CPSC 425: Computer Vision

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Lecture Notes 2016/2017 Term 2
Menu March 2, 2017

Topics:
  RANSAC
  Hough transform

Reading:
  Today:  Forsyth & Ponce (2nd ed.) 10.4.2, 10.1
  Next:   Forsyth & Ponce (2nd ed.) 7.1.1, 7.2.1, 7.4, 7.6

Reminders:
  Assignment 5 due Tuesday, March 8
  www:  http://www.cs.ubc.ca/~little/cpsc425/
  piazza:  https://piazza.com/ubc.ca/winterterm22015/cpsc425/
Today’s “Fun” Example: PR2 Rides the Elevator

Point your web browser to the YouTube video at

http://www.youtube.com/watch?v=6NPgToTOClw

for a demo of the PR2 robot platform from Willow Garage

Video credit: UBC Hackathon Team
June, 2012
Lecture 13: Re-cap

Four steps to SIFT feature generation:

1. Scale-space representation and local extrema detection

2. Keypoint localization
   — select stable keypoints (threshold on magnitude of extremum, ratio of principal curvatures)

3. Keypoint orientation assignment

4. Keypoint descriptor
   — vector with $8 \times 4 \times 4 = 128$ dimensions
Lecture 13: Re-cap

Object recognition with SIFT:

1. Match each keypoint independently to database of known keypoints extracted from “training” examples

2. Identify clusters of (at least) 3 matches that agree on an object and a similarity pose
   — use generalized Hough transform

3. Check each cluster found by performing detailed geometric fit of affine transformation to the model
Suppose we are fitting a line to a dataset that consists of 50% outliers.

We can fit a line using two points.

If we draw pairs of points uniformly at random, what fraction of pairs will consist entirely of ‘good’ data points (inliers)?
Fitting a model to noisy data

- Suppose we are fitting a line to a dataset that consists of 50% outliers

- We can fit a line using two points

- If we draw pairs of points uniformly at random, then about 1/4 of these pairs will consist entirely of ‘good’ data points (inliers)

- We can identify these good pairs by noticing that a large collection of other points lie close to the line fitted to the pair

- A better estimate of the line can be obtained by refitting the line to the points that lie close to the line
RANSAC (RANdom SAmple Consensus)

1. Randomly choose minimal subset of data points necessary to fit model (a sample)

2. Points within some distance threshold, $t$, of model are a consensus set. Size of consensus set is model’s support

3. Repeat for $N$ samples; model with biggest support is most robust fit
   — Points within distance $t$ of best model are inliers
   — Fit final model to all inliers

Slide credit: Christopher Rasmussen
Example 1:

Figure credit: Hartley & Zisserman
Example 1 (cont’d):

Two samples and their supports for line fitting

Figure credit: Hartley & Zisserman
Algorithm 10.4
This was Algorithm 15.4 in Forsyth & Ponce (1st ed.)

Algorithm 15.4: RANSAC: fitting lines using random sample consensus

Determine:
- \( n \) — the smallest number of points required
- \( k \) — the number of iterations required
- \( t \) — the threshold used to identify a point that fits well
- \( d \) — the number of nearby points required to assert a model fits well

Until \( k \) iterations have occurred
- Draw a sample of \( n \) points from the data uniformly and at random
- Fit to that set of \( n \) points
- For each data point outside the sample
  - Test the distance from the point to the line against \( t \); if the distance from the point to the line is less than \( t \), the point is close
- If there are \( d \) or more points close to the line
  - then there is a good fit. Refit the line using all these points.

end
- Use the best fit from this collection, using the fitting error as a criterion

RANSAC: Fitting Lines Using Random Sample Consensus
RANSAC: How Many Samples?

Let $\omega$ be the fraction of inliers (i.e., points on line)

Let $n$ be the number of points needed to define hypothesis ($n = 2$ for a line in the plane)

Suppose $k$ samples are chosen

The probability that a single sample of $n$ points is correct is
RANSAC: How Many Samples?

Let $\omega$ be the fraction of inliers (i.e., points on line)

Let $n$ be the number of points needed to define hypothesis ($n = 2$ for a line in the plane)

Suppose $k$ samples are chosen

The probability that a single sample of $n$ points is correct is

$$\omega^n$$

The probability that all samples fail is
RANSAC: How Many Samples?

Let $\omega$ be the fraction of inliers (i.e., points on line)

Let $n$ be the number of points needed to define hypothesis ($n = 2$ for a line in the plane)

Suppose $k$ samples are chosen

The probability that a single sample of $n$ points is correct is

$$\omega^n$$

The probability that all samples fail is

$$(1 - \omega^n)^k$$

Choose $k$ large enough (to keep this below a target failure rate)
RANSAC: \( k \) Samples Chosen (\( p = 0.99 \))

<table>
<thead>
<tr>
<th>Sample size</th>
<th>Proportion of outliers</th>
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<td></td>
<td>5%</td>
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<td>8</td>
<td>5</td>
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Table credit: Hartley & Zisserman
After RANSAC

- RANSAC divides data into inliers and outliers and yields estimate computed from minimal set of inliers

- Improve this initial estimate with estimation over all inliers (e.g., with standard least-squares minimization)

- But this may change inliers, so alternate fitting with re-classification as inlier/outlier
Example 2:

(a) 

(b) 

Figure credit: Hartley & Zisserman
Example 3: Automatic Matching of Images

- How to get correct correspondences without human intervention?
- Can be used for image stitching or automatic determination of epipolar geometry

Figure credit: Hartley & Zisserman
Example 3 (cont’d): Feature Extraction

- Find features in pair of images using Harris corner detector
- Assumes images are roughly the same scale

≈ 500 corner features found in each image

Figure credit: Hartley & Zisserman
Example 3 (cont’d): Finding Feature Matches

- Select best match over threshold within a square search window (here ±320 pixels) using SSD or (normalized) cross-correlation for small patch around the corner

Figure credit: Hartley & Zisserman
Example 3 (cont’d): Initial Match Hypotheses

268 matched features (over SSD threshold) superimposed on left image (pointing to locations of corresponding feature in right image)

Figure credit: Hartley & Zisserman
Example 3 (cont’d): Outliers & Inliers after RANSAC

- $n$ is 4 for this problem (a homography relating 2 images)
- Assume up to 50% outliers
- 43 samples used with $t = 1.25$ pixels

117 outliers

151 inliers

Figure credit: Hartley & Zisserman
Example 3 (cont’d): Final Matches

final set of 262 matches

Figure credit: Hartley & Zisserman
Discussion of RANSAC

- **Advantages:**
  - General method suited for a wide range of model fitting problems
  - Easy to implement and easy to calculate its failure rate

- **Disadvantages:**
  - Only handles a moderate percentage of outliers without cost blowing up
  - Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)

- The Hough transform can handle high percentage of outliers
Example: Photo Tourism

- Takes as input unstructured collections of photographs and reconstructs each photo’s viewpoint and a sparse 3D model of the scene
- Uses both SIFT and RANSAC

Figure credit: Snavely et al. 2006
Framework for Next Topic

Problem:
Fitting a model to data (in the presence of noise and outliers)

Key Idea(s):
Use a parametric model to represent the “object” of interest. Let the data “vote” for instances of the model the data are consistent with. Recognize specific instances of the model when there is sufficient support for that instance.

Practical Detail(s):
The combinatorics of (naive) Hough voting are not in our favour when the complexity of the model (i.e., number of parameters) increases
Fitting a Model

Suppose we want to fit a model to a set of tokens

— e.g. A line fits well to a set of points. This is unlikely to be due to chance, so we represent the points as a line.

— e.g. A 3D model can be scaled, rotated and translated to closely fit a set of points or line segments. If it fits well, the object is recognized.
Fitting a Model is Difficult

Difficulties arise owing to:

- Extraneous data: clutter or multiple models
  - We do not know what is part of the model
  - Can we fit models with a few parts when there is significant background clutter?

- Missing data: only some parts of model are present

- Noise

- Computational cost:
  - Not feasible to check all combinations of features by fitting a model to each possible subset
The Hough Transform

Idea of Hough transform:

For each token
   Vote for all models to which the token could belong
Return models that get many votes

Example: For each point, vote for all lines that could pass through it;
the true lines will pass through many points and so receive many votes
(Brief) Digression: Line in the Plane

Let $\theta$ be the (counter-clockwise) angle, $0 \leq \theta < 2\pi$, that the line makes with respect to the $x$-axis.

Let $r \geq 0$ be the perpendicular (i.e., closest) distance of the line from the origin.

The equation of the line is

$$x \sin \theta - y \cos \theta + r = 0$$
The Hough Transform for Lines

Idea: Each point votes for the lines that pass through it

- A line is the set of points, \((x, y)\), such that
  \[x \sin \theta - y \cos \theta + r = 0\]

- Different choices of \(\theta, r\) give different lines
The Hough Transform for Lines

**Idea:** Each point votes for the lines that pass through it

- A line is the set of points, \((x, y)\), such that
  \[x \sin \theta - y \cos \theta + r = 0\]

- Different choices of \(\theta, r\) give different lines

- For any \((x, y)\) there is a one parameter family of lines through this point. Just let \((x, y)\) be constants and for each value of \(\theta\) the value of \(r\) will be determined

- Each point enters votes for each line in the family

- If there is a line that has lots of votes, that will be the line passing near the points that voted for it
Example 4:

Tokens

Votes

Horizontal axis is $\theta$
Vertical Axis is $r$

Forsyth & Ponce (2nd ed.) Figure 10.1 (Top)