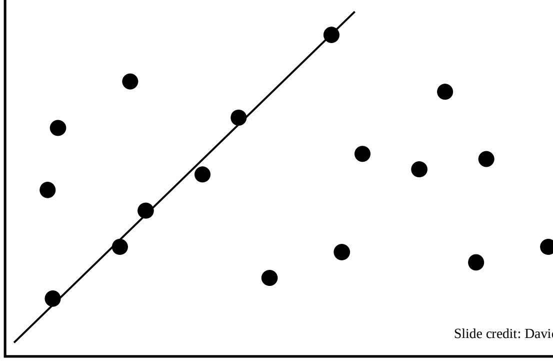
Fitting a Model to Data
Reading: 15.1, 15.5.2

- Cluster image parts together by fitting a model to some selected parts
- **Examples:**
  - A line fits well to a set of points. This is unlikely to be due to chance, so we represent the points as a line.
  - A 3D model can be rotated and translated to closely fit a set of points or line segments. It it fits well, the object is recognized.

Line Grouping Problem



Slide credit: David Jacobs

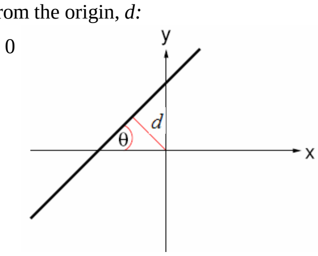
This is difficult because of:

- Extraneous data: clutter or multiple models
  - We do not know what is part of the model?
  - Can we pull out models with a few parts from much larger amounts of background clutter?
- Missing data: only some parts of model are present
- Noise
- **Cost:**
  - It is not feasible to check all combinations of features by fitting a model to each possible subset

Equation for a line

- Representing a line in the usual form, $y = mx + b$, has the problem that m goes to infinity for vertical lines
- A better choice of parameters for the line is angle, θ, and perpendicular distance from the origin, d:

$$x \sin \theta - y \cos \theta + d = 0$$



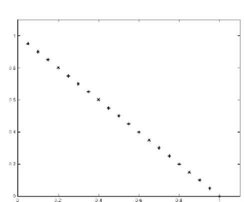
The Hough Transform for Lines

- **Idea:** Each point votes for the lines that pass through it.
- A line is the set of points (x, y) such that

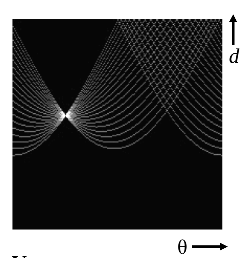
  $$x \sin \theta - y \cos \theta + d = 0$$

- Different choices of θ, d give different lines
- For any (x, y) there is a one parameter family of lines through this point. Just let (x,y) be constants and for each value of θ the value of d will be determined.
- Each point enters votes for each line in the family
- If there is a line that has lots of votes, that will be the line passing near the points that voted for it.

The Hough Transform for Lines

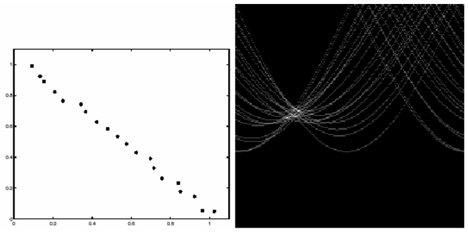


Tokens

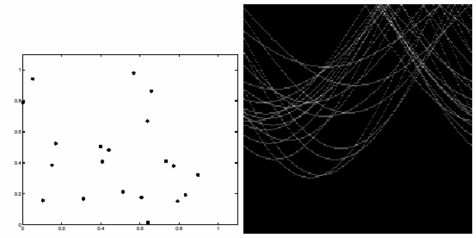


Votes

Hough Transform: Noisy line

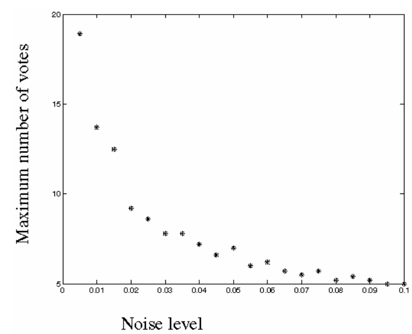


tokens **votes**

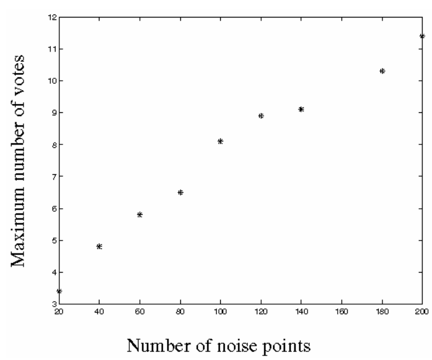


Mechanics of the Hough transform

- Construct an array representing θ, *d*
- For each point, render the curve (θ, *d*) into this array, adding one vote at each cell
- Difficulties
  - how big should the cells be? (too big, and we merge quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - Count the peaks in the Hough array
  - Treat adjacent peaks as a single peak
- Which points belong to each line?
  - Search for points close to the line
  - Solve again for line and iterate



Fewer votes land in a single bin when noise increases.



Adding more clutter increases number of bins with false peaks.

More details on Hough transform

- It is best to vote for the two closest bins in each dimension, as the locations of the bin boundaries is arbitrary.
  - By “bin” we mean an array location in which votes are accumulated
  - This means that peaks are “blurred” and noise will not cause similar votes to fall into separate bins
- Can use a hash table rather than an array to store the votes
  - This means that no effort is wasted on initializing and checking empty bins
  - It avoids the need to predict the maximum size of the array, which can be non-rectangular

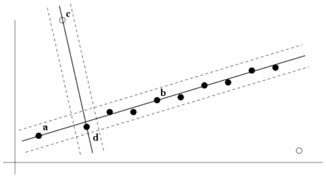
When is the Hough transform useful?

- The textbook wrongly implies that it is useful mostly for finding lines
  - In fact, it can be very effective for recognizing arbitrary shapes or objects
- The key to efficiency is to have each feature (token) determine as many parameters as possible
  - For example, lines can be detected much more efficiently from small edge elements (or points with local gradients) than from just points
  - For object recognition, each token should predict scale, orientation, and location (4D array)
- **Bottom line:** The Hough transform can extract feature groupings from clutter in linear time!

RANSAC (RANdom SAmple Consensus)

1. Randomly choose minimal subset of data points necessary to fit model (a *sample*)
2. Points within some distance threshold t of model are a *consensus set*. Size of consensus set is model's *support*
3. Repeat for N samples; model with biggest support is most robust fit
   - Points within distance t of best model are inliers
   - Fit final model to all inliers

Two samples
and their supports
for line-fitting



from Hartley & Zisserman

Slide: Christopher Rasmussen

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
n — the smallest number of points required
k — the number of iterations required
t — the threshold used to identify a point that fits well
d — the number of nearby points required
to assert a model fits well
Until k iterations have occurred
Draw a sample of n points from the data
uniformly and at random
Fit to that set of n points
For each data point outside the sample
Test the distance from the point to the line
against t; if the distance from the point to the line
is less than t, the point is close
end
If there are d or more points close to the line
then there is a good fit. Refit the line using all
these points.
end
Use the best fit from this collection, using the
fitting error as a criterion

RANSAC: How many samples?

How many samples are needed?

Suppose w is fraction of inliers (points from line).

n points needed to define hypothesis (2 for lines)

k samples chosen.

Probability that a single sample of n points is correct:

$$w^n$$

Probability that all samples fail is:

$$(1-w^n)^k$$

Choose k high enough to keep this below desired failure rate.

RANSAC: Computed k (p = 0.99)

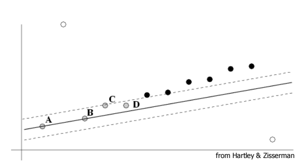
Sample size	Proportion of outliers						
n	**5%**	**10%**	**20%**	**25%**	**30%**	**40%**	**50%**
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

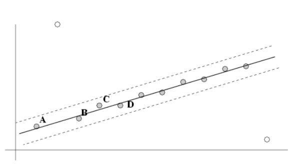
adapted from Hartley & Zisserman

Slide credit: Christopher Rasmussen

After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers
- Improve this initial estimate with estimation over all inliers (e.g., with standard least-squares minimization)
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier





from Hartley & Zisserman

Slide credit: Christopher Rasmussen

Automatic Matching of Images

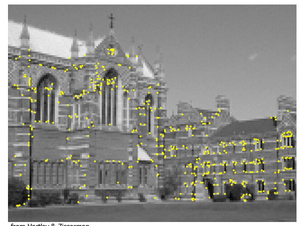
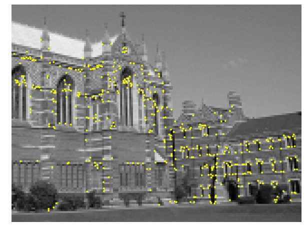
- How to get correct correspondences without human intervention?
- Can be used for image stitching or automatic determination of epipolar geometry


from Hartley & Zisserman

Slide credit: Christopher Rasmussen

Feature Extraction

- Find features in pair of images using Harris corner detector
- Assumes images are roughly the same scale (we will discuss better features later in the course)

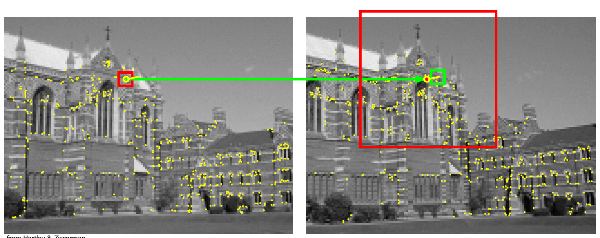
from Hartley & Zisserman

~500 features found

Slide credit: Christopher Rasmussen

Finding Feature Matches

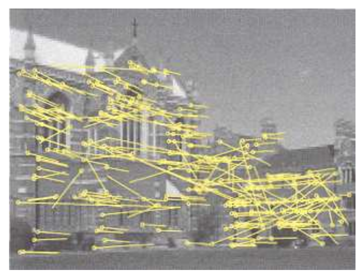
- Select best match over threshold within a square search window (here 300 pixels^2) using SSD or normalized cross-correlation for small patch around the corner



from Hartley & Zisserman

Slide credit: Christopher Rasmussen

Initial Match Hypotheses

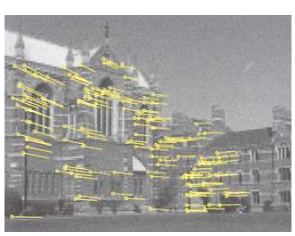


268 matched features (over SSD threshold) in left image pointing to locations of corresponding right image features

Slide credit: Christopher Rasmussen

Outliers & Inliers after RANSAC

- n is 4 for this problem (a homography relating 2 images)
- Assume up to 50% outliers
- 43 samples used with t = 1.25 pixels

from Hartley & Zisserman

117 outliers

151 inliers

Discussion of RANSAC

- **Advantages:**
  - General method suited for a wide range of model fitting problems
  - Easy to implement and easy to calculate its failure rate
- **Disadvantages:**
  - Only handles a moderate percentage of outliers without cost blowing up
  - Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)
- The Hough transform can handle high percentage of outliers, but false collisions increase with large bins (noise)