# Optical thin film synthesis program based on the use of Fourier transforms

J. A. Dobrowolski and D. Lowe

In Sossi's formulation of the Fourier transform method of optical multilayer design the refractive-index profile is derived for an inhomogeneous layer of infinite extent having the desired spectral transmittance. This layer is then approximated by a finite system of discrete homogeneous layers. Because it does not make any assumptions about the refractive indices, thicknesses, or number of layers, it is the most powerful analytical method proposed so far. The method has been programmed for a computer and combined with other numerical design procedures. With the program it is possible to design filters with almost any desired transmittance characteristics using realistic refractive indices.

## I. Introduction

There exist two basic approaches to the synthesis of optical multilayer systems with irregular spectral characteristics. The numerical synthesis methods[1,2]—refinement, search, evolution, substractive method—do not make any assumptions about the thicknesses or refractive indices of the layers which may be absorbing or dispersive. The construction parameters can be easily constrained to be within reasonable limits. Generally any property of the multilayer that can be evaluated can be specified. But in some instances considerable expenditure of computer time is needed to obtain a satisfactory solution.

In the analytical synthesis methods[3,4] a series expansion of the desired spectral transmittance or reflectance curve is solved for the construction parameters of the required multilayer system. To make solution possible certain simplifying assumptions have to be made about either the thicknesses or refractive indices of the nonabsorbing, nondispersive layers. Solutions can be obtained quickly, but they sometimes call for refractive indices that cannot be realized in practice.

One particular analytical method of thin film synthesis is based on the use of Fourier transforms (FT). Delano[5,6] in 1966 seems to be the first to have described the basic principles of the method, although he does make reference to unpublished work by R. J. Pegis. In papers published in 1974 and 1976, Sossi[7,8] once again focused attention on the FT method. The computer program developed at the NRCC follows fairly closely his formulation of the solution. The method is a significant advance over the previously described analytical approaches because it is the first that does not make any assumptions about the number, thicknesses, or refractive indices of the individual films of the resulting multilayer coatings.

In this paper the properties of the FT thin film synthesis method and of the resulting layer systems are discussed at some length. The full potential of this design approach is only realized when the refractive indexes of the resulting multilayers are brought under control, and when the method is programmed for a computer and combined with numerical design procedures. This has been accomplished in the computer program developed at the NRCC.

## II. Basic Equations

For the purpose of this paper it is sufficient to quote without proof the essential equations necessary for the implementation of Sossi's formulation of the FT thin film synthesis method. For their derivation the interested readers are referred to the original work.[7,8]

Using as a basis two papers[9,10] on the calculation of the spectral transmission and reflection coefficients of an inhomogeneous layer, Sossi showed[8] that it is possible to relate the spectral transmittance of an inhomogeneous layer to its refractive-index profile $n(x)$ by the following approximate expression:

$$\int_{-\infty}^{\infty} \frac{dn}{dx} \cdot \frac{1}{2n} \cdot \exp(ikx) \cdot dx = Q(k) \cdot \exp[i\phi(k)] = f(k), \quad (1)$$

where $k = 2\pi/\lambda$. In the above equation $Q(k)$ is a suitable even function of the desired transmittance $T$ ex-

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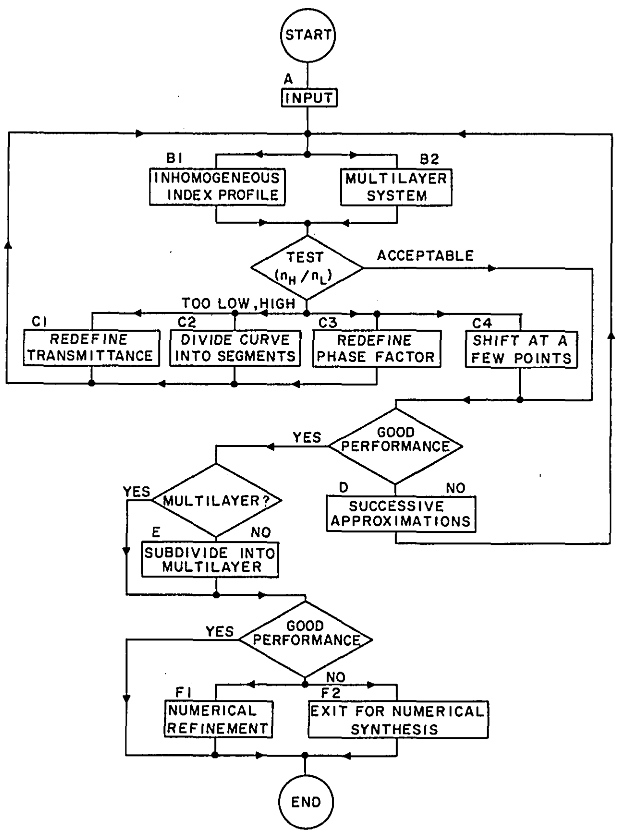


Fig. 1. Block diagram of the flow of calculations.

pressed as a function of the wave vector $k$. The variable $x$ is twice the optical path and is defined to be

$$x = 2 \int_0^z n(u) \cdot du, \tag{2}$$

where $z$ is the geometrical coordinate within the layer. $\phi(k)$ is a phase factor that must be an odd function to ensure that $n(x)$ is real. Note that $\phi$ is *not* the phase change on reflection or transmission that the incident radiation undergoes at the layer.

By applying a Fourier transformation to Eq. (1) and by integration with respect to $x$ one obtains

$$n(x) = \exp \left[ \frac{2}{\pi} \int_0^\infty \frac{Q(k)}{k} \sin \left[\phi(k) - kx\right] \cdot dk \right\}. \tag{3}$$

This is the refractive-index profile of a nonabsorbing nondispersive inhomogeneous layer of infinite extent with the desired spectral performance. The reader may feel that this is not an auspicious start, but we shall see below that with a few approximations and some refinement, practical multilayer designs having complex properties can be obtained with this method.

It is too difficult to derive an exact expression for $Q(k)$. Although there are many ways in which the function may be defined, success is contingent on a good approximation of $Q(k)$. For instance, Delano[5,6] uses the definition $Q(k) = (R/T)^{1/2}$. The definition used in Sossi's papers and adopted here,

$$Q(k) = \left\{ 1/2 \left[ \frac{1}{T(k)} - T(k) \right] \right\}^{1/2} \tag{4}$$

appears to yield better results. Sossi suggests[8] that there might be yet other definitions of $Q(k)$ leading to more accurate solutions. However, one advantage of the present definition is that the integration of Eq. (3) need only be carried out over the wave vector interval $k_L \le k \le k_H$ in which the desired spectral transmittance curve differs from unity.

Analytical integration of Eq. (3) is not possible for any but the simplest desired spectral transmittance curves $T(k)$. For most other cases numerical integration by computer is straightforward.

## III. Outline of the Program

A computer program for the synthesis of optical multilayer coatings based on the above equations has been developed and is being run at the NRCC on a PDP-11/40 RSTS time-sharing minicomputer system. Output may be obtained in printed or graphical form.

The computer program is written in FORTRAN IV language. Although an automatic sequencing of the calls of the various subroutines is possible, a conversational use of the program is preferred—the user can call different subroutines at will. A brief explanation of a possible flow of calculations will now be discussed with the aid of the schematic block diagram of Fig. 1. (The labels of the various blocks point to the parts of Sec. IV of this article, where a more detailed discussion of their operation is to be found.)

First the desired spectral transmittance $T(k)$ and the phase factors $\phi(k)$ are entered (Sec. IV.A).

Next the refractive-index profile $n(x)$ of an inhomogeneous layer whose transmittance approximates $T(k)$ is evaluated (Sec. IV.B.1). There is a possibility of calculating directly the construction parameters of a homogeneous multilayer system with the required transmittance (Sec. IV.B.2).

Several remedial measures can be taken if the ratio of the highest to the lowest refractive index of the resulting profile exceeds the maximum experimentally realizable value. The refractive index can be controlled by either redefining the transmittance outside the spectral region of interest (Sec. IV.C.1), by dividing the required curve into several segments (Sec. IV.C.2), or by redefining the phase factor (Sec. IV.C.3) and then recalculating the refractive-index profile. If the refractive indices lie outside the practical limits only at a few points, it is possible to bring them back into the permitted range by introducing discontinuous steps in the refractive-index profile without affecting the performance of the system in the spectral region of interest (Sec. IV.C.4).

Once the refractive indices lie within acceptable limits the performance of the system can be improved by successive iterations of the refractive-index profile calculations (Sec. IV.D).

An inhomogeneous layer with a satisfactory spectral transmittance is then subdivided into a multilayer system consisting of a number of homogeneous layers

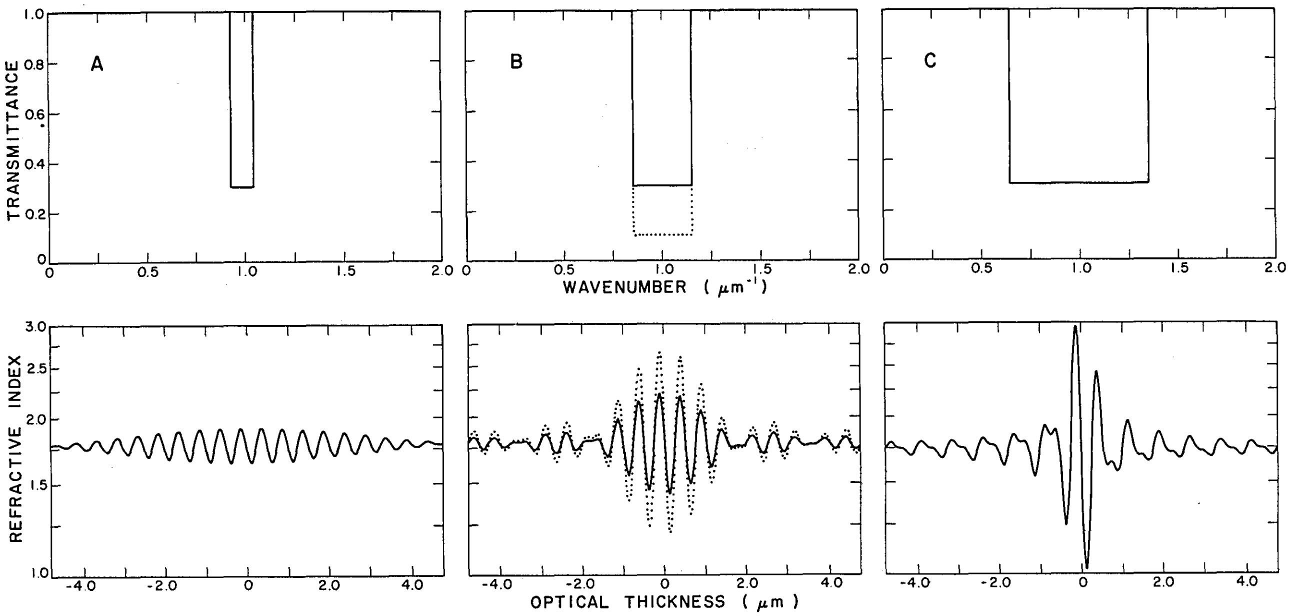


Fig. 2. Refractive-index profiles for filters with different bandwidths and rejections.

(Sec. IV.E). The reason for this operation is, of course, the fact that although inhomogeneous layers have been investigated frequently in the past,[11,12] they are much more difficult to construct.

The above operation will usually result in a deterioration of the performance. This latter can be upgraded again either through the use of a built-in numerical refinement routine (Sec. IV.F.1) or by calculations with an external numerical synthesis program (Sec. IV.F.2).

Results obtained with the program are presented in this paper as sets of two diagrams. The desired and the achieved transmittances and the phase factor will be plotted against wavelength ($\lambda$) or wavenumber ($\lambda^{-1}$) in one of the diagrams. The second diagram will display the corresponding refractive-index profile as a function of the optical thickness. This latter, in terms of the parameter $x$ of Eq. (2), is given by $nt = (x/4\pi)$. The origin of the thickness scale is at the center of the layer system. The spectral properties of the inhomogeneous layer are evaluated by approximating it with a homogeneous layer system consisting of about 400 layers and analyzing that system with conventional matrix methods. The refractive index is plotted on a logarithmic scale. Before plotting it is multiplied by

$$(1.35 \times 2.35/n_L n_H)^{1/2}, \tag{5}$$

where $n_L$, $n_H$ are the lowest and the highest refractive indices calculated from Eq. (3). Such a normalization centers the refractive indices onto the range of refractive indices of coating materials available for the visible part of the spectrum, yet it does not change the spectral transmittance curve.[13]

## IV. Details of the Method

### A. Specification of the Problem

All the input to the program is in free format. The operation of a program of this type has to be controlled by a number of parameters. These have been predefined to assume values that are suitable for an average problem in the visible part of the spectrum. It is possible, however, to change such parameters at any time during the calculations.

The only mandatory input is the required spectral performance. It is a property of analytical synthesis methods that it is necessary to define the transmittance curve not only in the spectral region of interest, but over the whole wavenumber spectrum. This can be variously viewed as a nuisance or as an additional degree of freedom with which to achieve the desired result. In the present program it is assumed at first that there is unit transmittance everywhere outside the range of interest. The advantage of such a course is that this reduces the range of integration in Eq. (3) (and hence computation time) to a minimum. Transmittances (and optionally, transmittance tolerances) need, therefore, only be entered for a number of wavelengths or wavenumbers within the spectral region of interest. Linear interpolation is used to provide additional intermediate points. The phase factor $\phi(k)$ will be assumed to be zero unless it is otherwise defined.

It is important to appreciate the effect of problem statement on the resulting refractive-index profile. For example, the refractive-index profiles of minus filters with a 70% rejection and with bandwidths of 10%, 30%, and 70% are shown in Figs. 2(A)–2(C). It will be seen

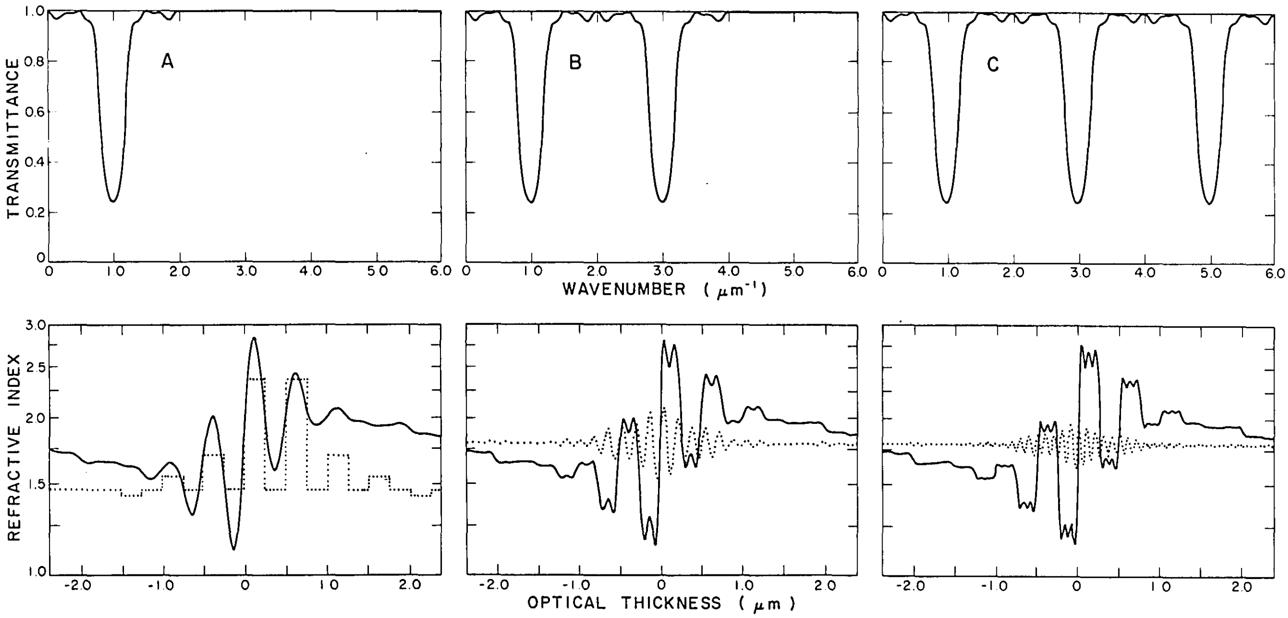


Fig. 3. Effect of higher wavenumber harmonics of the primary transmission curve on the refractive-index profile.

that the refractive-index ratio $(n_H/n_L)$ increases and that the optical thickness of the inhomogeneous layer with significant refractive-index variations decreases with the bandwidth. In this and in subsequent examples a phase factor $\phi(k) = 0$ was used, unless otherwise stated.

It follows from Eq. (4) that one may not specify zero transmittance. In actual practice one may approach zero as closely as one wishes, but only at the expense of increasing the refractive-index ratio $(n_H/n_L)$. This is illustrated by the dotted curves of Fig. 2(B) corresponding to a minus filter with a 90% rejection.

Specifying different transmittance values at high wavenumbers outside the spectral region of interest does not lead to different generic solutions. Suppose that the transmittance is specified in several distinct and nonoverlapping spectral regions $A, B, C, \ldots$. It follows from Eq. (3) that the refractive-index profile of the combined transmittance curve is given by

$$n(x) = n_A(x) \cdot n_B(x) \cdot n_C(x) \ldots, \tag{6}$$

where $n_A(x)$, $n_B(x)$, $n_C(x) \ldots$ are the refractive-index profiles corresponding to the transmission regions $A, B, C \ldots$. The superposition or refractive-index profiles with higher frequency modulations onto the profile $n_A$ will result in relatively small modifications of the latter and not in major changes. This is illustrated in Figs. 3(A)–3(C) where the transmittance curves specified consist of 1, 2, and 3 successive minima of a multilayer minus filter whose construction parameters are given by the dotted curve of Fig. 3(A). The dotted curves of Figs. 3(B) and 3(C) are the refractive-index profiles of the second and third harmonics of the original curve.

It follows also from Eq. (6) that specifying transmittance features on the low wavenumber side of the spectral region of interest will have a significant effect on the refractive-index profile. One extreme application of this principle is as follows. The FT method usually yields solutions in which the multilayer filter is embedded between two media of essentially the same index. One way to match such a filter into another medium is with a suitable set of antireflecting layers found with a numerical synthesis method. But it is also easy to specify a substrate in the FT method by calling for a constant transmittance at low wavenumbers given by

$$T = 1 - \left(\frac{n_s - 1}{n_s + 1}\right)^2, \tag{7}$$

where $n_s$ is the refractive index of the substrate. Since a thin film cannot affect the transmittance at low wavenumbers, the solution yields a semiinfinite film of the appropriate index, i.e., the substrate.

### B. Determination of the Thin Film System

#### 1. *Refractive-Index Profile Determination for an Inhomogeneous Layer*

After all the necessary input has been completed the refractive-index profile of a suitable inhomogeneous layer of infinite extent can be calculated from Eq. (3). Numerical integration based on the trapezoid rule is used. This essential part of the synthesis process requires on the average less than 30 sec of CPU time and costs less than three cents on our computer installation.

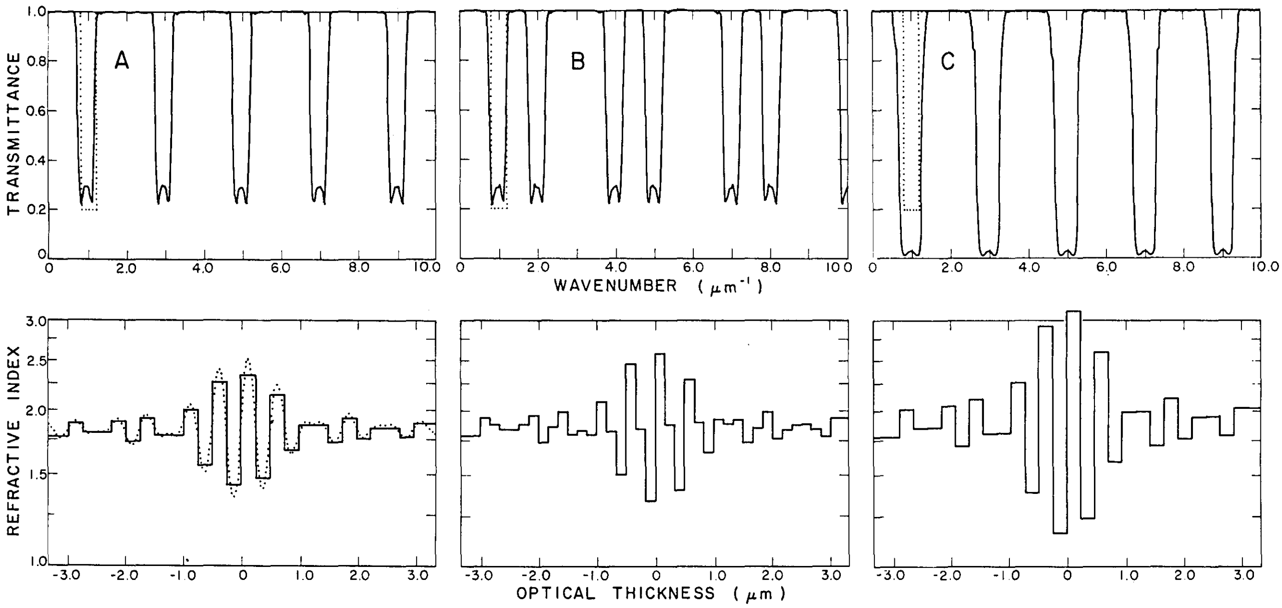


Fig. 4. Homogeneous layer solutions obtained by the use of the replicating function with wavenumber displacements of 2 $\mu m^{-1}$, 1.5 $\mu m^{-1}$, and 1 $\mu m^{-1}$, respectively.

*2. Determination of a Homogeneous Multilayer System*

Frequently the transmittance may be allowed to depart from unity at wavenumbers higher than those of the spectral regions of interest. In such cases it is possible to bypass the determination of an inhomogeneous layer and to obtain the parameters of a homogeneous multilayer system with the desired spectral characteristics.

Note that in Fig. 3 the refractive-index profile became increasingly more angular with the addition of higher order odd harmonics of the desired spectral transmittance curve. This will be true in general. Let us suppose that the desired spectral transmittance curve can be completely defined in the wave vector interval $o \leq k \leq p$ and that the transmittance is unity outside this range. Let $p$ be the wave vector displacement between adjacent harmonics, and let $m$ such harmonics be added to the original curve. By using the Fourier transform shift theorem[14] and by summation it can be shown that the resulting refractive-index profile will be given by

$$n(x) = \exp\left(\frac{1}{\pi}\int_o^p Q(k)\cdot\sum_{m=0}^{m}\left\{\frac{\sin[\phi(k)-(mp+k)x]}{(mp+k)}\right\}\cdot dk\right). \tag{8}$$

This profile can be readily subdivided into a set of homogeneous layers.

But the desired transmission curve can also be readily repeated throughout the whole wave vector spectrum at intervals $p$ through the use of the ш replicating function.[14] Using the properties of this function and the convolution theorem[14] the following expression for the refractive-index profile is obtained:

$$n(x) = \exp\left[\frac{4}{p}\sum_{m=-\infty}^{\infty} F\left(\frac{m}{p}\right)\right], \tag{9}$$

where $F(x)$ is the Fourier transform of $f(k)$ [Eq. (1)]. In the above expression $n(x)$ is evaluated only for values $x = (m/p)$, $m = 0, \pm1, \pm2, \ldots$. For intermediate values of $x$ the refractive index $n$ has constant values. Clearly it takes much less time to evaluate this equation.

The method is illustrated by the design of a narrowband rejection filter with a minimum transmission of 20% [dotted curve, Fig. 4(A)]. The dotted curve in the lower diagram represents the refractive-index profile of an inhomogeneous layer obtained when Eq. (3) is applied to the above specification. The full lines in this diagram represent the construction parameters and the transmittance of the homogeneous multilayer system obtained from Eq. (9) with a value of $p = 4\pi$. Figures 4(B) and 4(C) show that with a suitable choice of $p$ ($3\pi$, $2\pi$ in these instances) rather complicated transmittance patterns can be obtained.

It has thus been demonstrated that the FT synthesis method can, in certain circumstances, be made to yield homogeneous multilayer solutions to problems. But, since with this approach the optical thicknesses of all the layers are an integer multiple of some basic thickness, a certain generality is lost.

## C. Refractive-Index Control

The FT thin film synthesis method will often result in solutions that call for refractive indices that lie outside the range of values of available materials. If such violations are small, their rectification can be deferred until later. But if not, to end up with a practical multilayer system it will be necessary to modify such solu-

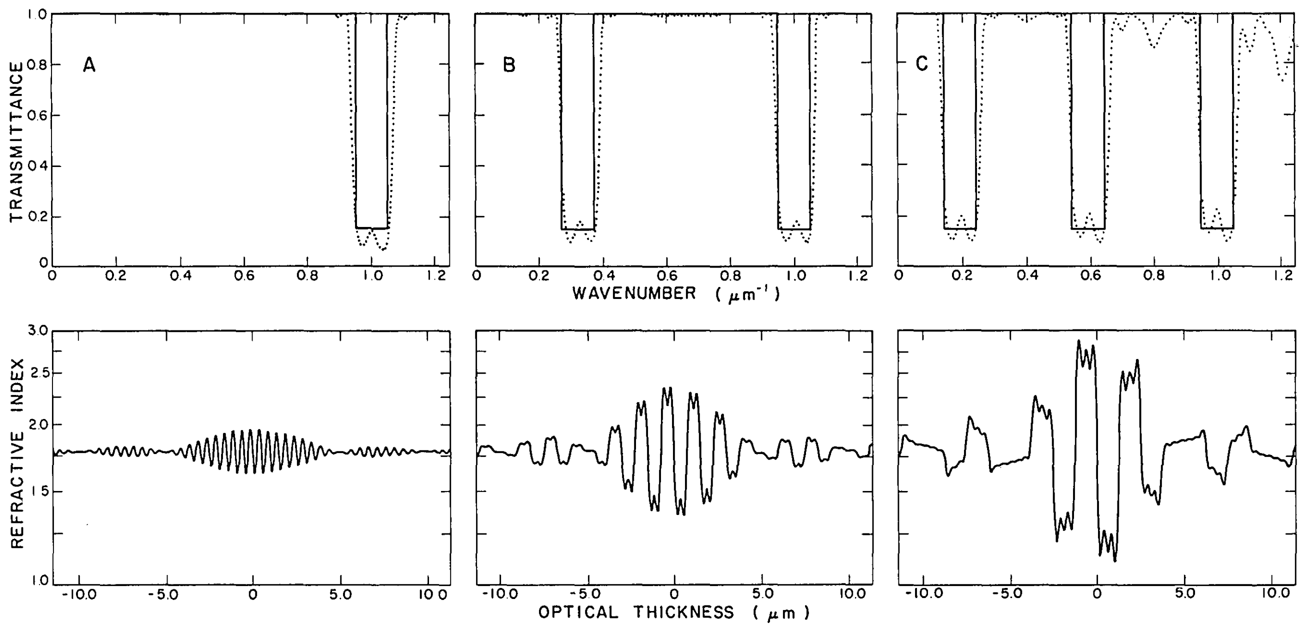


Fig. 5. Effect of lower wavenumber harmonics of the primary transmission curve on the refractive-index profile.

tions or to suitably rephrase the problem and start from the beginning. There are several different ways of controlling the refractive index.

*1. Modification of the Spectral Transmittance Curve*

The refractive-index profile can be modified to a certain extent by suitably specifying the transmittance at wavenumbers lower than those corresponding to the spectral region of interest. For example, it is possible to increase the refractive index ratio ($n_H/n_L$) and to reduce the number of layers (but not the over-all thickness) of a system giving a narrowband rejection in the visible part of the spectrum by specifying lower order harmonics of the required transmittance curve [Figs. 5(A)–5(C)].

*2. Division of Transmittance Curve into Several Segments*

A technique used in the minus filter method of thin film design[15] can also be employed. Suppose that the refractive-index profiles $n_A(x)$, $n_B(x)$, $n_C(x)\ldots$ yield transmittances $T_A(k)$, $T_B(k)$, $T_C(k)\ldots$, which are unity everywhere except in distinct and nonoverlapping spectral regions $A, B, C\ldots$. It has been shown[15] that the combined transmittance curve $T(k) = T_A(k)\cdot T_B(k)\cdot T_C(k)\ldots$ is obtained not only with an inhomogeneous layer having a refractive-index profile given by Eq. (6), but also with the refractive-index profile

$$n(x) = n_A(x) + n_B(x) + n_C(x) + \ldots, \qquad (10)$$

where the above summation is taken to mean that the multilayers are deposited onto one another. The approach can be used, for example, to reduce the refractive-index ratio ($n_H/n_L$) of the refractive-index profile of a broadband attenuator by specifying the rejection region in two parts. This was done for the case of a 50% broadband attenuator. The results were not very spectacular—it was difficult to reduce by refinement all the ripples in the transmittance curve introduced by this procedure. It is possible though that the method might be useful for other cases.

*3. Redefinition of the Phase Factor*

The most satisfactory method of controlling the refractive index is through the control of the shape of the refractive-index profile by means of the phase factor $\phi(k)$. It follows from Eq. (3) that the refractive-index profile $n(x)$, when plotted on a logarithmic scale, is symmetric or skew symmetric about the point $x = 0$ when the phase factor assumes constant values of 0 or $\pi/2$, respectively. The modulation of the refractive-index profile is largest around $x = 0$ and increases with the general departure of the desired transmittance curve from unity. The envelope of the refractive-index profile does not materially depend on the value of the constant phase factor. To illustrate the above points, refractive-index profiles were evaluated using Eq. (3) for a 50% attenuator for a 3:1 wavelength range, for phase factors $\phi(k) = 0$ and $\pi/2$ [Figs. 6(A) and 6(B)].

From the construction point of view, refractive-index profiles with more rectangular envelopes would be very desirable. One would expect them to result in lower refractive-index ratios. The clue to the control of the refractive-index profiles is the phase factor $\phi(k)$. The

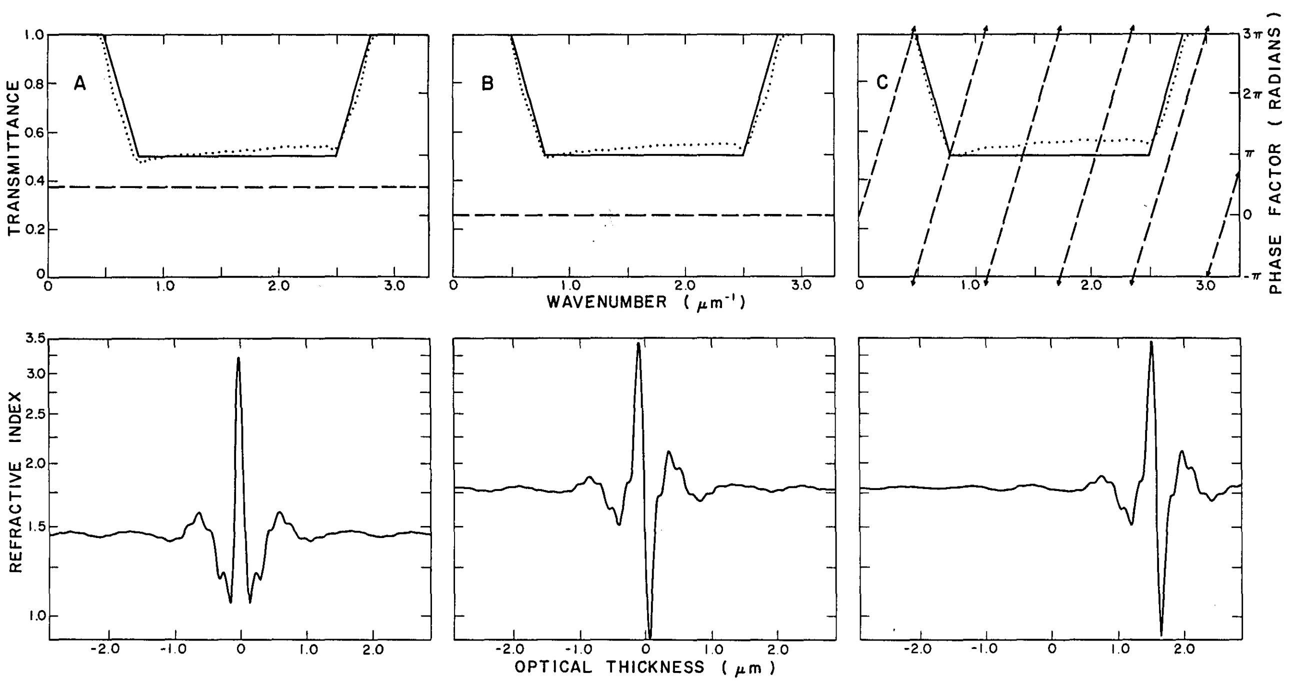


Fig. 6. Effect of the phase factor on the refractive-index profile.

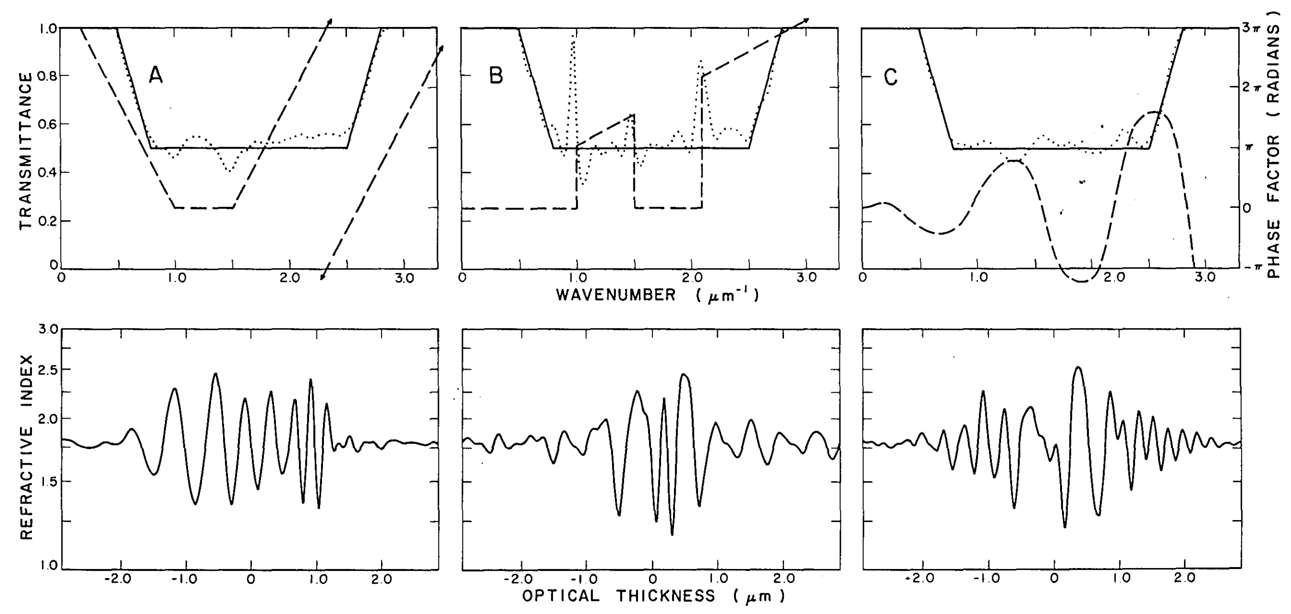


Fig. 7. Use of the phase factor to control the refractive-index profile.

results of some preliminary numerical experiments will now be presented.

It follows from Eq. (3) that a phase factor of the type

$$\phi(k) = ck, \tag{11}$$

where $c$ is a constant, merely results in a displacement $c$ of the refractive-index profile along the $x$ axis [Fig. 6(C)].

Suppose that in any given problem we divide the desired spectral transmittance curve into two parts such that each has a refractive-index profile with approximately the same value of $(n_H/n_L)$. Suppose also that the average optical path between successive maxima of the two refractive-index profiles is $2c$. Then if we define the phase factors for the two parts of the curve to be $2ck$, 0, the centers of symmetry of the two refractive-index profiles will be displaced from one another sufficiently for the central maxima of one to coincide with the central minima of the other, and vice versa. It

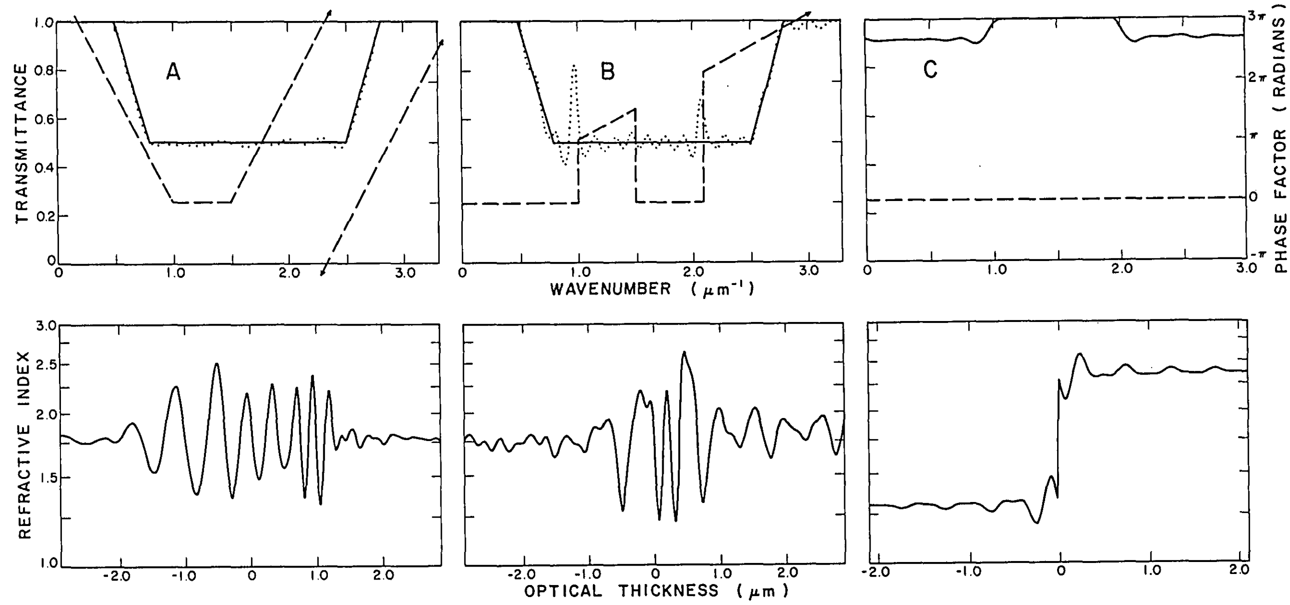


Fig. 8. Illustration of refinement by successive approximations (A), (B) and of refractive-index steps that do not change the transmittance in a designated wavenumber range (C).

follows from Eq. (6) that the ratio $(n_H/n_L)$ for the combined system should be considerably reduced. This is illustrated in Fig. 7(B).

A more rectangular refractive-index profile will clearly be obtained if the refractive-index profiles of the two parts of the transmittance curve are displaced by larger amounts, or even if the curve is divided into more than two segments, each with a different definition of the phase factor. This is illustrated in Fig. 7(A). The minus filter approach mentioned previously [Eq. (10)] may be regarded as being a special case of the above in which the two refractive-index profiles have been sheared clear of each other.

It is also evident that when different phase factors are assigned to different parts of the transmittance curve, there will be regions in the inhomogeneous layer that are more closely associated with these parts than with others. One visible evidence of this will be the variation in the thicknesses of the individual films of the resulting multilayer. This may or may not be considered an advantage at the time of manufacture. One way of avoiding this effect is to interleave two or more sets of transmission regions with different phase factors [Fig. 7(B)].

We noticed that for refractive-index profiles for which nonconstant phase factors have been specified, the calculated transmittances differed significantly from the desired values at those wavenumbers at which discontinuities occurred in the derivative of the phase factor [Fig. 7(B)]. Although these discrepancies can be corrected by refinement (see Sec. IV.D), it seems desirable to define a continuously variable phase factor. A definition we experimented with,

$$\phi(k) = ck \sin(hk), \tag{12}$$

where $c$ and $h$ are constants, gave encouraging results [Fig. 7(C)]. Clearly more work is needed in this area.

### 4. *Control at Individual Points*

Sossi showed[8] that it is possible to increase or decrease the refractive-index profile in one or more intervals without affecting the resulting transmittance $T(k)$ in the wavevector region $k_L \le k \le k_H$. The refractive-index profile is changed from $n(x)$ to $n_1(x)$ in the following way:

$$n_1(x) = N(x) \cdot n(x), \tag{13}$$

where $N(x)$ is given by

$$N(x) = \exp\left(2 \sum_{j=1}^{m} c_j\right) \prod_j \exp\{Si[k_L(x - x_j)] - Si[k_H(x - x_j)]\}. \tag{14}$$

$Si(x)$ is the integral of $[\sin(t)/t]$ from $t = 0$ to $t = x$.

In the above $x_j$ are the points at which refractive index discontinuities occur, and the constants $c_j$ determine their magnitudes. Figure 8(C) is an example of the use of this method to introduce a refractive-index step in a homogeneous medium of infinite extent without affecting the transmittance in a certain spectral region. Unfortunately $N(x)$ does not always converge rapidly to constant values. We found our embodiment of the method useful only for the simplest cases. In more complicated examples we obtained better results with other methods.

## D. Refinement by Successive Approximations

Suppose that when the refractive-index profile of an inhomogeneous layer was evaluated from Eq. (3) the numerical integration took place over a sufficiently close

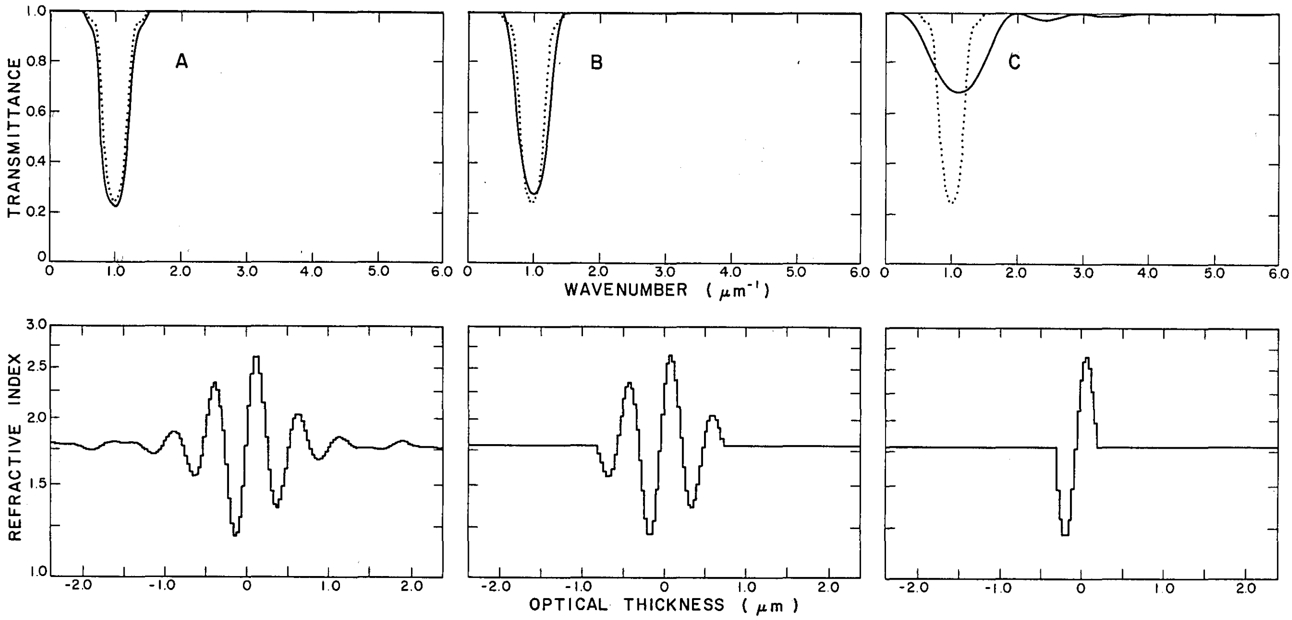


Fig. 9. Effect of limiting the optical thickness of the inhomogeneous layer on the transmittance curve.

wavenumber net, and that when its performance was evaluated with conventional matrix methods the layer was subdivided into a large number of very thin films. The performance of such a layer, or, for that matter, of a homogeneous multilayer system obtained from Eqs. (8) or (9) may still not reasonably match the desired spectral transmittance. This will be because of the approximate nature of the definition of $Q(k)$ [Eq. (4)]. We have observed significant departures for values of $T$ less than 0.40 and for narrowband features. In the future better definitions for $Q(k)$ might be found. In the meantime it is an easy matter to improve the performance by a successive approximation technique.[7]

The method consists of evaluating the quantity $Q_A(k)$ corresponding to the new transmittance. The quantity $[Q(k) - Q_A(k)]$ is then added to the value of $Q(k)$ used previously. Equations (3), (8), or (9) are then resolved to obtain a new refractive-index profile. This procedure is fast and can be repeated a number of times. Figures 8(A) and 8(B) represent the results obtained when the systems of Figs. 7(A) and 7(B) were corrected in this way.

## E. Subdivision into a Multilayer System

Once the spectral transmittance curve is satisfactory, the synthesis process is complete for a multilayer system evaluated with Eq. (9). But an inhomogeneous layer has still to be subdivided into a finite number of discrete layers. Control over the calculations is retained through the specification of a minimum index difference and a maximum layer thickness.

There are two aspects to this process. The inhomogeneous layer of infinite extent has to be reduced to a layer of finite thickness. Fortunately, for all the cases examined the deterioration in the performance was not serious providing that modulations of the refractive index in excess of 5% of the ratio $n_H/n_L$ were retained. This is illustrated in Figs. 9(A)–9(C). How closely the refractive-index profile must be approximated within this range depends on the wavenumber region over which the desired transmittance curve is defined. The thickness steps will, of course, depend on the steepness of the refractive-index profile; but a good first estimate for the average thickness would be $0.125/k_H$. The effect of different degrees of approximation of the extent of the inhomogeneous layer are shown in Figs. 10(A)–10(C). In the interest of reducing the total number of layers in the system it may be desirable to accept a significant deterioration of the transmittance and to refine the resulting construction parameters by one or more of the methods described below.

## F. Numerical Refinement and Synthesis Methods

The spectral transmittance of the homogeneous multilayer system which approximates the inhomogeneous layer may, if the number of layers is reasonably small, depart considerably from the desired transmittance curve. We found that numerical refinement methods[16] applied to such systems gave excellent results.

Unlike the successive approximation method of Sec. IV.D, they utilize merit functions that are based only on the spectral region of interest. The performance outside that region is allowed to vary, thus introducing extra degrees of freedom. The merit functions are defined in terms of transmittances and their tolerances, as described in Ref. 16. This is a more direct approach than the use of the function $Q(k)$. Matrix methods are

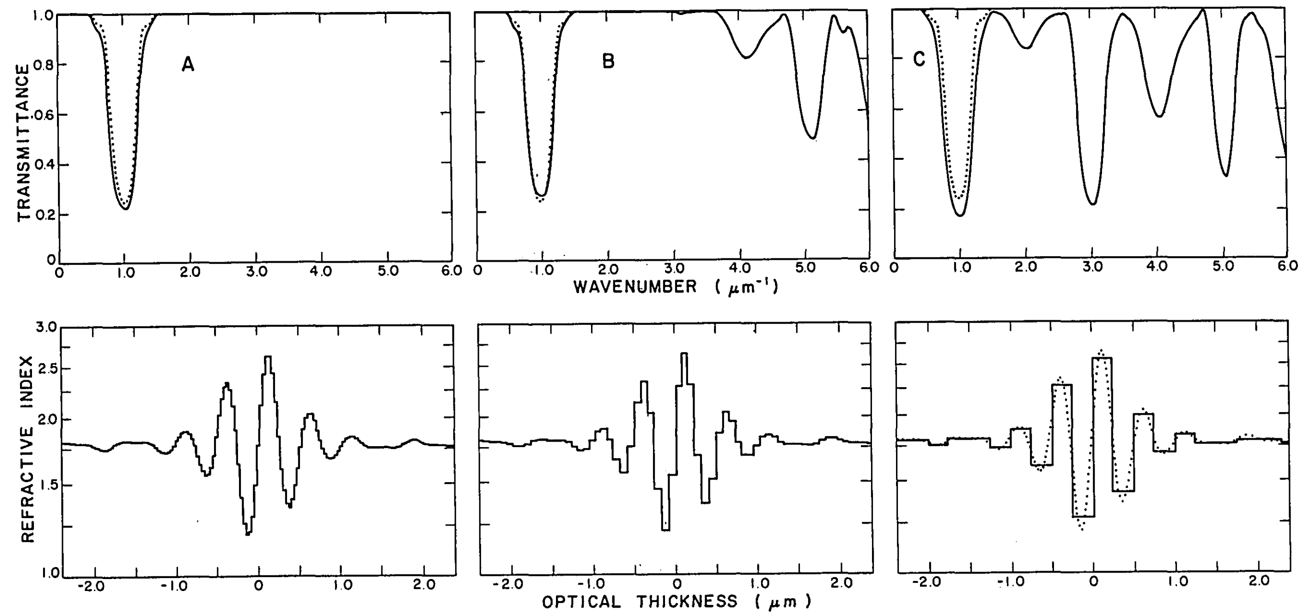


Fig. 10. Effect of different approximations of the inhomogeneous layer on the transmittance curve.

used to calculate the transmittance of the system. Partial products of the layer matrices are stored to reduce computation time. The parameters of the system in the present version of the program are varied one at a time to minimize the merit function. At all stages of the refinement complete control is exercised over the construction parameters of the system. With the successive correction method further refinement is needed once the inhomogeneous layer is approximated by a multilayer system to bring the refractive indices in line with those corresponding to real materials. Numerical refinement methods offer some control over the over-all fit to the desired spectral transmittance curve.

The FT synthesis method is particularly powerful when combined with the numerical complete search or gradual evolution synthesis methods.[16] These easily correct any errors that may have arisen due to the approximations inherent in the FT method or the degradation in performance due to the approximation of the inhomogeneous layer by a homogeneous multilayer system of finite extent. The method of gradual evolution is particularly effective in matching the system to other media. This method has been used in the example of Fig. 11.

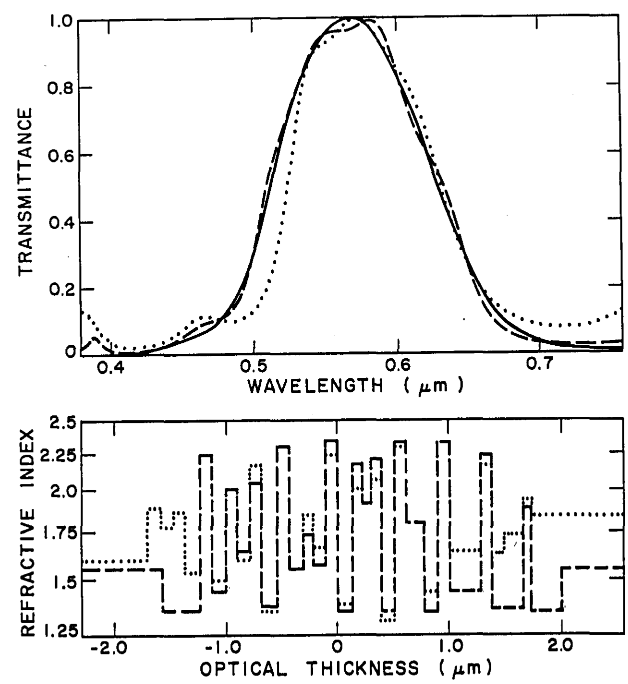


Fig. 11. $\bar{y}_\lambda$ filter.

## V. Examples

The examples in this paper are intended only for the comparison of solutions obtained with the FT method with those obtained with other thin film synthesis programs. Dispersion of the refractive indices and residual absorption within the dielectric layers have not been allowed for since they do not materially affect the solution. Any refractive indices lying within reasonable upper and lower limits were accepted. In practical systems the number of different refractive indices would have to be reduced. This would necessitate further calculations with numerical synthesis methods.

### A. Comb Filter

The design of a comb filter with alternate 500-Å wide transmission and rejection bands in the 0.4–0.7-$\mu$m

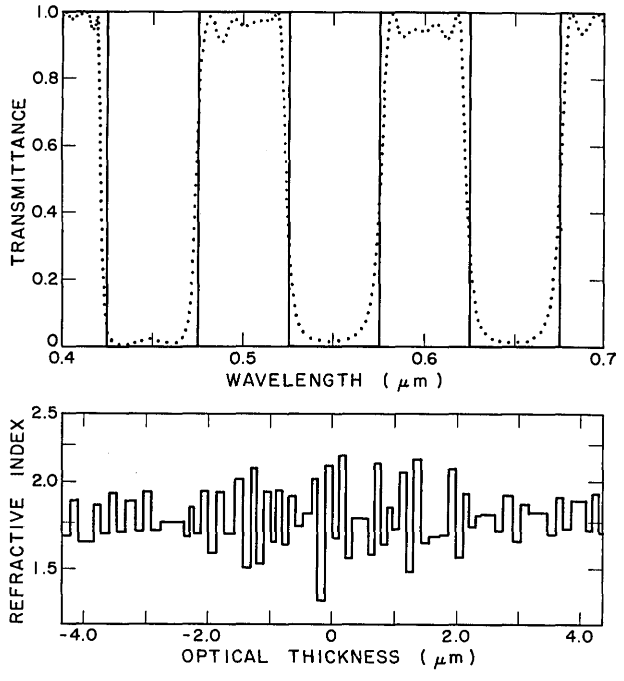


Fig. 12. Comb filter.

spectral region has been obtained previously with the numerical minus filter method of thin film design.[15] The performance of the sixty-one-layer filter of total optical thickness of approximately 25 μm was very good. The FT method yielded a solution with an equivalent performance consisting of sixty-two layers, but with a total optical thickness of only 8.5 μm (Fig. 12). This is clearly a superior solution in view of the substantial reduction in the over-all thickness of the filter. It was obtained by assuming $\phi(k) = 0$ when calculating the refractive-index profile. The initial solution was refined numerically to achieve the performance shown.

### B. $\bar{y}\lambda$ Tristimulus Filter

In Refs. 15 and 17 programs for the synthesis of multilayer coatings based on gradual evolution and on the use of minus filters were used to design $\bar{y}_\lambda$ tristimulus filters. The former method yielded a solution that consisted of seventeen layers and two absorption filters. The calculated peak transmittance of the combined filter was 87%, and the maximum departure from the required shape was ±4%. The minus filter method yielded a twenty-four-layer solution without the use of auxiliary glass filters. The calculated peak transmittance was 100%, and the maximum departure from the required curve was ±5%.

The results obtained for the same problem with the FT method are shown in Fig. 11. After applying the successive approximations refinement method the inhomogeneous layer was divided into twenty-nine layers (dotted curves). Note that the substrate and medium indices were 1.59 and 1.84, respectively. To match the layer system into media of index 1.55 and to improve its performance, the layer system was reduced to twenty-two layers by combining layers with similar indices and by removing several outside layers. The results obtained after calculations with a program for thin film synthesis by gradual evolution are represented in Fig. 11 by dashed lines. The system consists now of twenty-six layers and has a spectral transmittance curve with a maximum departure of ±3% from the desired shape.

The layer systems given in Figs. 11 and 12 are not yet, of course, in a form suitable for experimental realization. Many more calculations are needed to reduce the number of different refractive indices and to allow for dispersion and absorption. This particular topic will not be discussed here.

## VI. Conclusions

The FT inhomogeneous layer synthesis method offers a good insight into the nature of thin film systems. For instance, the following conclusions can be drawn from the above about the design of a coating with an irregular and nonperiodic transmittance curve defined over a wide spectral region. The general shape of the refractive-index profile will be governed by the spectral transmittance curve at the low wavenumber end of the region of interest. The required transmittance curve at the higher wavenumbers will be obtained through a fine modulation of the coarse features of the refractive-index profile. The inhomogeneous layer will have to be subdivided into many layers if its high wavenumber performance is not to be degraded. When used with a nonconstant phase factor and combined with numerical refinement and synthesis techniques, the FT synthesis method is a very powerful tool for the design of multilayer coatings with unusual spectral transmittance characteristics. Programmed for a computer it

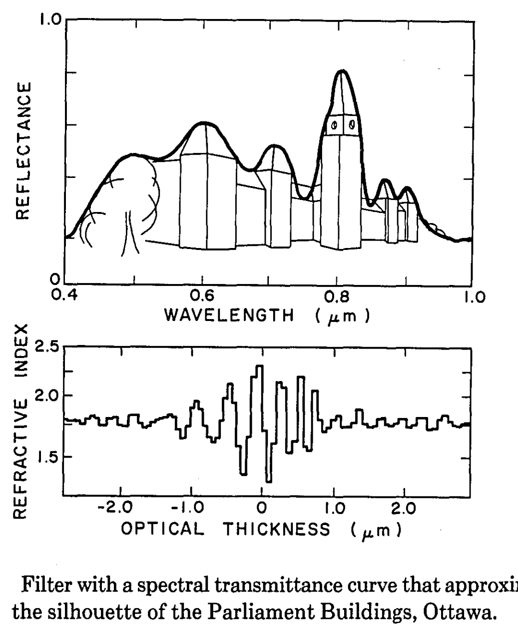


Fig. 13. Filter with a spectral transmittance curve that approximates the silhouette of the Parliament Buildings, Ottawa.

yields solutions to problems quickly and cheaply. For instance, all the calculations for the example of Fig. 12 (input, refractive index profile calculations, subdivision into layers, refinement, plotting) took 3 min and cost about $3 on our computer.

It is evident that, with the techniques described in this paper, it is possible, using realistic refractive indices, to design multilayer coatings with almost any desired spectral transmittance curves (Fig. 13). The practical realization of such filters is, of course, another matter; and this is the area where future progress must be made. In particular, the practicability of constructing inhomogeneous layers might be reconsidered in the future.

Solutions obtained with the FT method are not unique even when the transmittance curve is defined over the whole wavenumber region. The reason for this is that there are an infinite number of ways of specifying the phase factor. It has been shown that this quantity plays an important part in the control of the refractive-index profile. When the transmittance curve needs to be controlled only in a finite spectral region, the number of possible solutions increases still further. Examples have also been given to indicate how the refractive-index ratio ($n_H/n_L$) can be controlled through an appropriate specification of the transmittance curve at wavenumbers below those of the region of interest.

Although the FT method is so powerful, it cannot be applied to all classes of thin film problems. Solutions obtained with the present program have to be regarded as a starting point for further calculations with numerical refinement programs to allow for the dispersion and small residual absorptions of real dielectric materials. The FT method is of no help in the design of systems containing strongly absorbing layers. This is the case in an increasing number of problems. Numerical synthesis methods[16] must also be used whenever the resulting multilayer system must meet more complicated specifications, such as transmittances at several different angles of incidence, phase changes, and polarization properties.

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