Competitive Viral Marketing

Based on:
Why Competition?

• To be blunt, non-competitive is not super realistic.

• Marketing of products, spread of ideas, innovations, political campaigns – all involve competition.
A Model

- Core – independent cascade.
The Model

• Core – *independent cascade*.

\[ p_1, T_1 \quad p_3, T_3 \quad p_5, T_5 \]

Note: In this and subsequent models, we will use *color* as a metaphor for state.
The Model

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The Model

- What if competing campaigns succeeded in activating a node?

The activating campaign with smaller activation delay wins. E.g., suppose $T_2 < T_4$. Then …
The Model

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Note: Ties avoided elegantly using the notion of time delay for activation, which is a continuous random variable.
Properties of the Model

• Let $\sigma^i(S_i) := \sigma(S_i|S_1, ..., S_{i-1}, S_{i+1}, ..., S_n)$ denote the expected number of nodes activated with color $i$, given the seed sets $S_1, ..., S_n$.
• Can show $\sigma^i(.)$ is monotone and submodular in $S_i$, given fixed seed sets for other colors.
• Proof of monotonicity is trivial.
• For submodularity, it is analogous to that for the IC model.
• The last player can adopt a greedy strategy to achieve a $\left(1 - \frac{1}{e}\right)$-approximation to the optimal spread for its color.

Can we guarantee this for other players?
Discrete Time Models

• Recall, classical IC/LT models are discrete time.
• Use of continuous time in Bharathi et al.’s competitive IC model is mainly to avoid ties in activation by rival campaigns.
• Discrete time will bring us back to ties and we need tie-breaking rules.
• E.g., competitive IC model: what if two different companies succeed in activating a node?
• Several options exist.
(Discrete Time) CIC Model

• Consider two companies (+ and −) for simplicity: what if both + and − succeed?
  • +-dominance: in the event of a tie, + wins.
  • −-dominance: analogous.
• One of them chosen as winner at random w/ equal probability.
• Proportional to the relative # that is active in the previous time step and succeeded in activating \( v \): \[
\frac{|A_t^+(v)|}{|A_t^+(v)| + |A_t^-(v)|}.
\]
  – \( A_t^+(v) \) = set of +-nodes that succeeded in activating \( v \) at time \( t \).
  – Remember, every active node gets one shot at trying to activate each of its out-neighbors.
Successful attempts by blue-active in-neighbors.

Nodes that became + active at time $t$.

Nodes that became − active at time $t$.

Nodes not active at time $t$.

Successful attempts by red-active in-neighbors.

Prob. of bottom node turning blue at $t + 1 = 2/3$
Prob. of it turning red at $t + 1 = 1/3$.

Semantics $\equiv$ randomly permuting $A^+_{\xi}(v) \cup A^-_{\xi}(v)$, and assigning to $v$ at time $t + 1$ the color of the first node in this permutation.
Some generalizations/observations:

- Positive and negative influence probs could be different: e.g., say your neighbors are more easily convinced by any negative opinion from you, but are more cautious in reacting to positive opinion from you: $p^+(u, v) \neq p^-(u, v)$.
- Model semantics is still well-defined.
- Natural assumption: seed sets for diff. campaigns are disjoint: technically don’t need this!
CIC Model (concluded)

• How do the propagations happen?
  • $S_0^+ = $ seeds for color $+$ ;
  • $S_{t+1}^+ \leftarrow S_t^+$ ;
  • Add a node $v$ iff
    - $A_t^+(v) \neq \emptyset = A_t^-(v)$ or
    - There was a tie and $+$ won the tie.

• Continue until $\forall v: v$ is inactive, $A_t^+(v) = \emptyset = A_t^-(v)$, $t$ being the current time.
Competitive LT (CLT) Model

• General case: weights $w^+(u, v)$ and $w^-(u, v)$, for each link $(u, v)$.

• Each node picks activation thresholds $\theta_v^+$ and $\theta_v^-$ independently and uniformly at random from $[0,1]$.

• $v$ turns + if $\sum_{u \in \text{in}(v) \cap S_t^+} w^+(u, v) \geq \theta_v^+$ and $\sum_{u \in \text{in}(v) \cap S_t^-} w^-(u, v) < \theta_v^-$;
CLT Model (contd.)

• $\nu$ turns — if $\sum_{u \in N_{\text{in}}(\nu) \cap S_t^-} w^-(u, \nu) \geq \theta_{\nu}^-$ and $\sum_{u \in N_{\text{in}}(\nu) \cap S_t^+} w^+(u, \nu) < \theta_{\nu}^+$;

• When both thresholds are exceeded, apply a tie-breaker rule.

• $+/-$ dominance, fixed probability rules — analogous to CIC.

• Proportionate prob. case not fully explored in the literature, but has potential.
CIC – Equiv. Live-Edge Model

• Let $G = (V, E, p)$ be given prob. graph.
  • Toss coins associated with all arcs w.p. $p^+$ to generate one possible world $G^+_L$. $\rightarrow$ a deterministic graph.
  • Generate $G^-_L$ similarly; note, the generations are independent.
  • Predetermine random choices for tie-breaking in advance (just in case!):
    – If fixed prob., toss a coin for each node $v$ with that prob.: let $\tau_v$ remember that outcome.
    – For proportionate, $v$ permutes its in-neighbors in $G^+_L \cup G^-_L$ and remembers this permutation $\pi_v$. 
CIC – Equivalent Live-Edge Model

• Proportionate prob. tie-breaker: each node \(\nu\) permutes its in-neighbors in \(G_L^+ \cup G_L^-\) randomly.
  
• \(S_1^+ \leftarrow S_0^+;\)
  
• Add nodes that are out-neighbors of \(S_0^+\) but not out-neighbors of \(S_0^-\).
  
• If node \(\nu \in N^{out}(S_0^+) \cap N^{out}(S_0^-)\), then look up \(\tau_\nu\). (fixed prob. tie-breaker.)
  
• For proportionate tie-breaker, assign to \(\nu\) the state of the first node in \(S_0^+ \cup S_0^-\) in the random permutation \(\pi_\nu\) chosen beforehand by \(\nu\).
  
• Same principle for \(t \rightarrow t + 1\).
  
• Proof of equivalence between CIC and this live-edge model is similar to the classical case with some additional wrinkles. Key (additional) insight: order in which various coins are tossed (more generally, random trials performed) is irrelevant.
CLT – Equivalent Live-Edge Model

• Generate $G^+_L/G^-_L$ using the “classic” favorite in-neighbor selection method, independently for the positive and negative link weights.
• Predetermine random choices for tie-breaking in advance (just in case!):
  – For fixed prob., toss a coin for each node $v$ with that prob.: let $\tau_v$ remember that outcome.
• Proof of equiv. to CLT – simple extension of LT to its corresponding live-edge model.
What do we want to optimize?

- $\sigma^+(S_0^+, S_0^-) = \text{expected \#nodes that are positively activated at end of campaign, given seeds } S_0^+, S_0^-.$

- $\sigma^-(S_0^+, S_0^-) = \text{expected \#nodes that are negatively activated at end of campaign, given seeds } S_0^+, S_0^-.$

- Some interesting questions: Given graph, seed budget $k$, and negative seed set $S_0^-$, find the positive seed set $S_0^{+*}$ of size $\leq k$ that maximizes $\sigma^+(S_0^+, S_0^-)$
  - Symmetry between $+$ and $-.$
  - \text{Influence maximization under competition.}
  - How hard is IM under competition?
What else do we want to optimize?

• Given negative seeds, positive seeds tend to counteract their influence (and vice versa), i.e., $\sigma^-(S_0^+, S_0^-) \leq \sigma^-(\emptyset, S_0^-)$. We can thus ask, given the graph, negative seeds $S_0^-$ and budget $k$, find the positive seed set $S_0^{++}$ that maximizes the reduction: $\sigma^-(\emptyset, S_0^-) - \sigma^-(S_0^+, S_0^-)$. $\rightarrow$ influence blocking maximization. (“damage control” maximization!)
Influence Maximization under Competition

- $\sigma^+(S_0^+, S_0^-)$ -- monotone in $S_0^+$ for any fixed $S_0^-$. Proof exploits equiv. to live-edge model.
- What about submodularity?

Positive side:
"$S'' = \emptyset; T'' = \{s^+\}."
Consider adding $u$.

Negative side:
Seed set = $\{s^-\}$. 