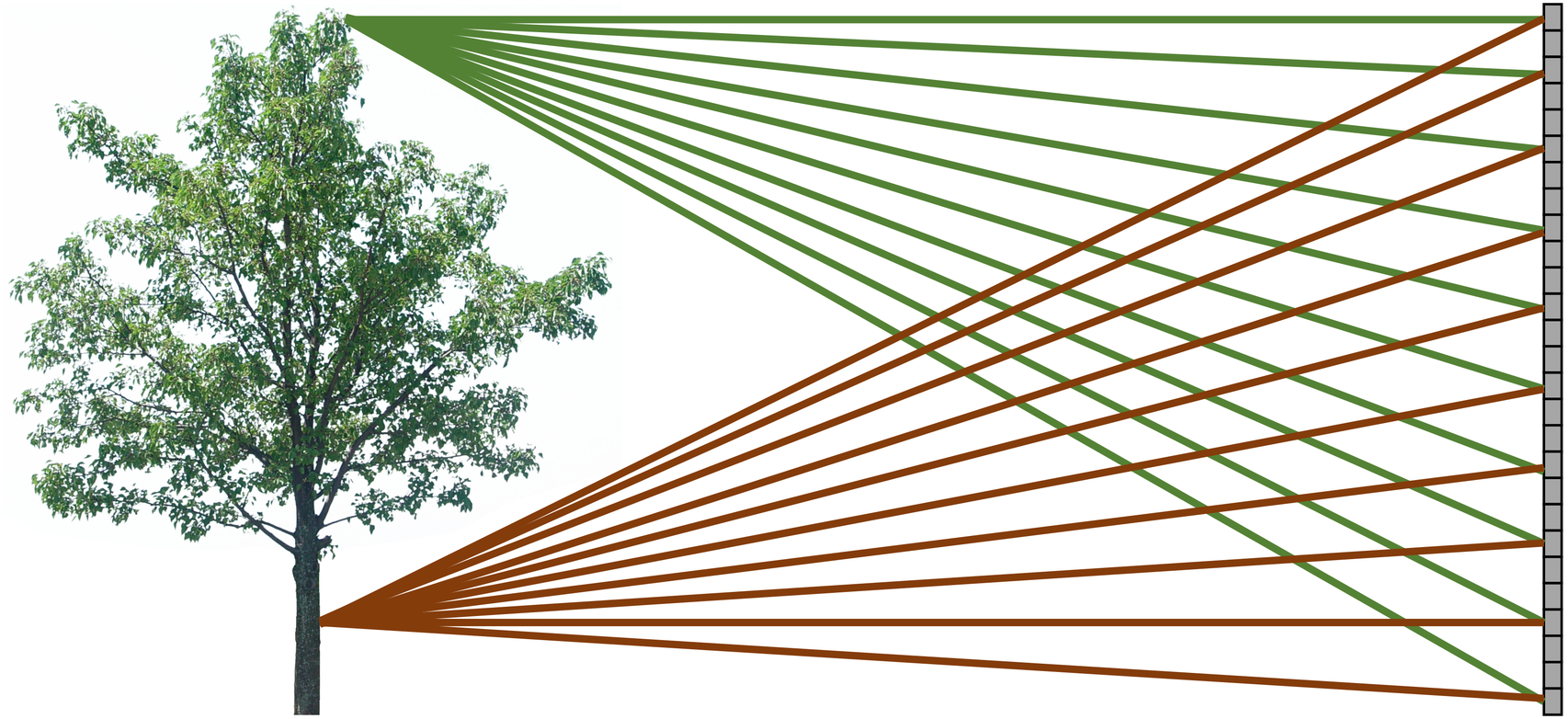

digital sensor (CCD or CMOS)

All scene points contribute to all sensor pixels

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Bare-sensor imaging

All scene points contribute to all sensor pixels

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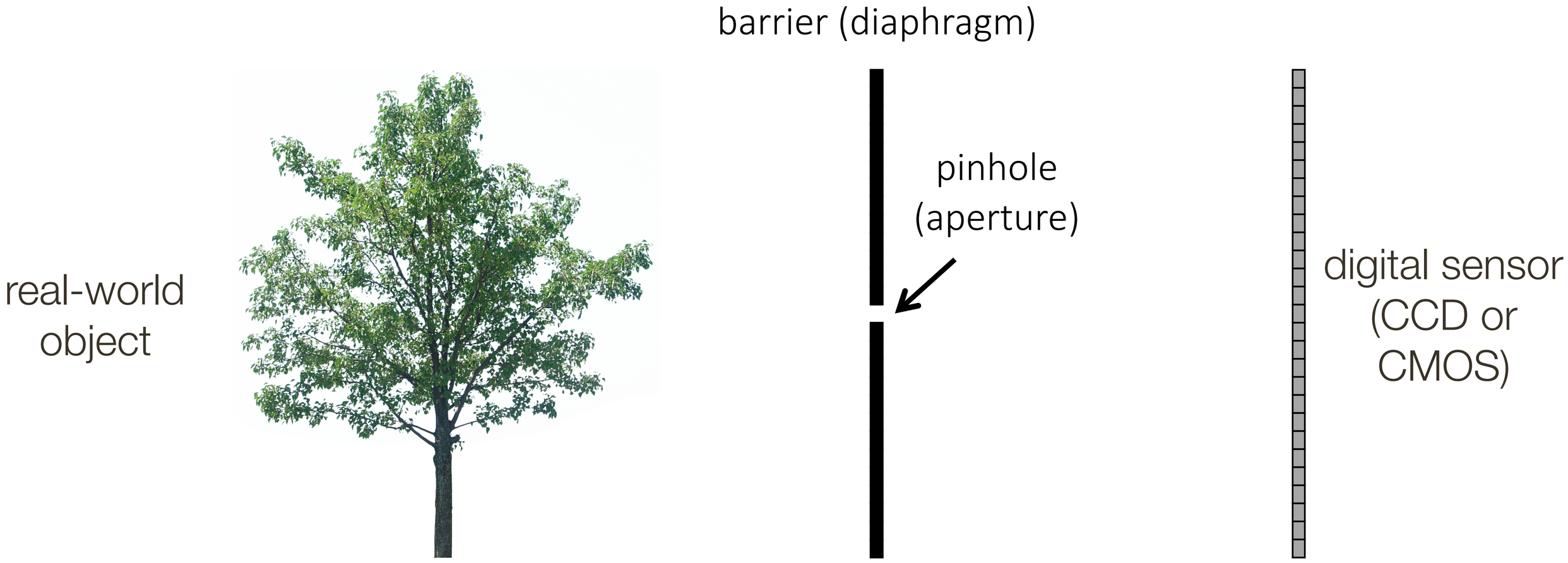
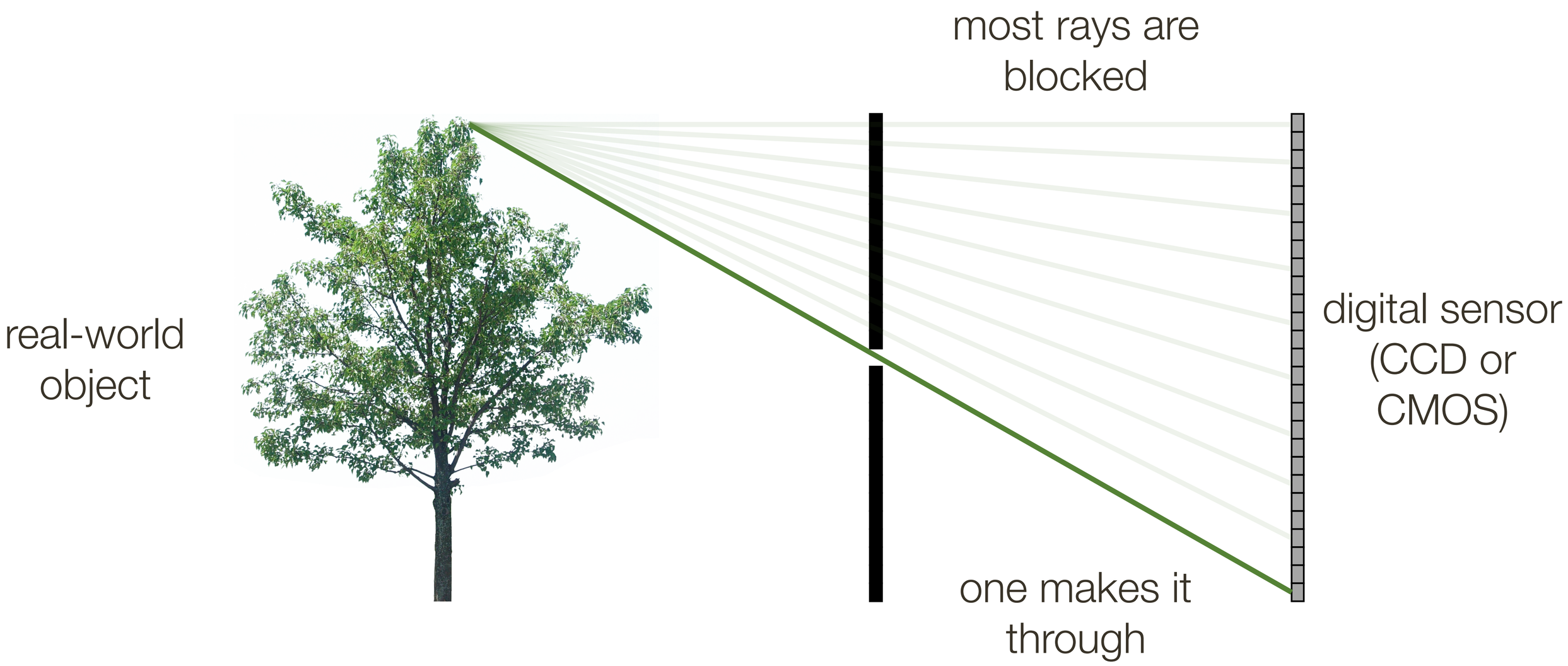
Pinhole Camera



What would an image taken like this look like?

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

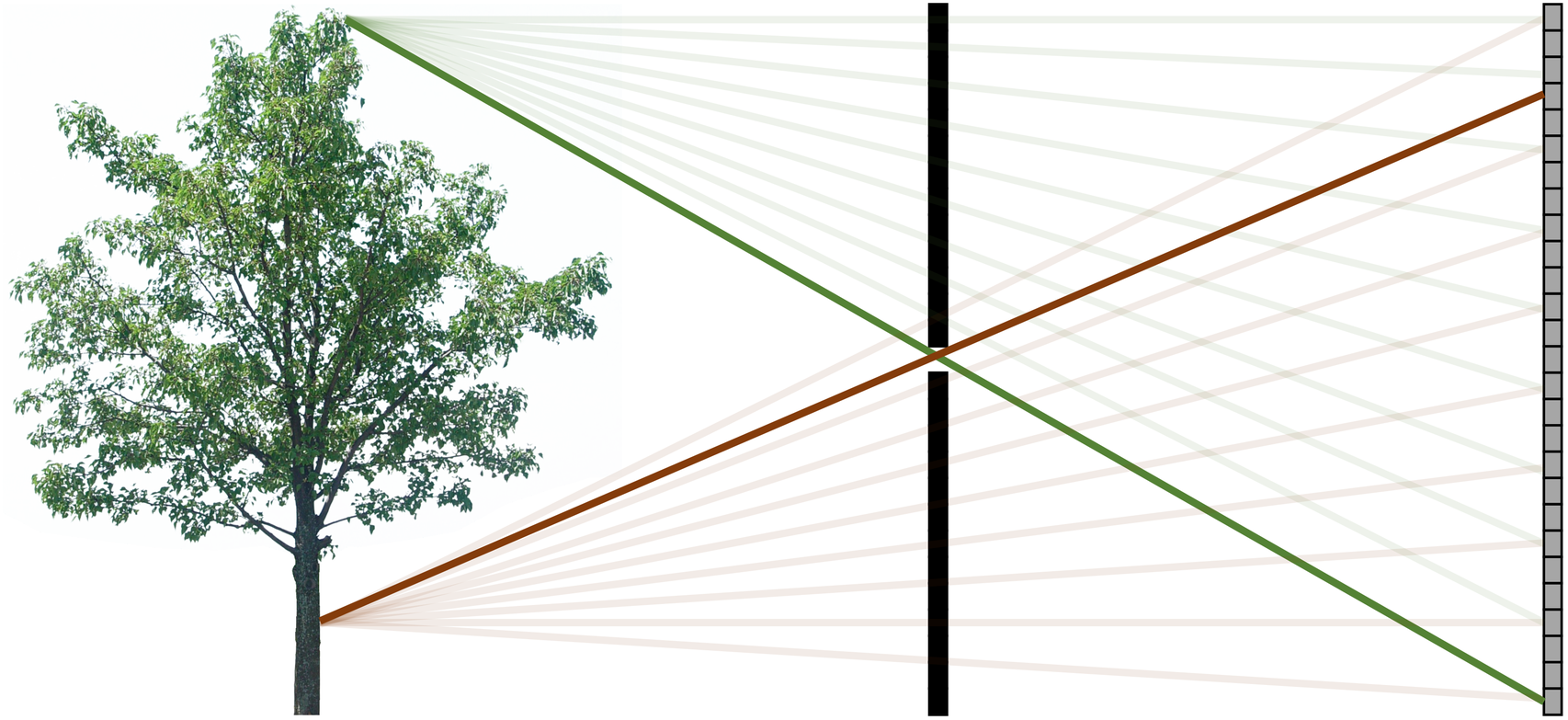
Pinhole Camera



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Pinhole Camera

real-world object



digital sensor (CCD or CMOS)

Each scene point contributes to only one sensor pixel

Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Camera Obscura (latin for “dark chamber”)

illum in tabula per radios Solis, quàm in cœlo contingit: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

Solis deliquium Anno Christi 1544. Die 24. Januarij Louanij

Sic nos exactè Anno .1544. Louanii eclipſim Solis obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

principles behind the pinhole camera or camera obscura were first mentioned by Chinese philosopher Mozi (Mo-Ti) (470 to 390 BCE)

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

Credit: John H., Hammond, “Th Camera Obscure, A Chronicle”

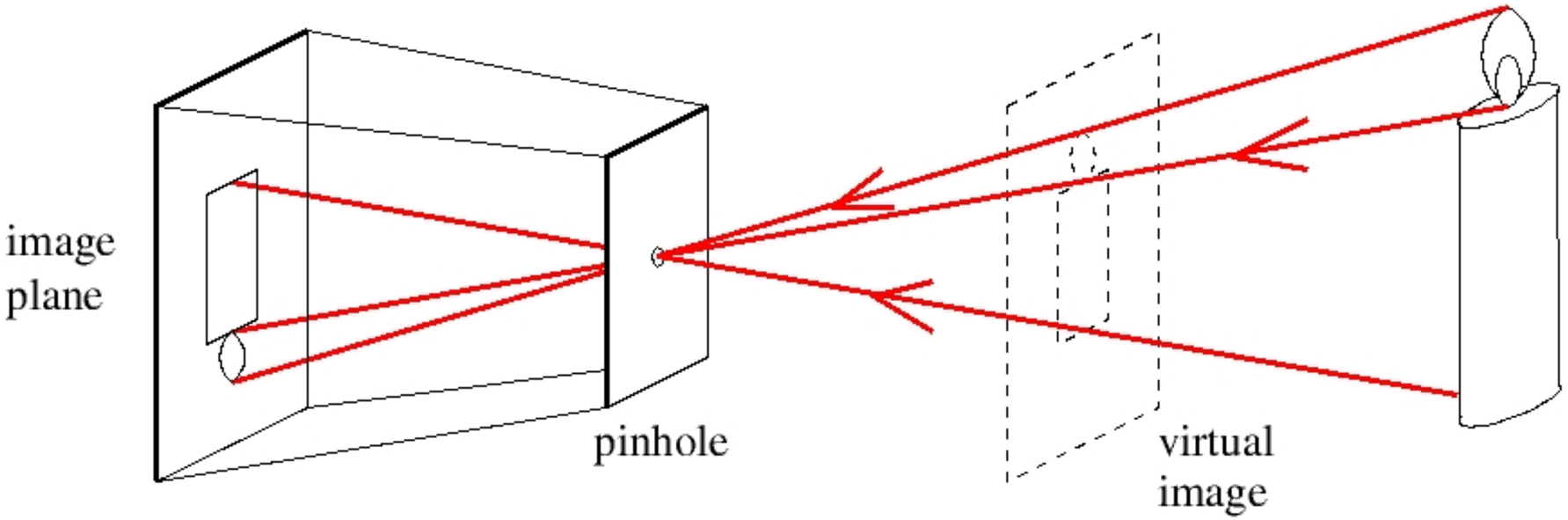
First **Photograph** on Record

La table servie

Credit: Nicéphore Niepce, 1822

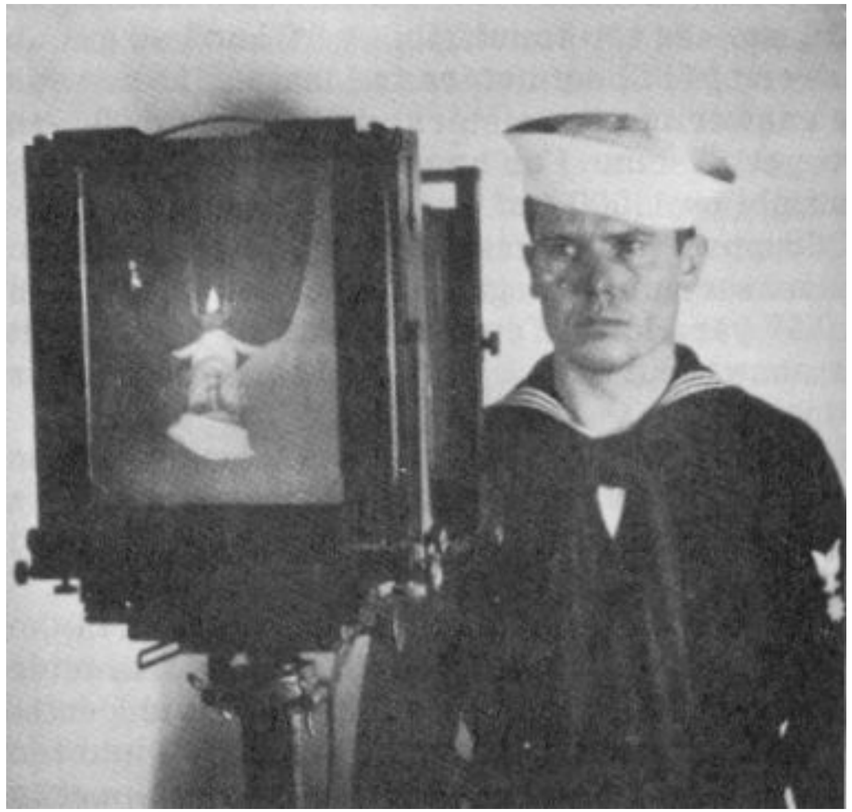
Pinhole Camera

A pinhole camera is a box with a small hall (**aperture**) in it



Forsyth & Ponce (2nd ed.) Figure 1.2

Image **Formation**



Forsyth & Ponce (2nd ed.) Figure 1.1

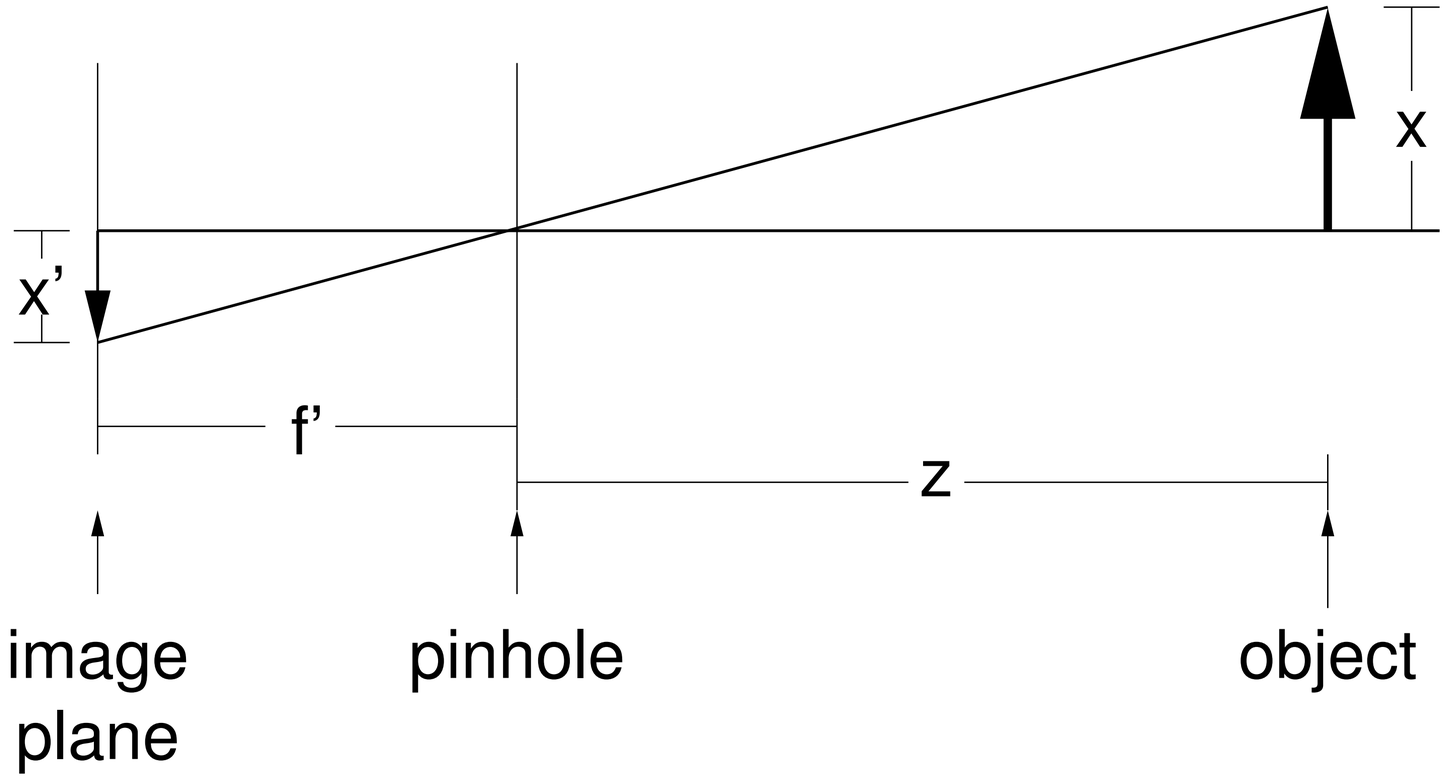
Credit: US Navy, Basic Optics and Optical Instruments. Dover, 1969

Accidental Pinhole Camera

Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Pinhole Camera (Simplified)

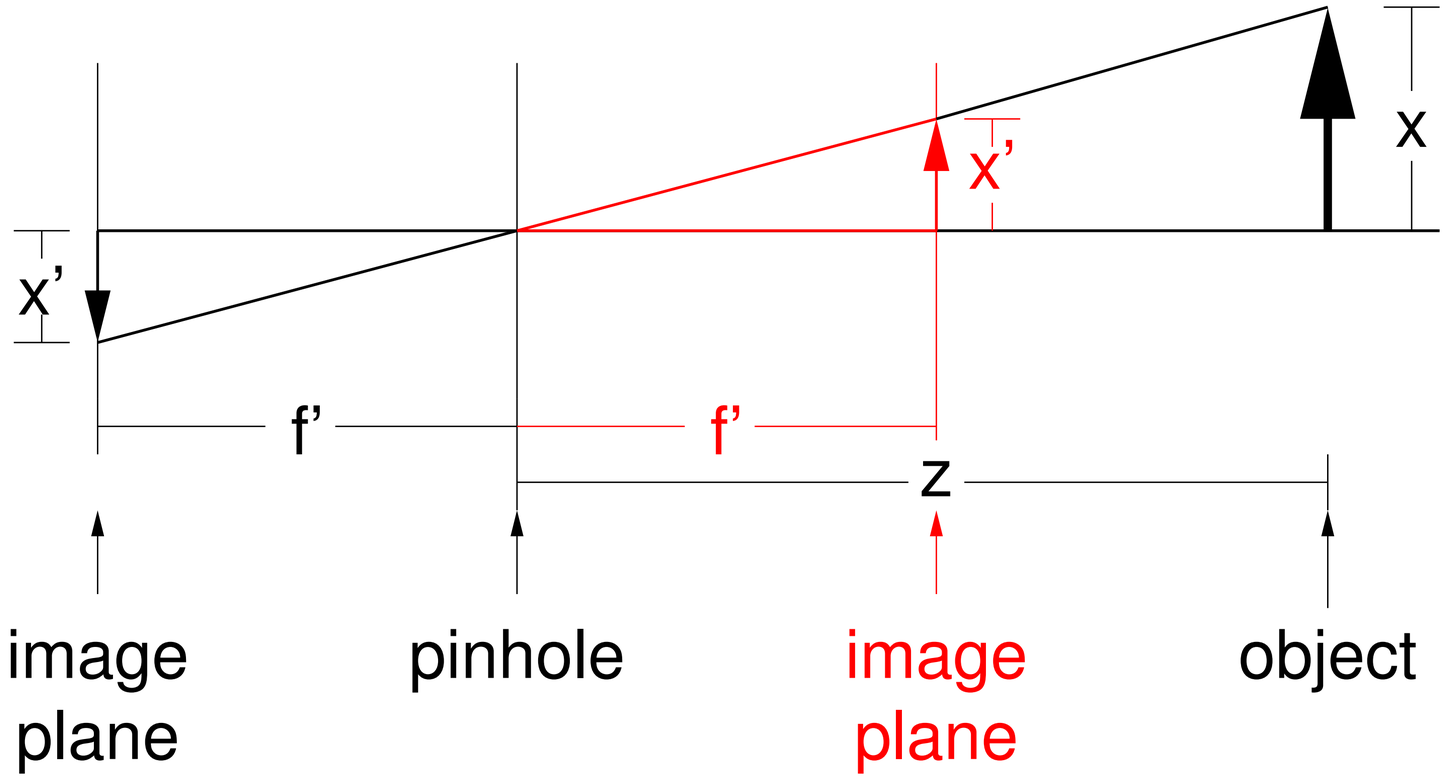
f’ is the **focal length** of the camera



Note: In a pinhole camera we can adjust the focal length, all this will do is change the **size** of the resulting image

Pinhole Camera (Simplified)

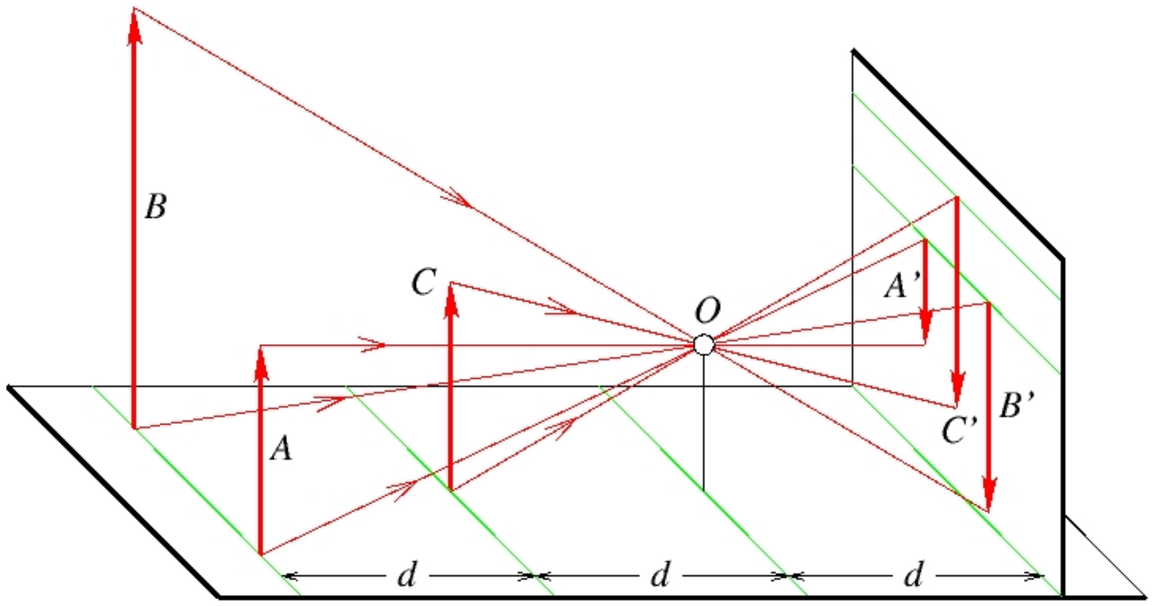
It is convenient to think of the **image plane** which is in from of the pinhole



What happens if object moves towards the camera? Away from the camera?

Perspective Effects

Far objects appear **smaller** than close ones

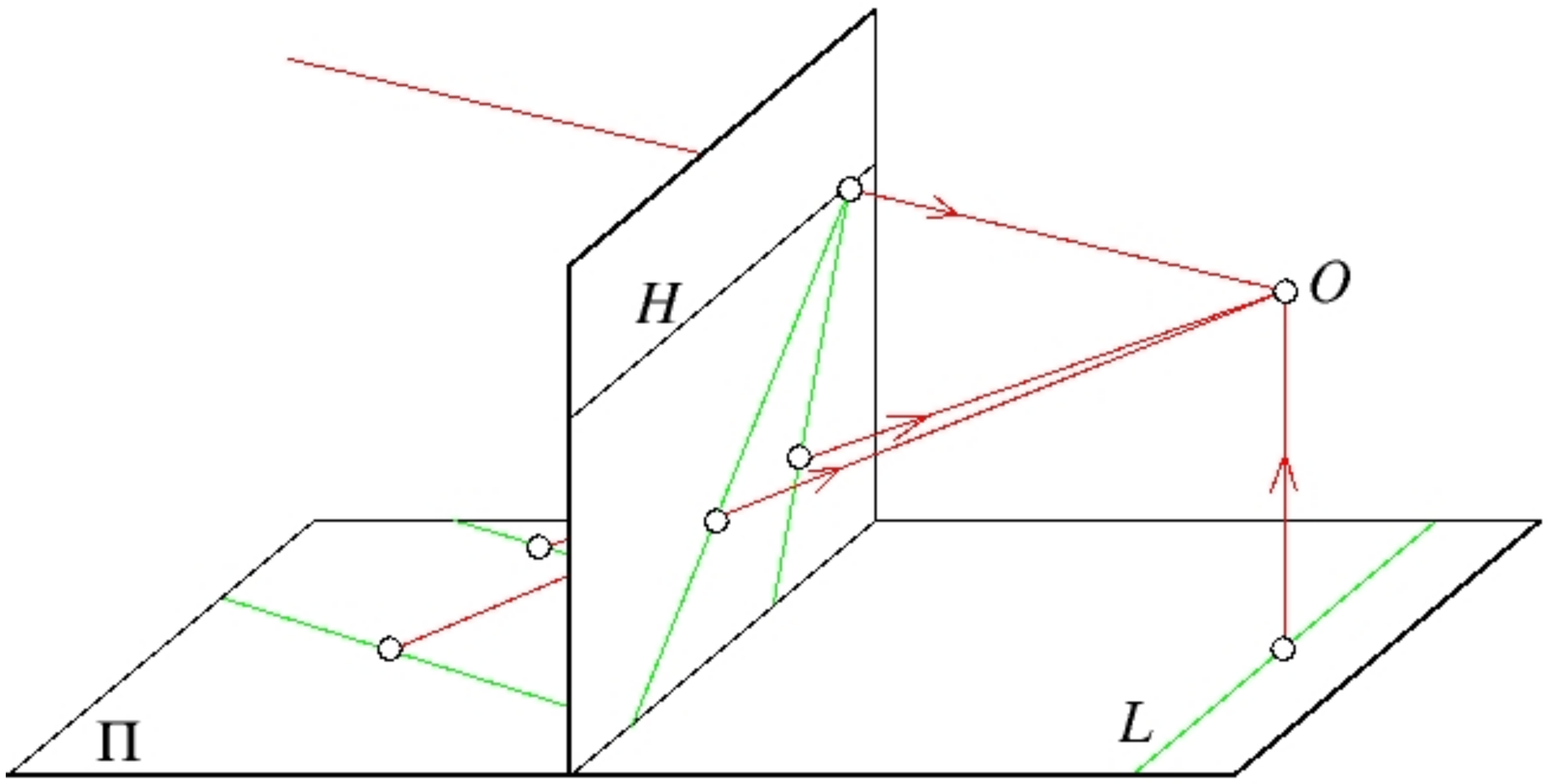


Forsyth & Ponce (1st ed.) Figure 1.3a

Size is **inversely** proportions to distance

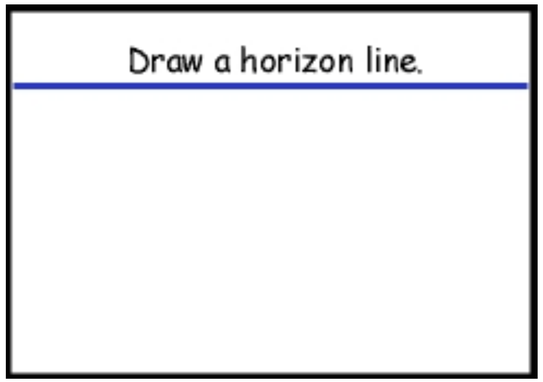
Perspective Effects

Parallel lines meet at a point (**vanishing point**)



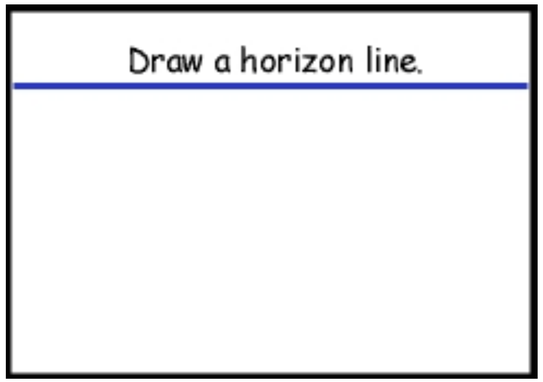
Forsyth & Ponce (1st ed.) Figure 1.3b

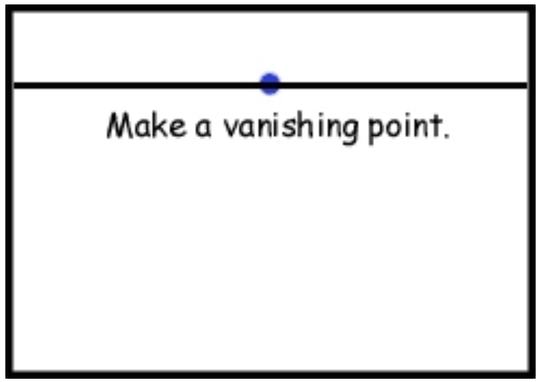
Vanishing Points



Slide Credit: David Jacobs

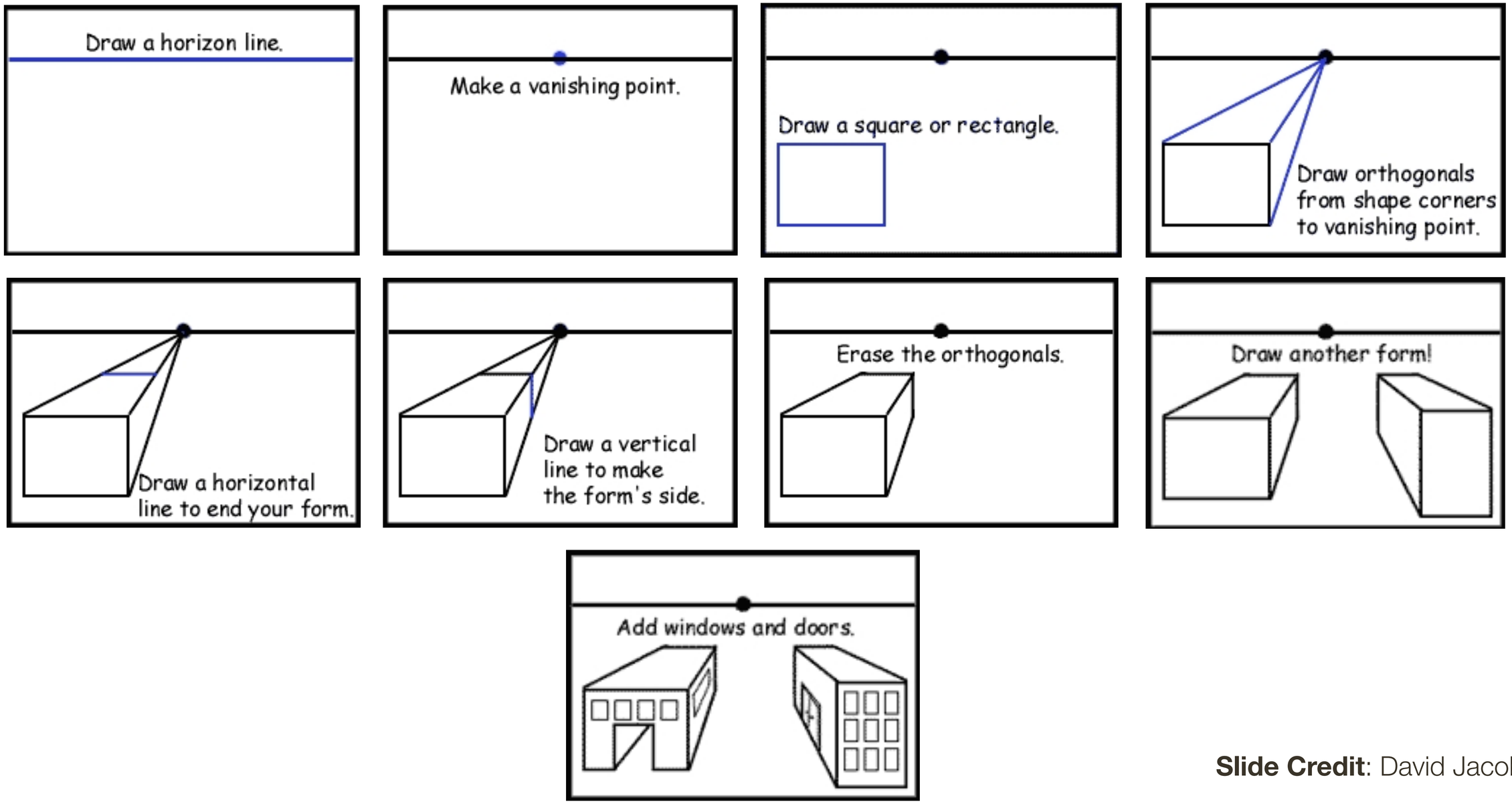
Vanishing Points





Slide Credit: David Jacobs

Vanishing Points



Slide Credit: David Jacobs

Vanishing Points

Each set of parallel lines meet at a different point

— the point is called **vanishing point**

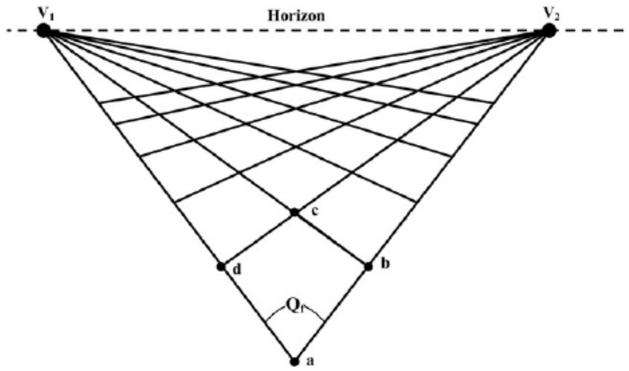
Vanishing Points

Each set of parallel lines meet at a different point

— the point is called **vanishing point**

Sets of parallel lines one the same plane lead to **collinear** vanishing points

— the line is called a **horizon** for that plane



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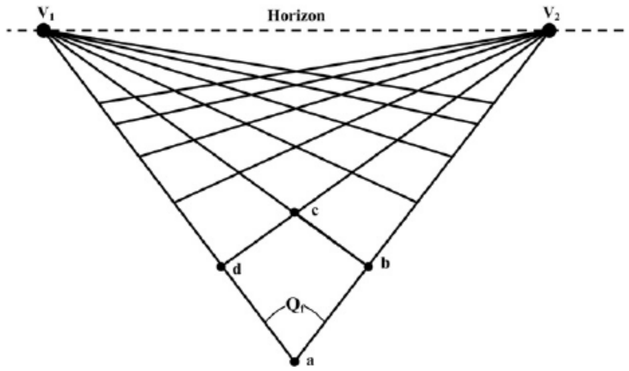


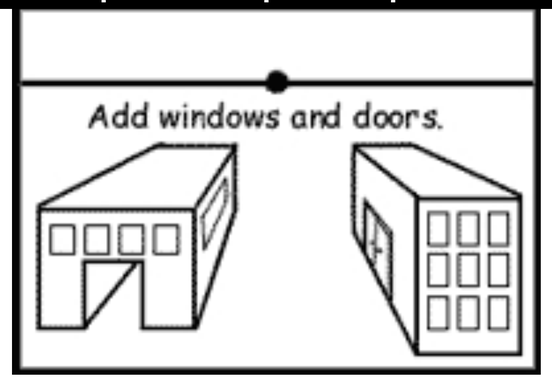
Good way to **spot fake images**

— scale and perspective do not work

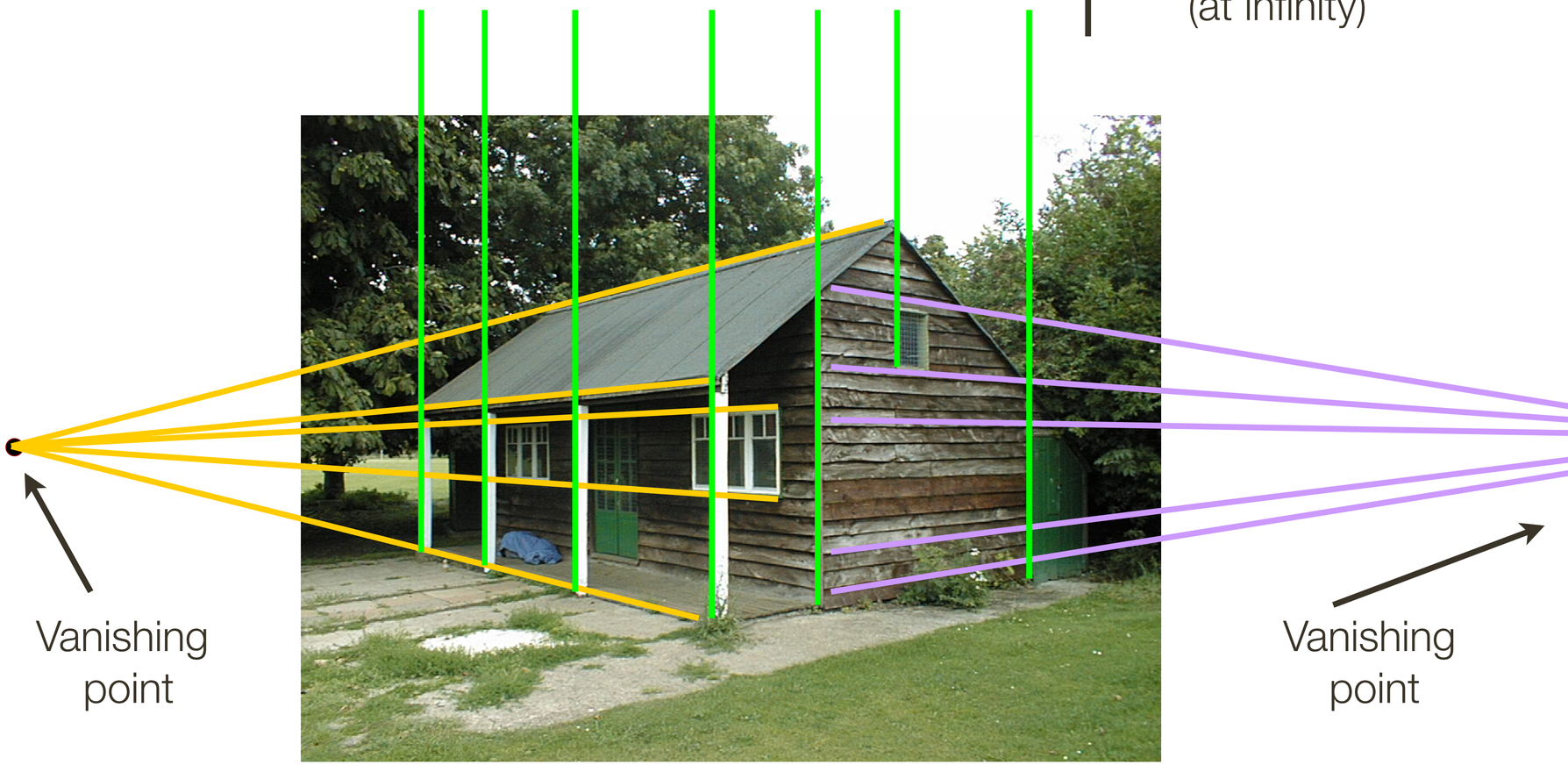
— vanishing points behave badly

Vanishing Points

One point perspective



Two point perspective



Slide Credit: Efros (Berkeley), photo from Criminisi

Perspective Aside

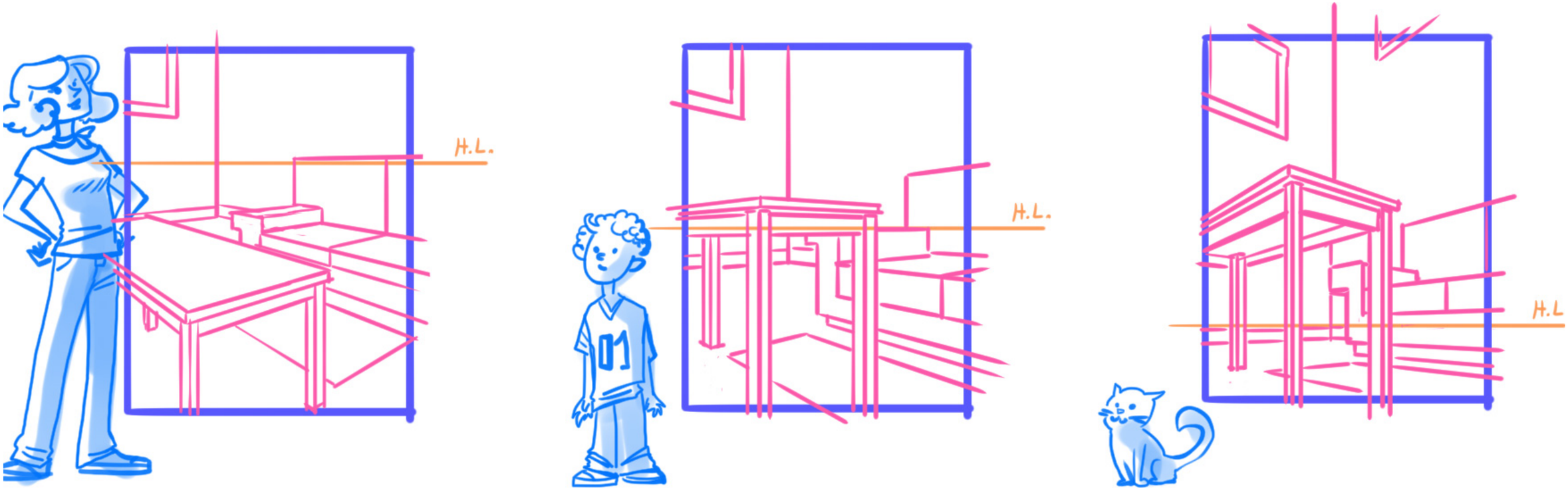
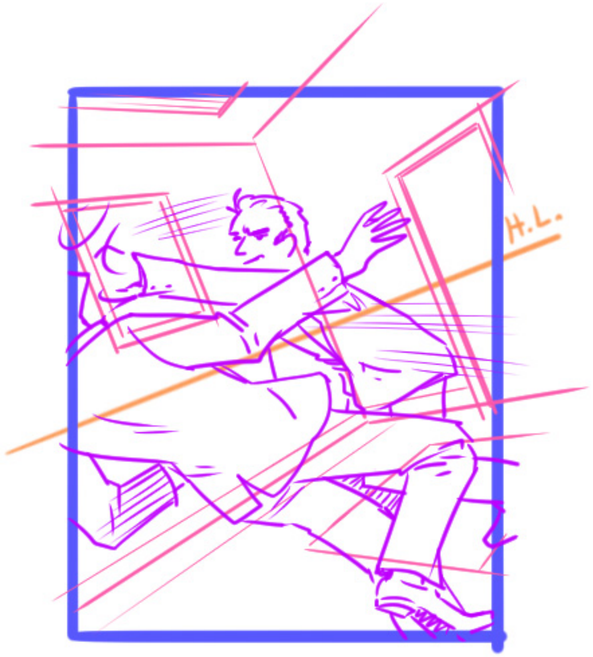


Image credit: http://www.martinacecilia.com/place-vanishing-points/

Perspective Aside



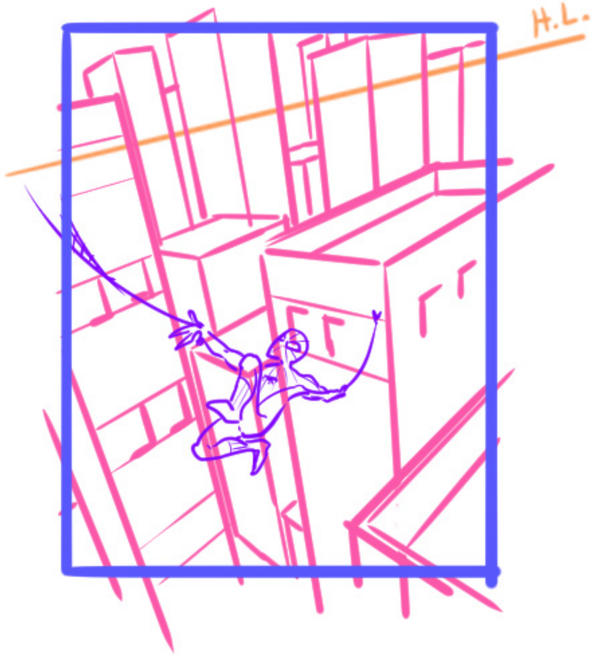


Image credit: http://www.martinacecilia.com/place-vanishing-points/

Properties of Projection

- Points project to points
- Lines project to lines
- Planes project to the whole or half image
- Angles are **not** preserved

Properties of Projection

— **Points** project to **points**

— **Lines** project to **lines**

— **Planes** project to the **whole** or **half** image

— Angles are **not** preserved

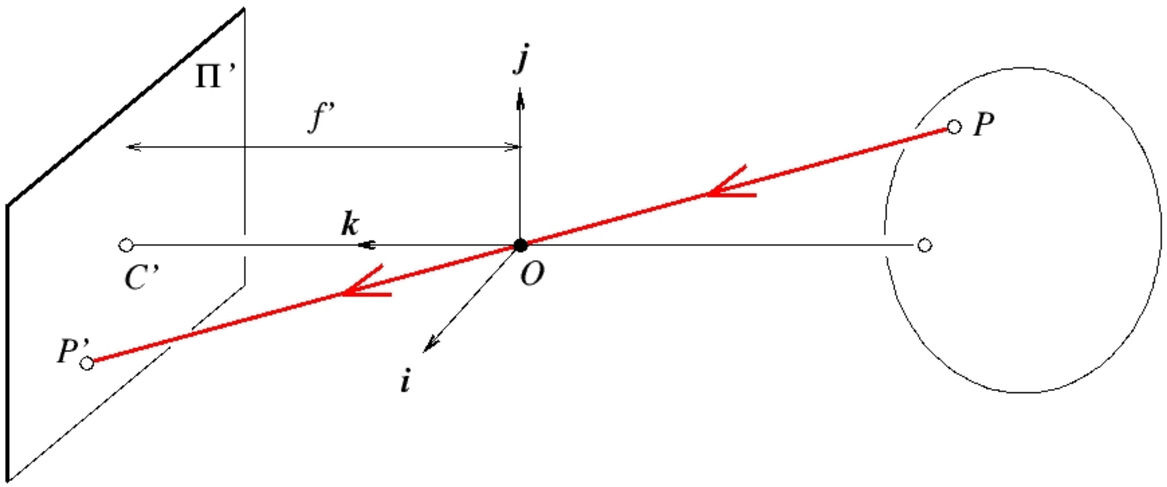
Degenerate cases

— Line through focal point projects to a point

— Plane through focal point projects to a line

Projection Illusion

Perspective Projection



Forsyth & Ponce (1st ed.) Figure 1.4

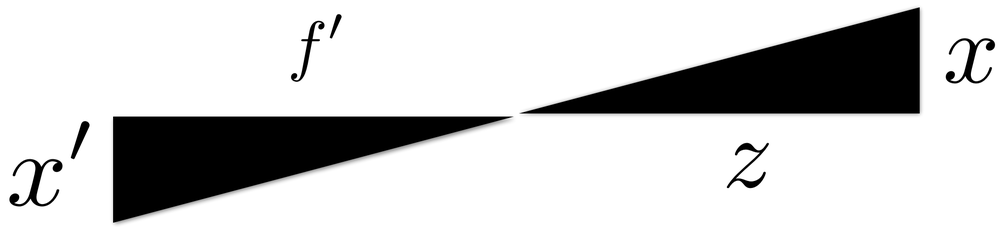
3D object point

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$ where

$$\boxed{\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Perspective Projection: Proof



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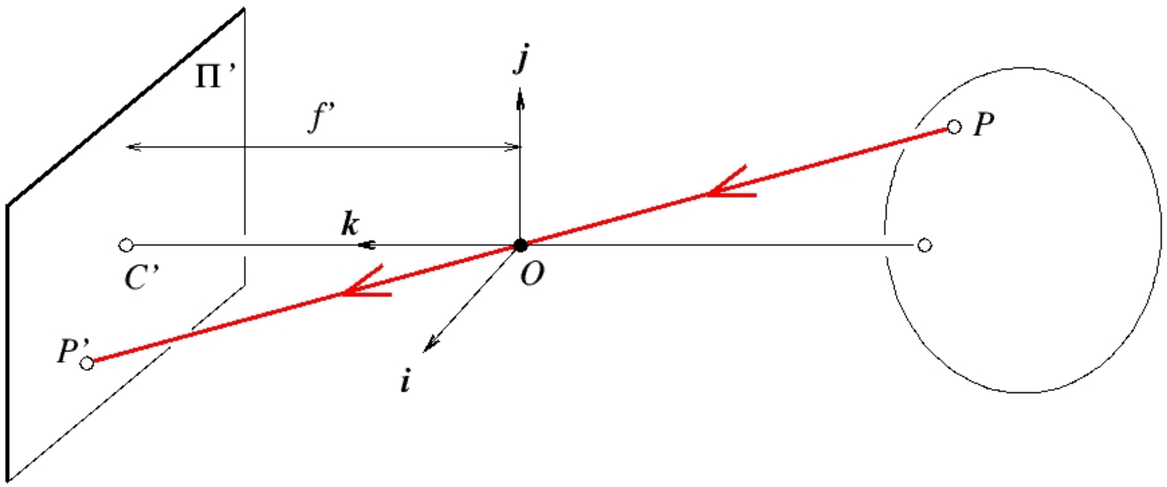
Perspective Projection: Proof

Camera Matrix



Forsyth & Ponce (1st ed.) Figure 1.4

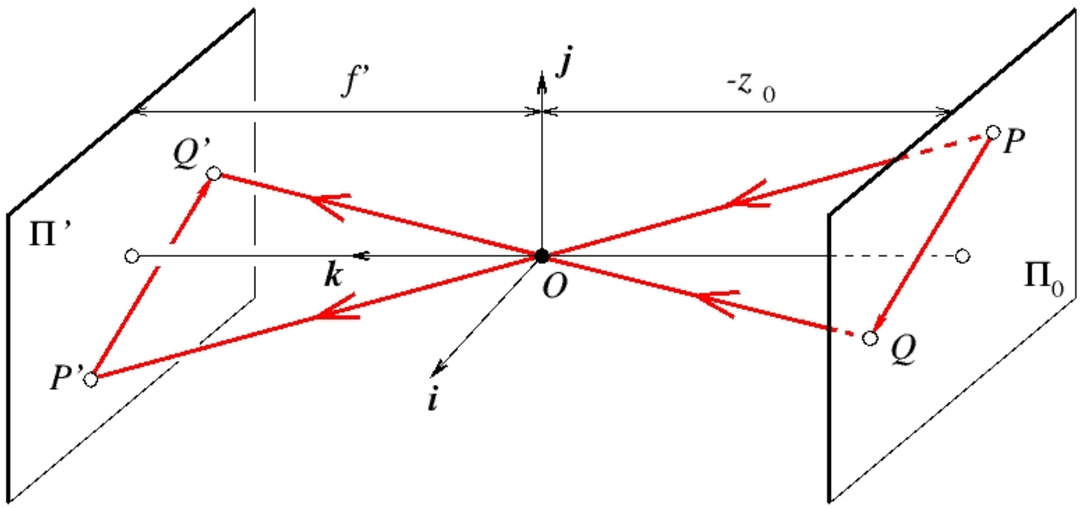
$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3D object point

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$ where $$\boxed{P' = \mathbf{C}P}$$

Note: this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

Weak Perspective

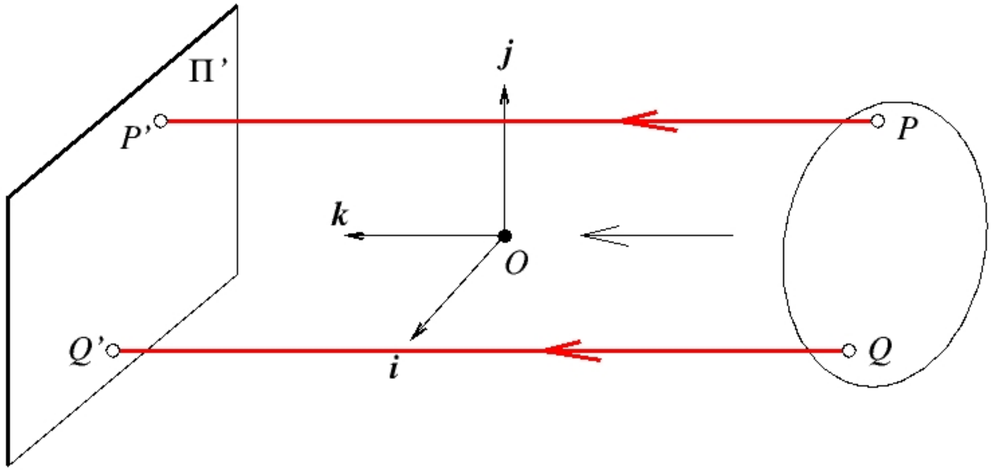


Forsyth & Ponce (1st ed.) Figure 1.5

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in Π_0 projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $\boxed{\begin{aligned} x' &= mx \\ y' &= my \end{aligned}}$ and $m = \dfrac{f'}{z_0}$

Orthographic Projection



Forsyth & Ponce (1st ed.) Figure 1.6

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $\boxed{\begin{aligned} x' &= x \\ y' &= y \end{aligned}}$

Summary of **Projection Equations**

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

Perspective	$x' = f' \frac{x}{z}$ $y' = f' \frac{y}{z}$	
Weak Perspective	$x' = m\,x$ $y' = m\,y$	$m = \frac{f'}{z_0}$
Orthographic	$x' = x$ $y' = y$	

Projection Models: Pros and Cons

Weak perspective (including orthographic) has simpler mathematics

— accurate when object is small and/or distant

— useful for recognition

Perspective is more accurate for real scenes

When **maximum accuracy** is required, it is necessary to model additional details of a particular camera

— use perspective projection with additional parameters (e.g., lens distortion)

Why **Not** a Pinhole Camera?

— If pinhole is **too big** then many directions are averaged, blurring the image

— If pinhole is **too small** then diffraction becomes a factor, also blurring the image

— Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

— Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time

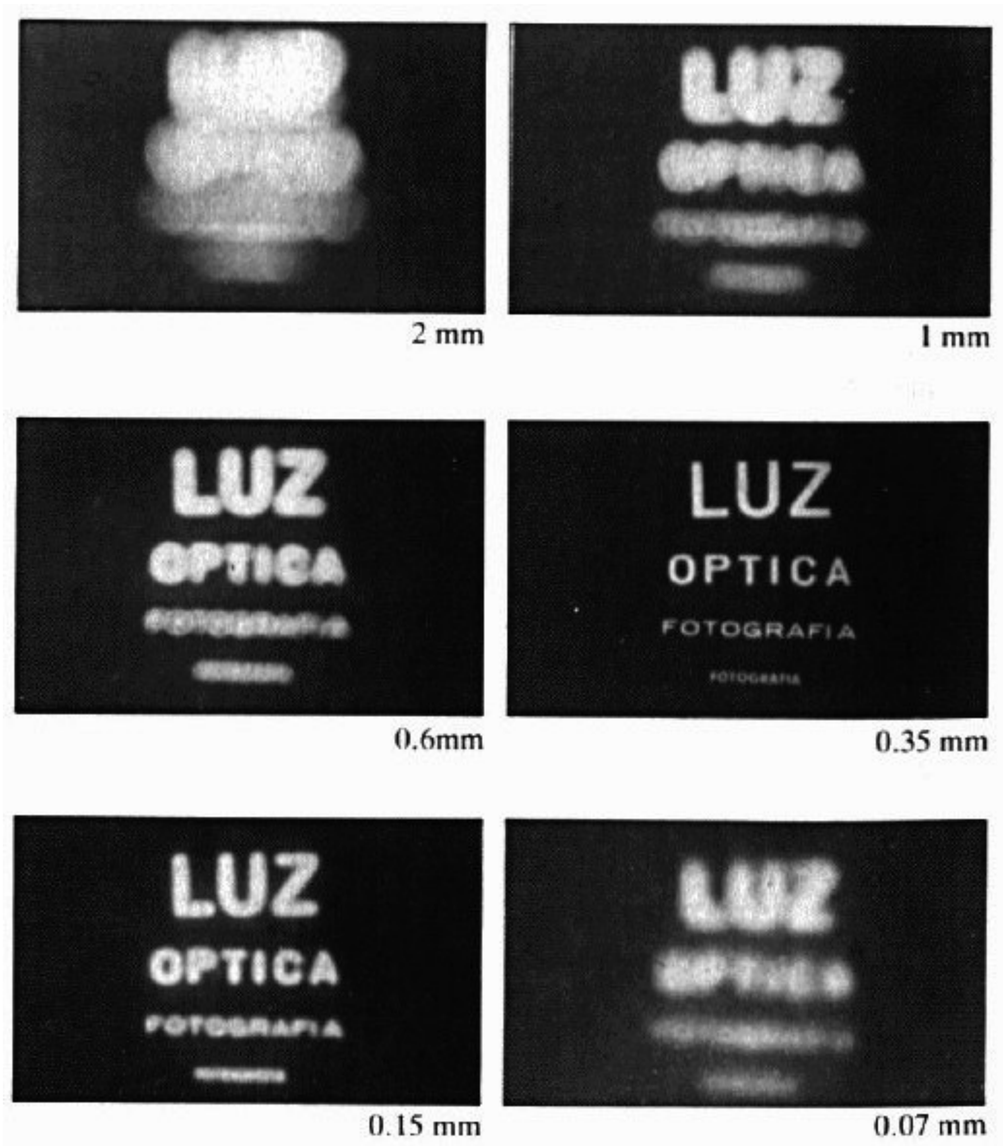
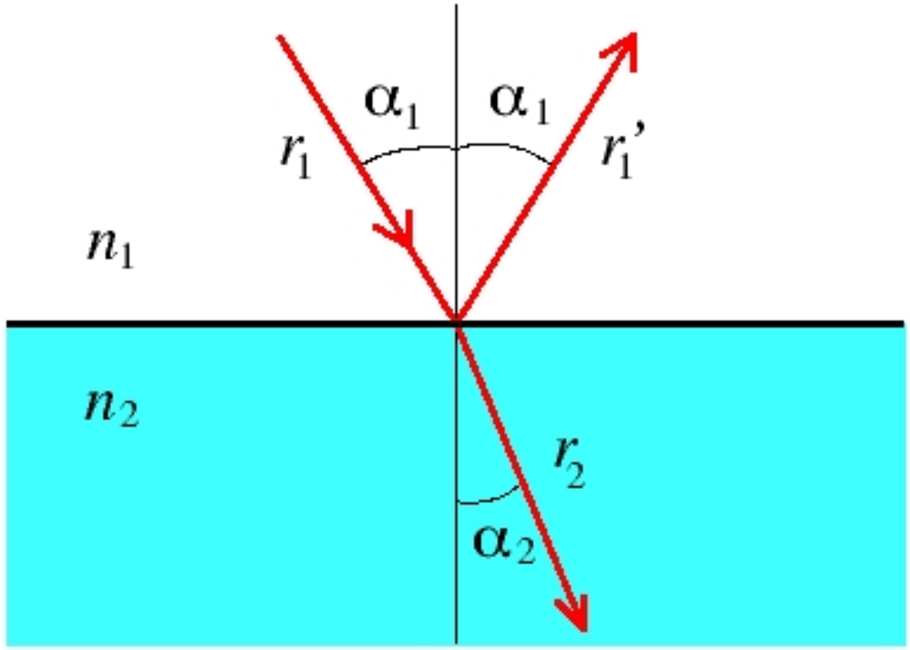


Image Credit: Credit: E. Hecht. “Optics,” Addison-Wesley, 1987

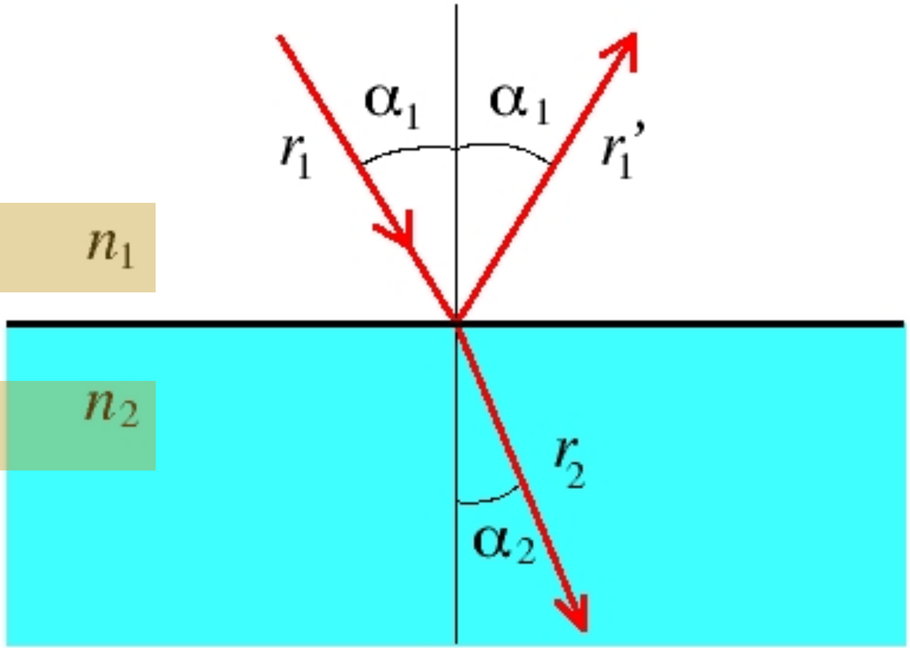
Snell's Law



$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Snell's Law

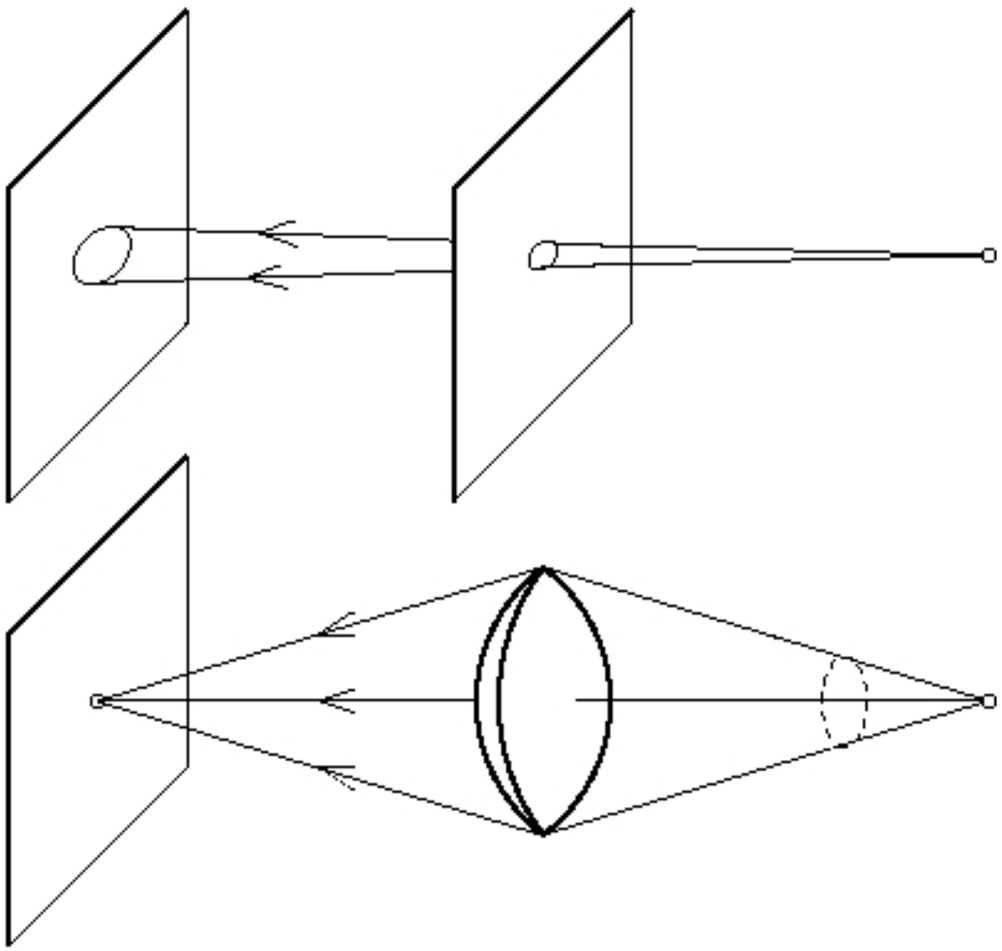
Index of **refraction**



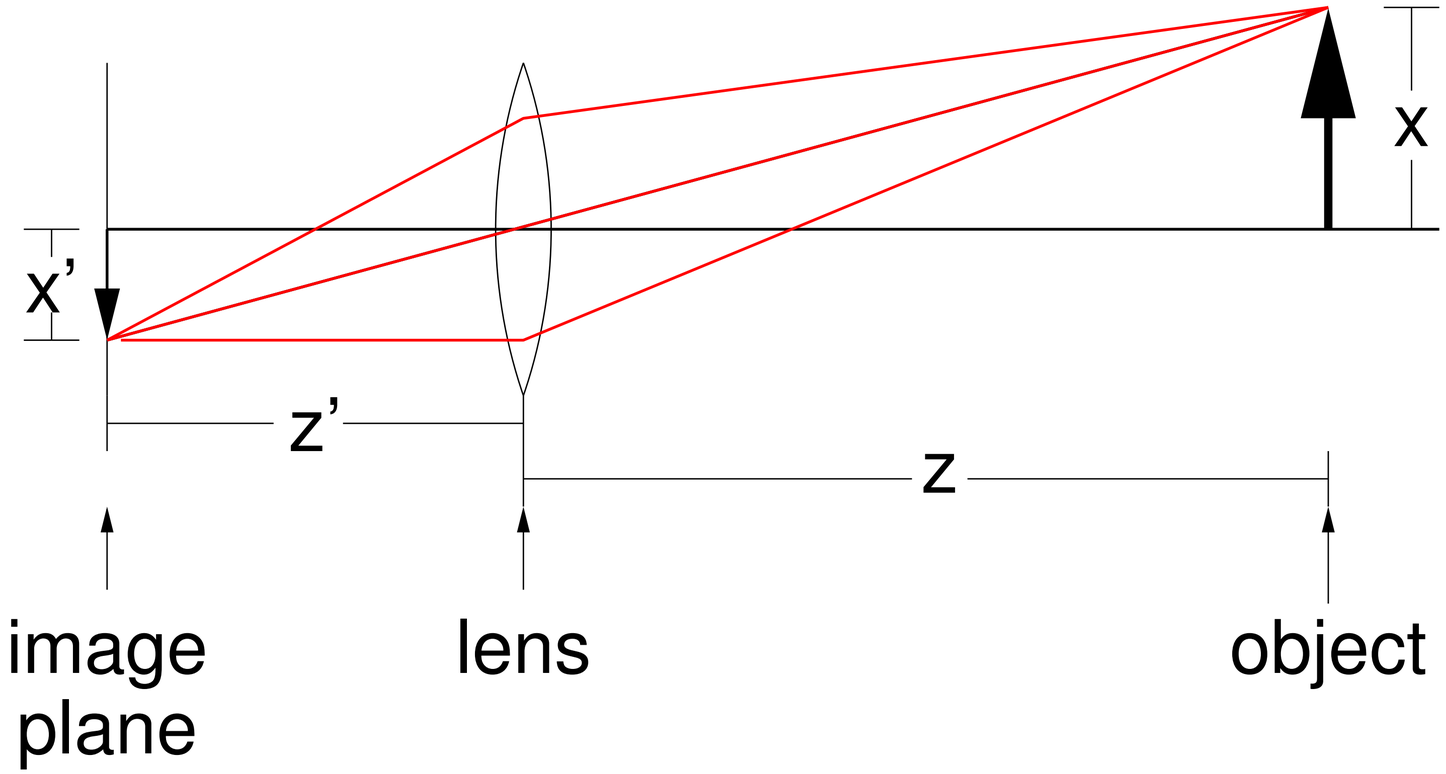
$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

Reason for **Lenses**

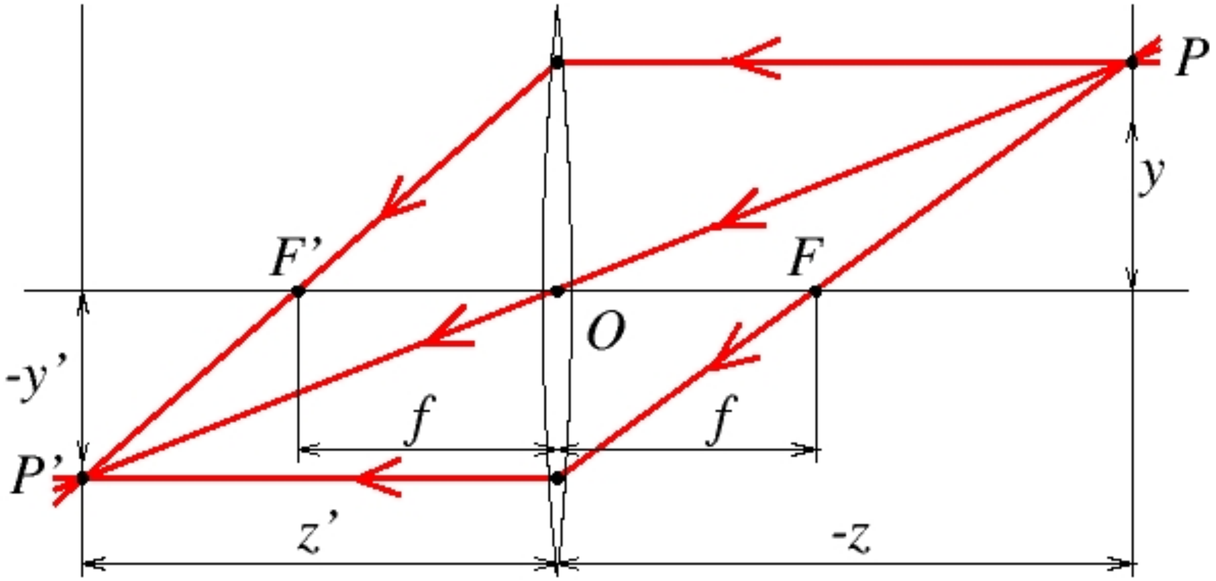
The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.



Pinhole Model (Simplified) **with Lens**



Thin Lens Equation



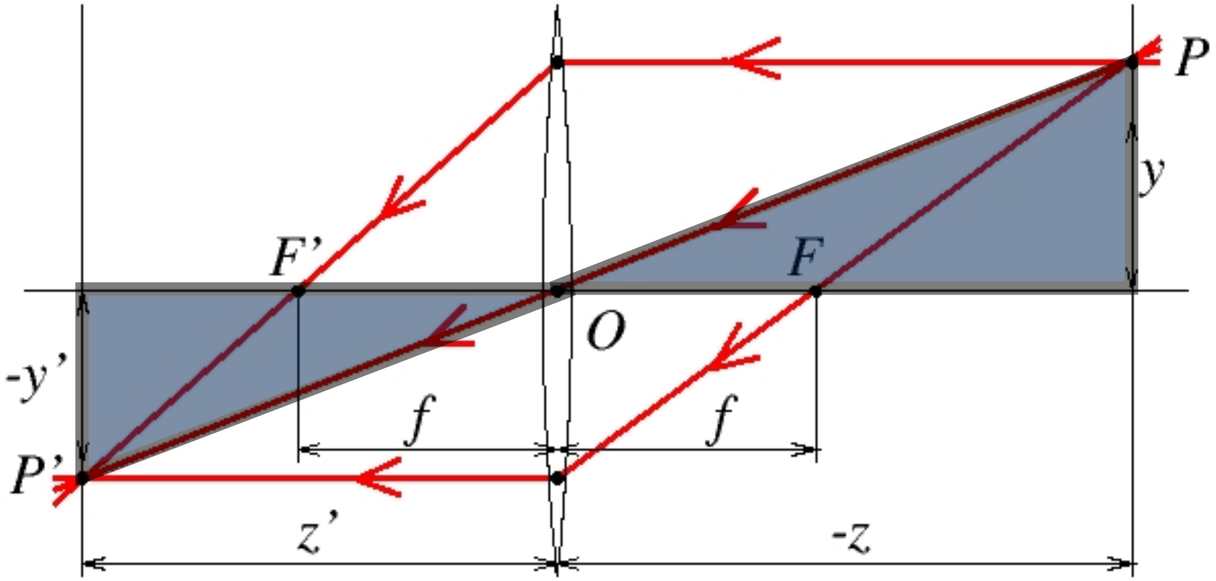
Forsyth & Ponce (1st ed.) Figure 1.9

$$\boxed{\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}}$$

Thin Lens Equation: Derivation

$$\frac{y}{-z} = \frac{-y'}{z'}$$

$$\frac{y}{y'} = \frac{z}{z'}$$



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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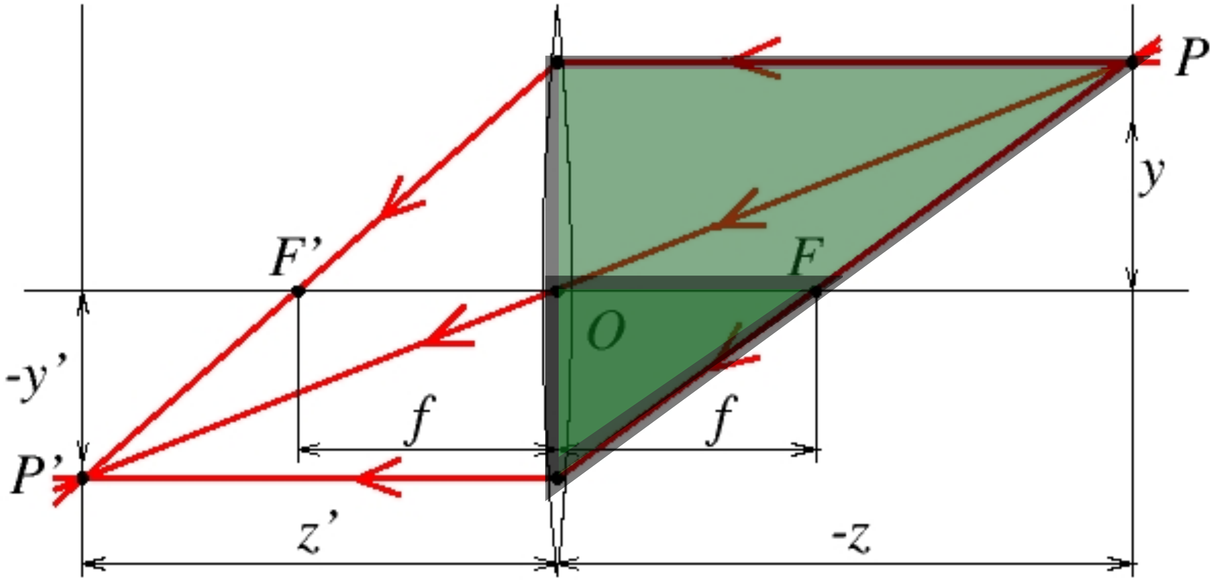

Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{-y'}{f} = \frac{y - y'}{-z}$$

$$\frac{1}{f} = \frac{y - y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{1}{z}$$

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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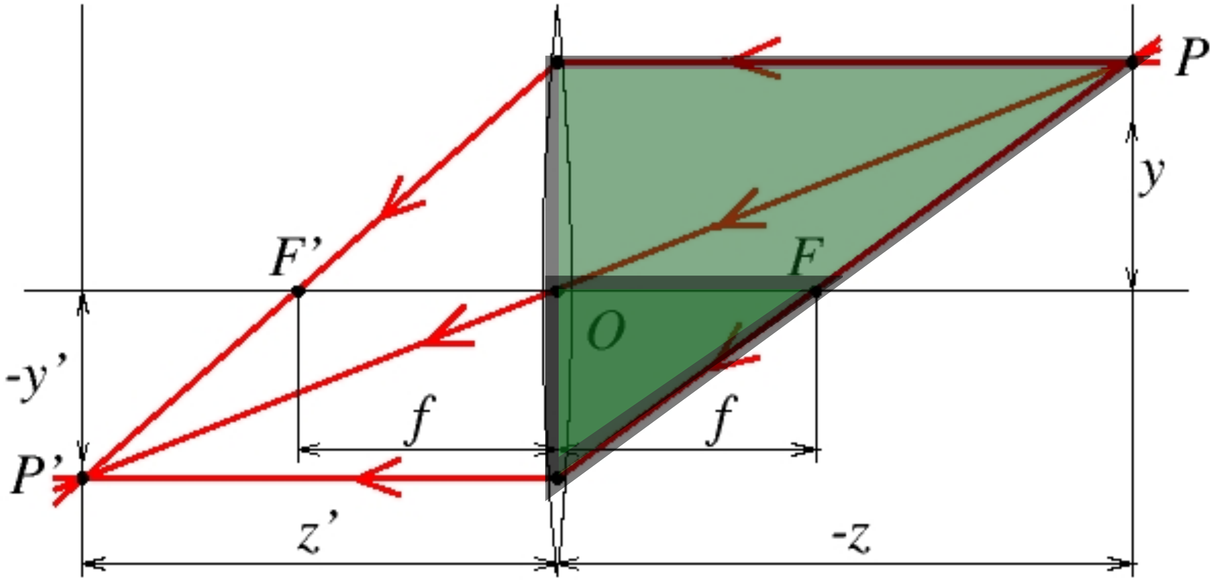


Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

$$\frac{-y'}{f} = \frac{y - y'}{-z}$$

$$\frac{1}{f} = \frac{y - y'}{zy'}$$

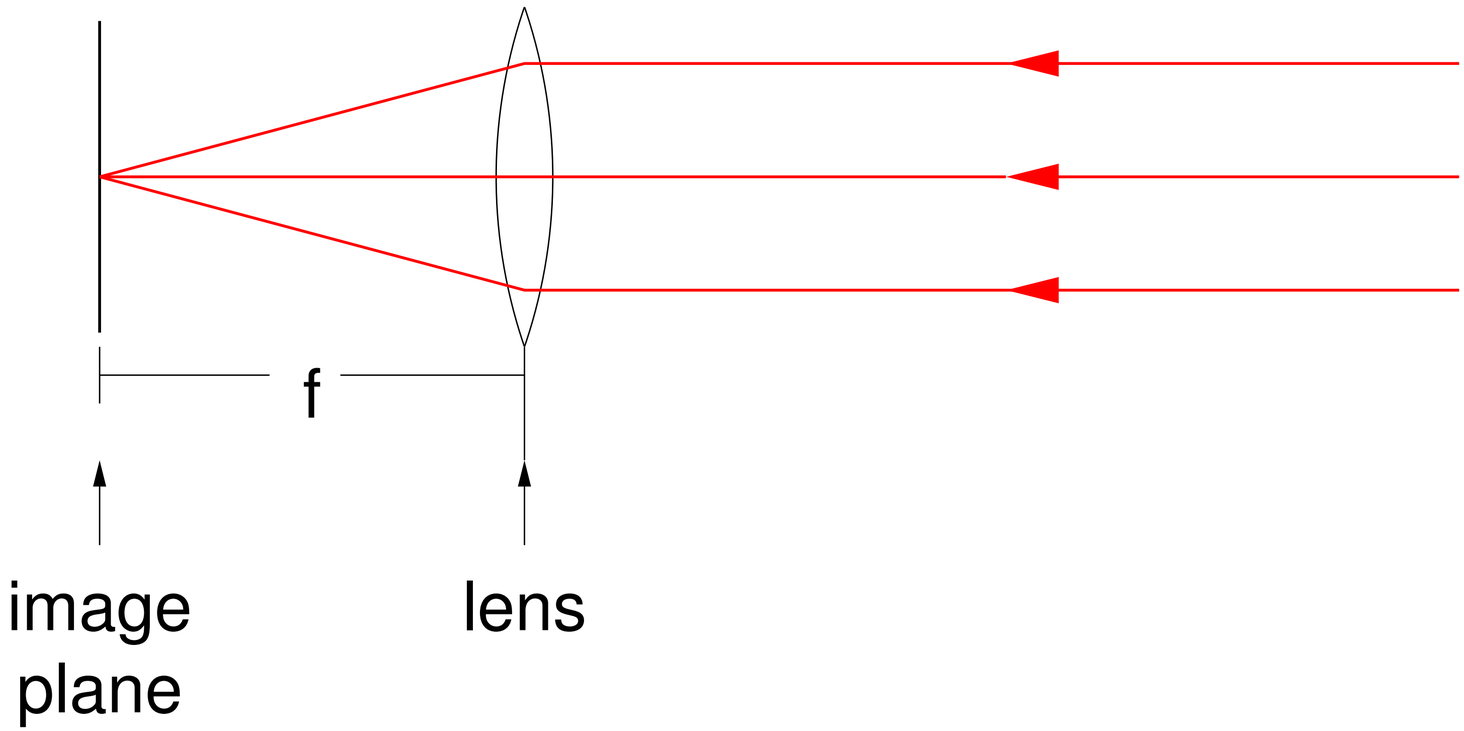
$$= \frac{y}{zy'} - \frac{y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{1}{z}$$

Substitute: $\frac{1}{f} = \frac{1}{\cancel{z}} \boxed{\frac{\cancel{z}}{z'}} - \frac{1}{z}$

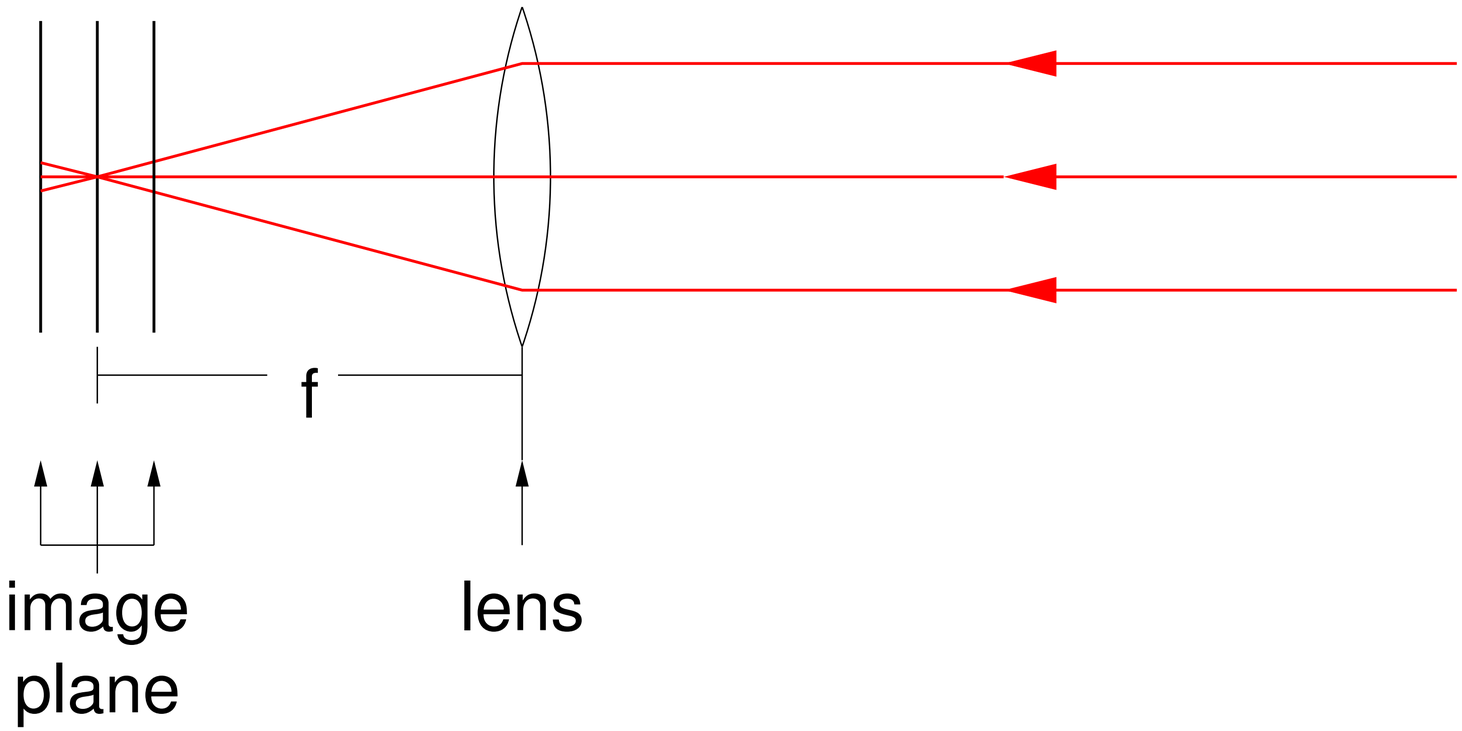
Focal Length

Another way of looking at the **focal length** of a lens. The incoming rays, parallel to the optical axis, **converge to a single point a distance f behind the lens**. This is where we want to place the image plane.

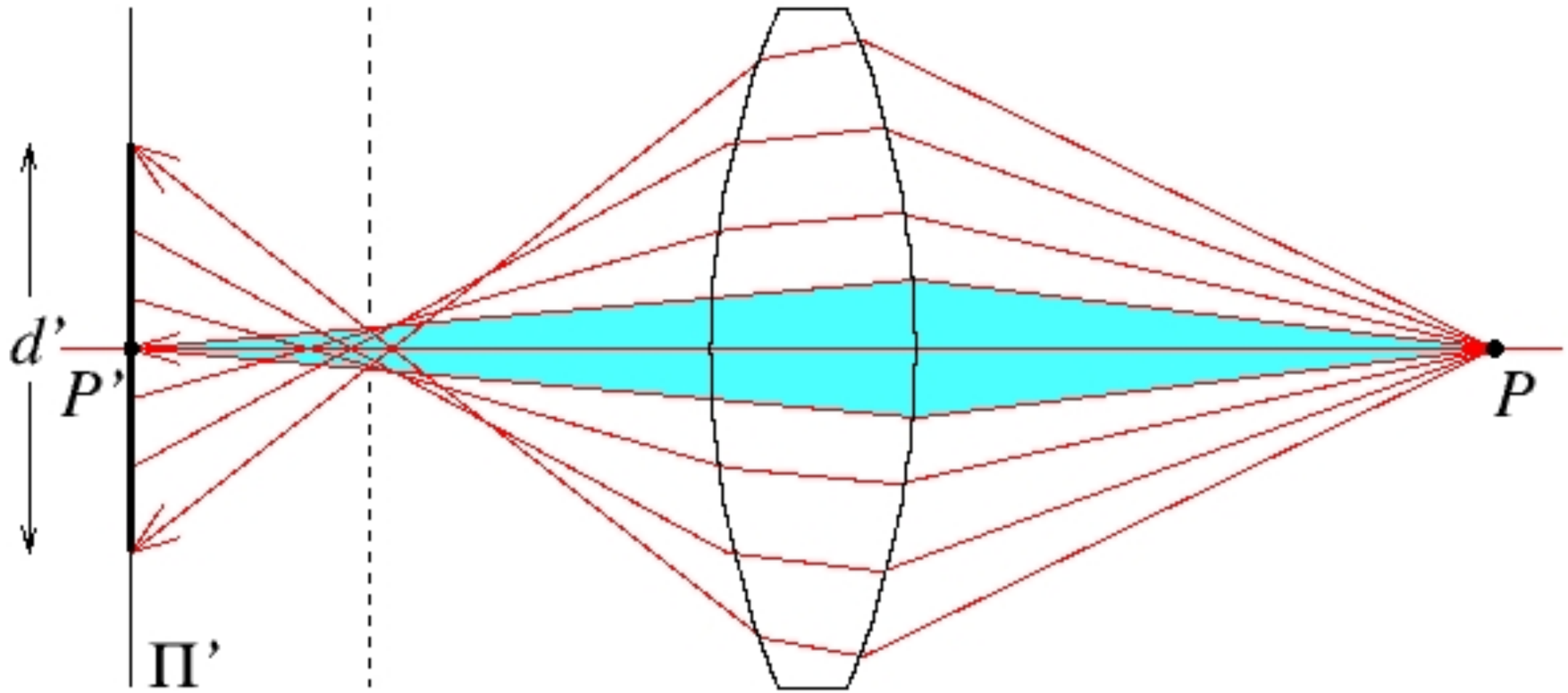


Out-of-Focus

The image plane is in the wrong place, either slightly closer than the required focal length, f, or slightly further than the required focal length, f.

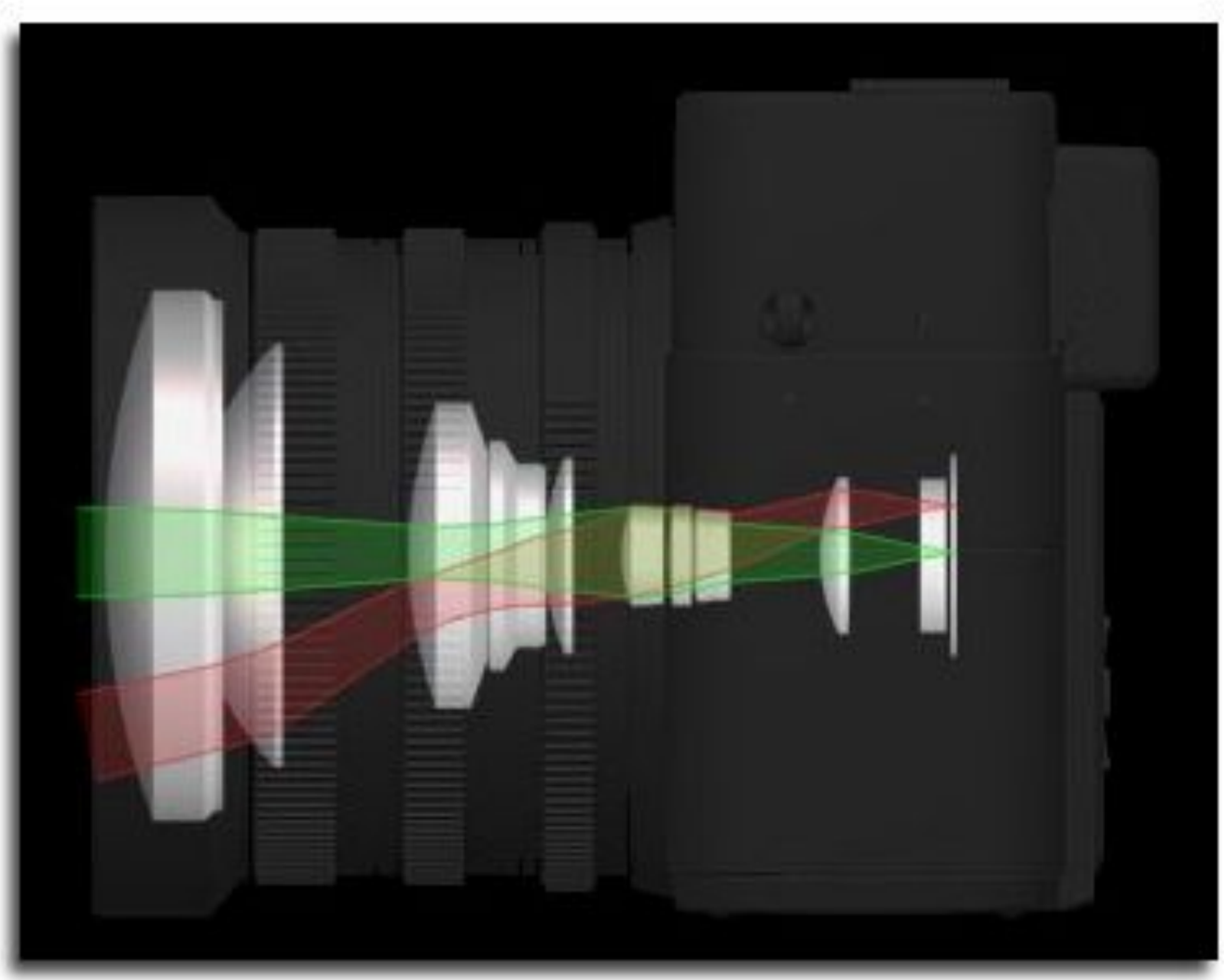


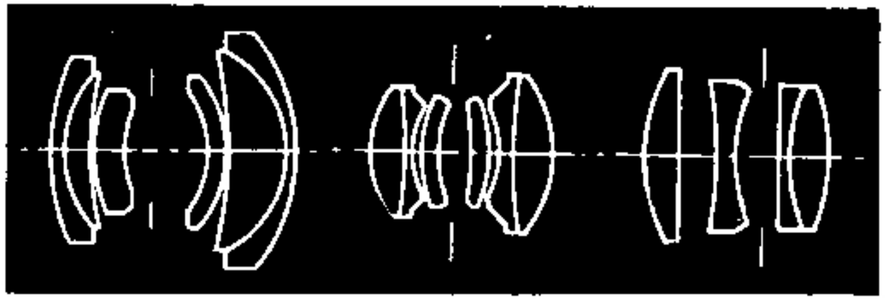
Spherical **Aberration**



Forsyth & Ponce (1st ed.) Figure 1.12a

Compound **Lens Systems**

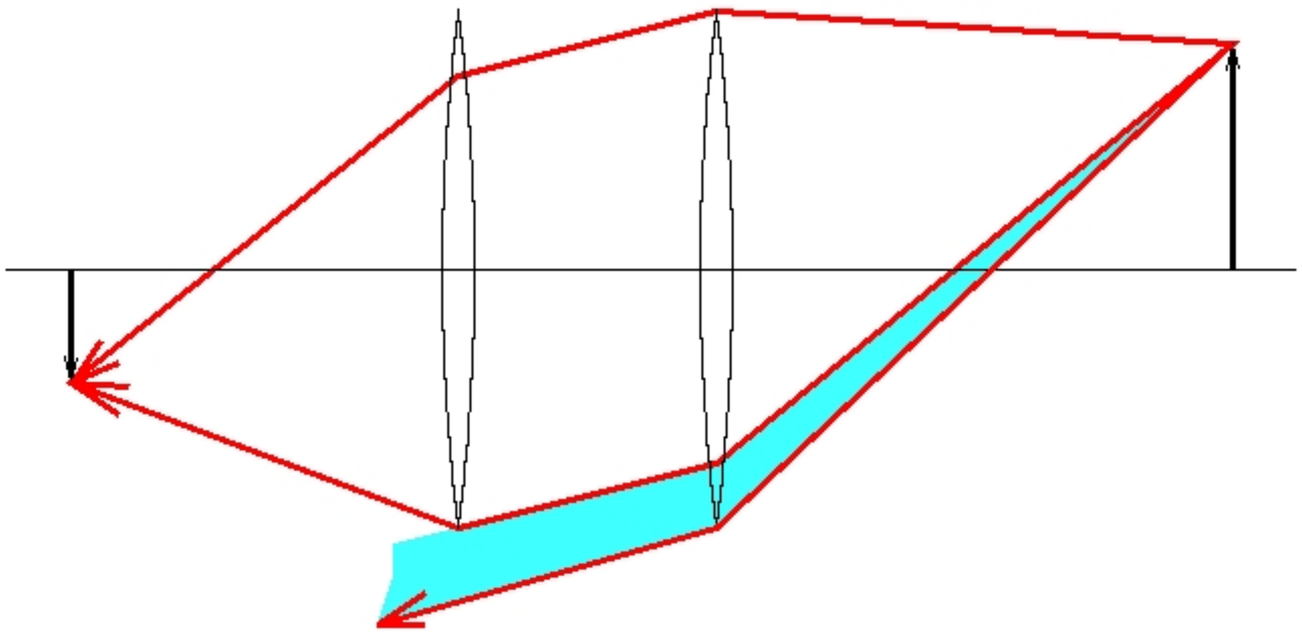




A modern camera lens may contain multiple components, including aspherical elements

Vignetting

Vignetting in a two-lens system



Forsyth & Ponce (2nd ed.) Figure 1.12

The shaded part of the beam **never reaches** the second lens

Vignetting

Image Credit: Cambridge in Colour

Chromatic **Aberration**

— Index of **refraction depends on wavelength**, λ, of light

— Light of different colours follows different paths

— Therefore, not all colours can be in equal focus

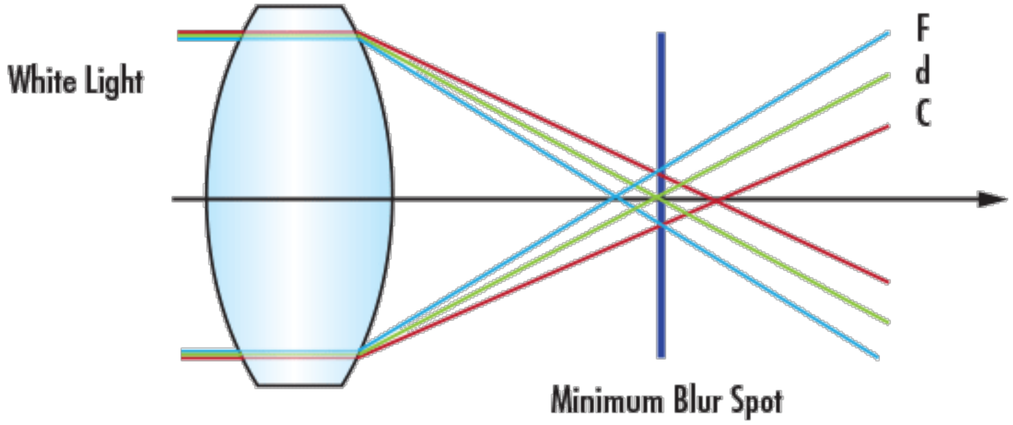


Image Credit: Trevor Darrell

Other (Possibly Significant) **Lens Effects**

Chromatic **aberration**

— Index of refraction depends on wavelength, λ, of light

— Light of different colours follows different paths

— Therefore, not all colours can be in equal focus

Scattering at the lens surface

— Some light is reflected at each lens surface

There are other **geometric phenomena/distortions**

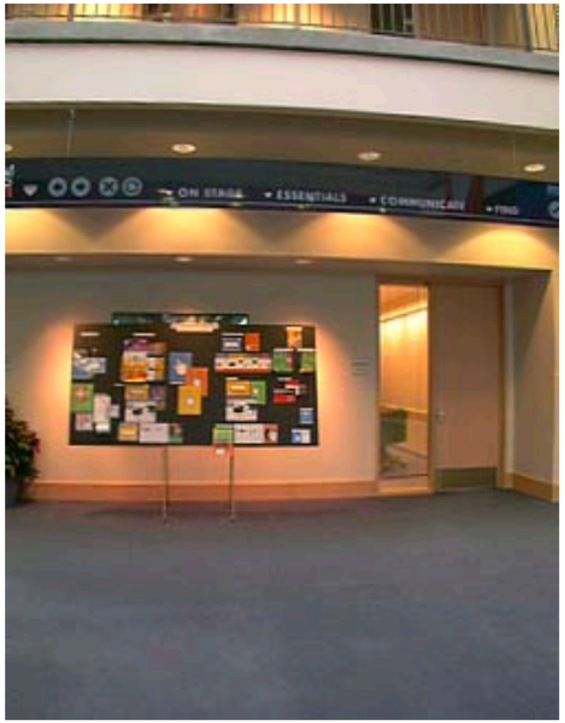
— pincushion distortion

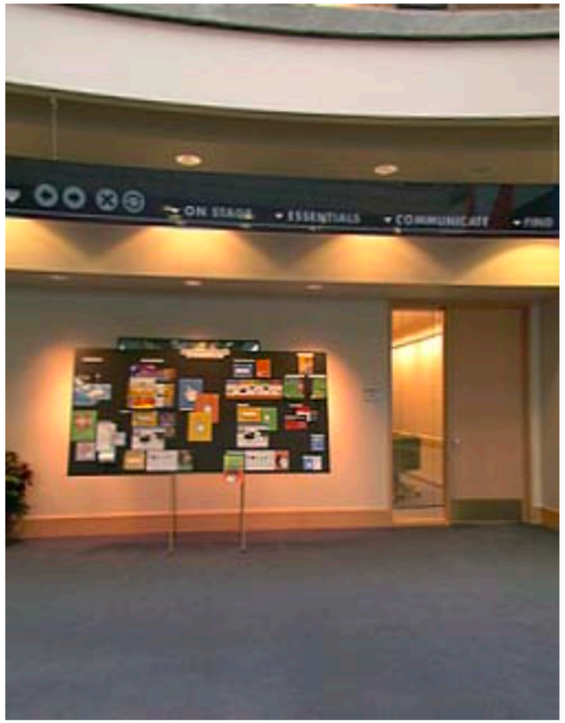
— barrel distortion

— etc

Lens **Distortion**

Fish-eye Lens





Szeliski (1st ed.) Figure 2.13

Lines in the world are no longer lines on the image, they are curves!

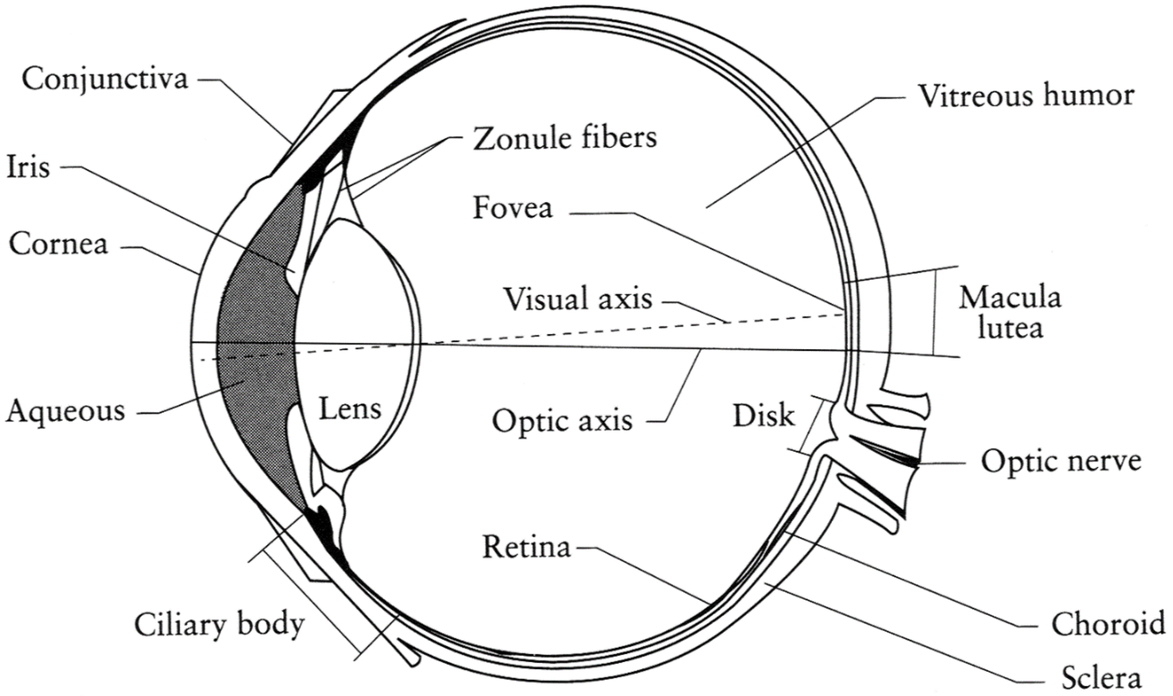
Human Eye

— The eye has an iris (like a camera)

— Focusing is done by changing shape of lens

— When the eye is properly focused, light from an object outside the eye is imaged on the retina

— The retina contains light receptors called rods and cones



pupil = pinhole / aperture

retina = film / digital sensor

Slide adopted from: Steve Seitz

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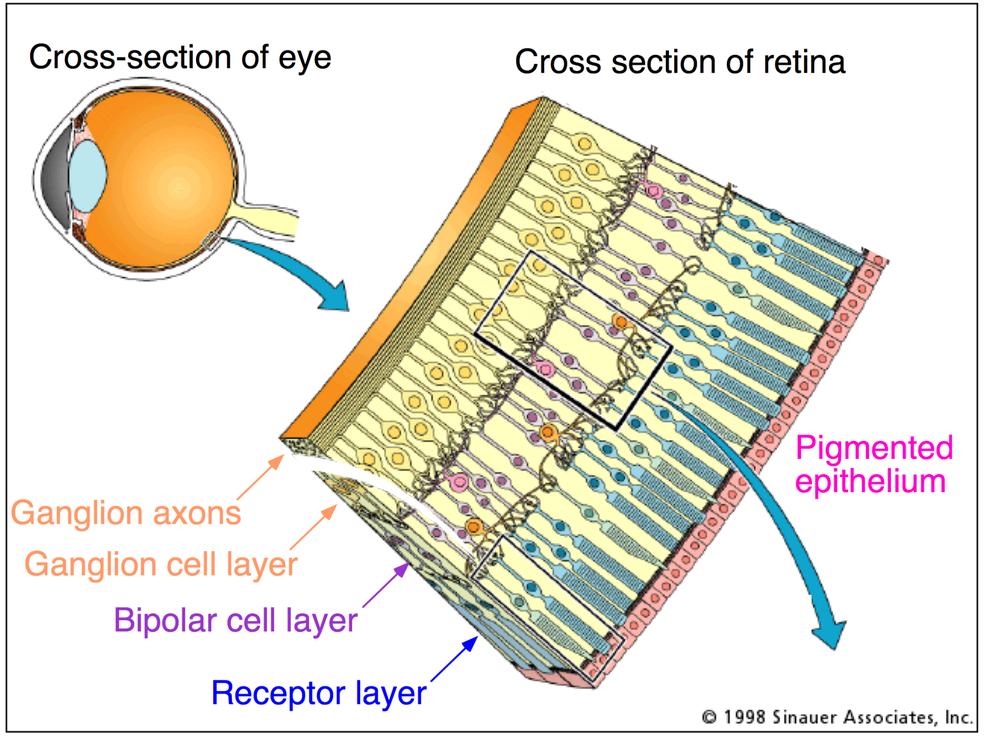


pupil = pinhole / aperture

retina = film / digital sensor

Slide adopted from: Steve Seitz

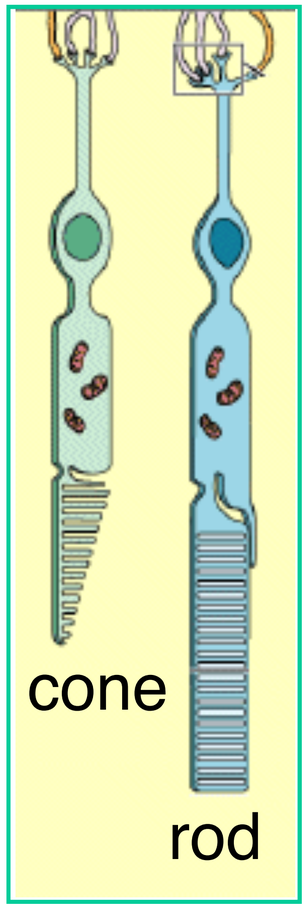
Two-types of **Light Sensitive Receptors**

Rods

75-150 million rod-shaped receptors
not involved in color vision, gray-scale vision only
operate at night
highly sensitive, can responding to a single photon
yield relatively poor spatial detail

Cones

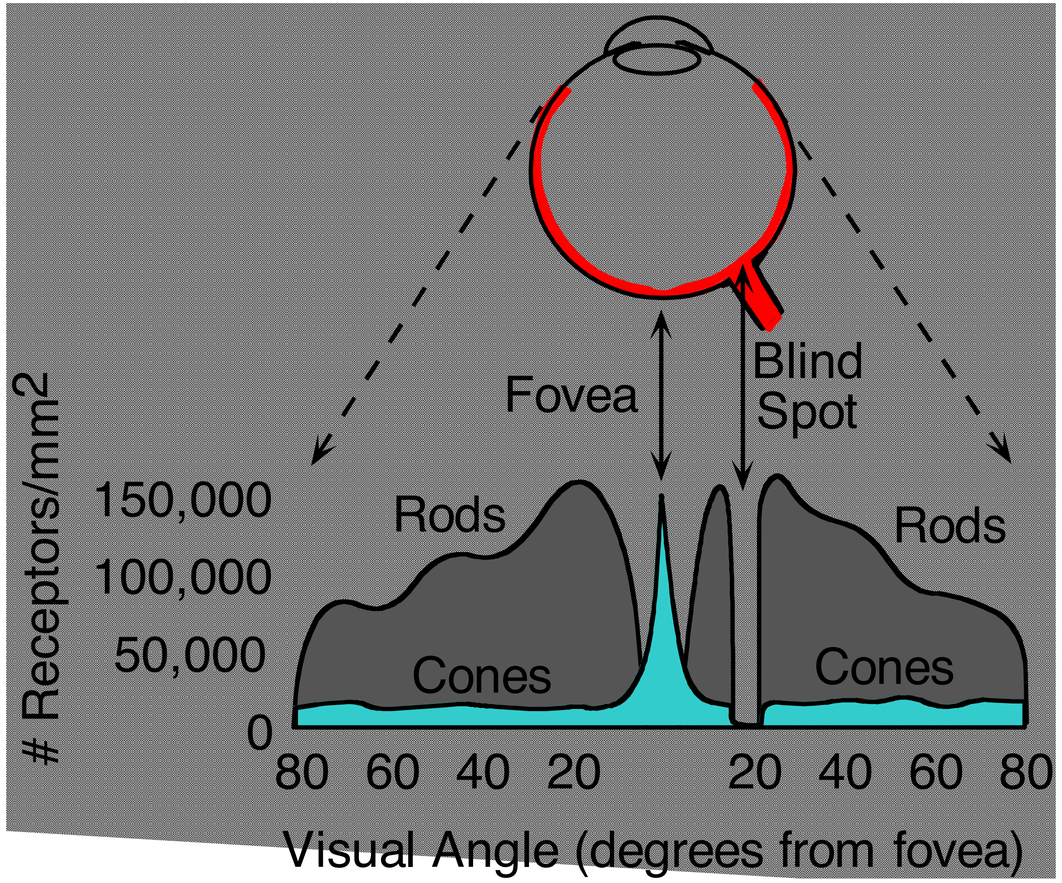
6-7 million cone-shaped receptors
color vision
operate in high light
less sensitive
yield higher resolution



Slide adopted from: James Hays

Human Eye

Density of rods and cones



Slide adopted from: James Hays

Lecture **Summary**

— We discussed a “physics-based” approach to image formation. Basic abstraction is the **pinhole camera**.

— **Lenses overcome limitations** of the pinhole model while trying to preserve it as a useful abstraction

— Projection equations: **perspective**, weak perspective, orthographic

— Thin lens equation

— Some “aberrations and **distortions**” persist (e.g. spherical aberration, vignetting)

— The **human eye** functions much like a camera

Reminders

Redings:

— **Today's** Lecture: Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3

— **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

Reminders:

— Complete **Assignment 0** (ungraded) by Wednsday, **September 12**

— **WWW**: http://www.cs.ubc.ca/~lsigal/teaching.html

— **Piazza**: piazza.com/ubc.ca/winterterm12018/cpsc425