Projection

Readings

• Szeliski 2.1
Projection

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Müller-Lyer Illusion

A clear and resounding victory

by Pravin Bhat

http://www.michaelbach.de/ot/sze_muelue/index.html
Let’s design a camera

• Idea 1: put a piece of film in front of an object
• Do we get a reasonable image?
Pinhole camera

Add a barrier to block off most of the rays

- This reduces blurring
- The opening known as the **aperture**
- How does this transform the image?
Camera Obscura

The first camera

- Known to Aristotle
- How does the aperture size affect the image?
Cameras

First photograph due to Niepce
First on record shown in the book - 1822
Basic abstraction is the pinhole camera
  • lenses required to ensure image is not too dark
  • various other abstractions can be applied
Shrinking the aperture

Why not make the aperture as small as possible?
  • Less light gets through
  • *Diffraction* effects...
Shrinking the aperture
The reason for lenses
Adding a lens

A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  - other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance
A lens focuses parallel rays onto a single focal point

- focal point at a distance $f$ beyond the plane of the lens
  - $f$ is a function of the shape and index of refraction of the lens
- Aperture of diameter $D$ restricts the range of rays
  - aperture may be on either side of the lens
- Lenses are typically spherical (easier to produce)
Thin lenses

Thin lens equation: \[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \]

- Any object point satisfying this equation is in focus
- What is the shape of the focus region?
- How can we change the focus region?
- Thin lens applet: [http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html](http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html) (by Fu-Kwun Hwang)
Depth of field

Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus

The eye

The human eye is a camera

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What’s the “film”?
  - photoreceptor cells (rods and cones) in the **retina**
A digital camera replaces film with a sensor array

- Each cell in the array is a Charge Coupled Device
  - light-sensitive diode that converts photons to electrons
  - other variants exist: CMOS is becoming more popular
Issues with digital cameras

Noise
- big difference between consumer vs. SLR-style cameras
- low light is where you most notice noise

Compression
- creates artifacts except in uncompressed formats (tiff, raw)

Color
- color fringing artifacts from Bayer patterns

Blooming
- charge overflowing into neighboring pixels

In-camera processing
- oversharpening can produce halos

Interlaced vs. progressive scan video
- even/odd rows from different exposures

Are more megapixels better?
- requires higher quality lens
- noise issues

Stabilization
- compensate for camera shake (mechanical vs. electronic)

More info online, e.g.,
  - http://www.dpreview.com/
Lens systems
Vignetting
Modeling projection

The coordinate system

- We will use the pin-hole model as an approximation
- Put the optical center (Center Of Projection) at the origin
- Put the image plane (Projection Plane) in front of the COP
  - Why?
- The camera looks down the negative z axis
  - we need this if we want right-handed-coordinates
Modeling projection

Projection equations

- Compute intersection with PP of ray from \((x, y, z)\) to COP
- Derived using similar triangles (on board)

\[
(x, y, z) \rightarrow (-d\frac{x}{z}, -d\frac{y}{z}, -d)
\]

- We get the projection by throwing out the last coordinate:

\[
(x, y, z) \rightarrow (-d\frac{x}{z}, -d\frac{y}{z})
\]
Pinhole cameras

Abstract camera model - box with a small hole in it

Pinhole cameras work in practice
Distant objects are smaller
Parallel lines meet

Common to draw film plane *in front* of the focal point. Moving the film plane merely scales the image.
Vanishing points

Each set of parallel lines (=direction) meets at a different point

- The vanishing point for this direction

Sets of parallel lines on the same plane lead to *collinear* vanishing points.

- The line is called the *horizon* for that plane

Good ways to spot faked images

- scale and perspective don’t work
- vanishing points behave badly
- supermarket tabloids are a great source.
Computer Vision - A Modern Approach
Set: Cameras
Homogeneous coordinates

Is this a linear transformation?
- no—division by z is nonlinear

Trick: add one more coordinate:

\[
(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]

homogeneous image coordinates
homogeneous scene coordinates

Converting \textit{from} homogeneous coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)
\quad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)
\]
Perspective Projection

Projection is a matrix multiply using homogeneous coordinates:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1/d & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z/d \\
1
\end{bmatrix} \Rightarrow (-d\frac{x}{z}, -d\frac{y}{z})
\]  

divide by third coordinate

This is known as **perspective projection**

- The matrix is the **projection matrix**
- Can also formulate as a 4x4 (today’s reading does this)

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & -1/d & 0
\end{bmatrix} \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z \\
-z/d
\end{bmatrix} \Rightarrow (-d\frac{x}{z}, -d\frac{y}{z})
\]  

divide by fourth coordinate
Perspective Projection

How does scaling the projection matrix change the transformation?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1/d & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
-z/d \\
1
\end{bmatrix}
\Rightarrow (\frac{-dx}{z}, \frac{-dy}{z})
\]

\[
\begin{bmatrix}
-d & 0 & 0 & 0 \\
0 & -d & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
-dx \\
-dy \\
z \\
1
\end{bmatrix}
\Rightarrow (\frac{-dx}{z}, \frac{-dy}{z})
\]
Orthographic projection

Special case of perspective projection
  - Distance from the COP to the PP is infinite

- Good approximation for telephoto optics
- Also called “parallel projection”: \((x, y, z) \rightarrow (x, y)\)
- What’s the projection matrix?

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix} \Rightarrow (x, y)\]
Orthographic ("telecentric") lenses

Navitar telecentric zoom lens

http://www.lhup.edu/~dsimanek/3d/telecent.htm
Variants of orthographic projection

Scaled orthographic

- Also called “weak perspective”

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1/d \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
= 
\begin{bmatrix}
x \\
y \\
z \\
1/d
\end{bmatrix} \Rightarrow (dx, dy)
\]

Affine projection

- Also called “paraperspective”

\[
\begin{bmatrix}
a & b & c & d \\
e & f & g & h \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]
Camera parameters

A camera is described by several parameters:

- Translation $T$ of the optical center from the origin of world coords
- Rotation $R$ of the image plane
- focal length $f$, principal point ($x'_c, y'_c$), pixel size ($s_x, s_y$)
- blue parameters are called “extrinsics,” red are “intrinsics”

Projection equation

$$
X = \begin{bmatrix}
    sx \\
    sy \\
    s
\end{bmatrix} = \begin{bmatrix}
    * & * & * \\
    * & * & * \\
    * & * & *
\end{bmatrix} \begin{bmatrix}
    X \\
    Y \\
    Z \\
    1
\end{bmatrix} = \Pi X
$$

- The projection matrix models the cumulative effect of all parameters
- Useful to decompose into a series of operations

$$
\Pi = \begin{bmatrix}
    -f s_x & 0 & x'_c \\
    0 & -f s_y & y'_c \\
    0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
    R_{3 \times 3} & 0_{3 \times 1} \\
    0_{1 \times 3} & 1
\end{bmatrix} \begin{bmatrix}
    I_{3 \times 3} & T_{3 \times 1} \\
    0_{1 \times 3} & 1
\end{bmatrix}
$$

- The definitions of these parameters are not completely standardized
  - especially intrinsics—varies from one book to another
Radial distortion of the image

- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens
Correcting radial distortion

from Helmut Dersch
Modeling distortion

Project \((\hat{x}, \hat{y}, \hat{z})\)
to “normalized”
image coordinates

\[
\begin{align*}
x'_n &= \frac{\hat{x}}{\hat{z}} \\
y'_n &= \frac{\hat{y}}{\hat{z}}
\end{align*}
\]

Apply radial distortion

\[
\begin{align*}
r^2 &= x'_n^2 + y'_n^2 \\
x'_d &= x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4) \\
y'_d &= y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)
\end{align*}
\]

Apply focal length
translate image center

\[
\begin{align*}
x' &= f x'_d + x_c \\
y' &= f y'_d + y_c
\end{align*}
\]

To model lens distortion

- Use above projection operation instead of standard
  projection matrix multiplication
Geometric properties of projection

Points go to points
Lines go to lines
Planes go to whole image
Polygons go to polygons

Degenerate cases
- line through focal point to point
- plane through focal point to line
Polyhedra project to polygons

(because lines project to lines)
Junctions are constrained

This leads to a process called “line labelling”

- one looks for consistent sets of labels, bounding polyhedra
- disadv - can’t get the lines and junctions to label from real images
Curved surfaces are much more interesting

Crucial issue:
outline is the set of points where the viewing direction is tangent to the surface
This is a projection of a space curve, which varies from view to view of the surface
Other types of projection

Lots of intriguing variants…
(I’ll just mention a few fun ones)
360 degree field of view...

Basic approach
- Take a photo of a parabolic mirror with an orthographic lens (Nayar)
- Or buy one a lens from a variety of omnicam manufacturers...
  - See http://www.cis.upenn.edu/~kostas/omni.html
Tilt-shift

http://www.northlight-images.co.uk/article_pages/tilt_and_shift_ts-e.html

Tilt-shift images from Olivo Barbieri and Photoshop imitations
Rotating sensor (or object)

Rollout Photographs © Justin Kerr
http://research.famsi.org/kerrmaya.html

Also known as “cyclographs”, “peripheral images”