CS 420: Advanced Algorithm Design and Analysis
Spring 2015 – Lecture 20

Department of Computer Science
University of British Columbia

March 19, 2015

Announcements

Assignments...

- Asst6/7...(due today)

Midterm III...

- Q/A session...March 24; 5:30-7:00; DMPT 110
- Exam...March 25; 5:30-7:00; DMPT 110
- ...on *all* course material up to and including March 19 lecture

Announcements (cont.)

Readings...

- matchings and network flows [Kleinberg&Tardos, Chapt. 7], [Cormen et al., Chapt. 26], [Dasgupta et al., Chapter 7]
- reductions and NP-hardness [Kleinberg&Tardos, Chapt. 8, 11], [Cormen et al., Chapt. 34,35]

Last class...

Reductions and relative hardness of problems

- The Cook-Levin theorem: establishing the first NP-hard problem
- more examples of reductions establishing **NP**-hardness and **NP**-completeness

Last class...

Satisfiability

The language SAT is defined as the set of all satisfiable Boolean expressions. Its restriction k-SAT is the set of all satisfiable Boolean expressions in k-CNF.

Note:

- 2-SAT is in **P** , since 2-SAT $\lesssim_{\mathrm{P}}$digraph_connectivity
- k-SAT $\lesssim_{\mathrm{P}}$SAT and SAT $\lesssim_{\mathrm{P}}$3-SAT
- SAT is in **NP**

Last class...

Theorem: VERTEX-COVER is **NP**-complete.

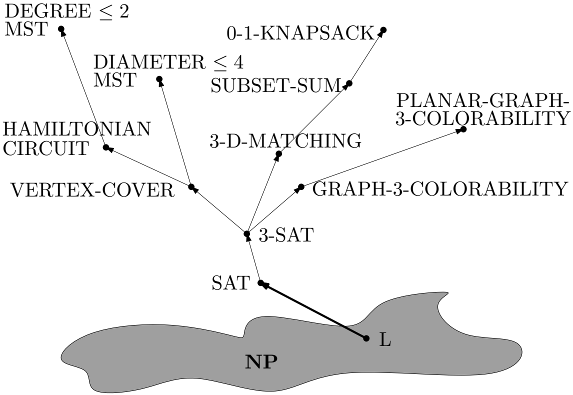
Proof:
We show that 3-SAT $\leq_{\mathrm{P}}$VERTEX-COVER

Last class.....

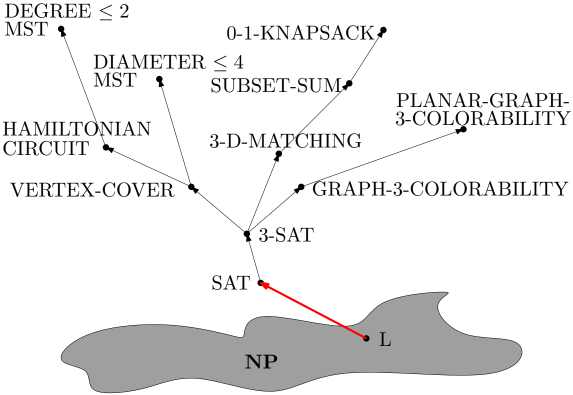
Building the **NP**-hardness reduction tree

- overview of reductions
- some selected examples

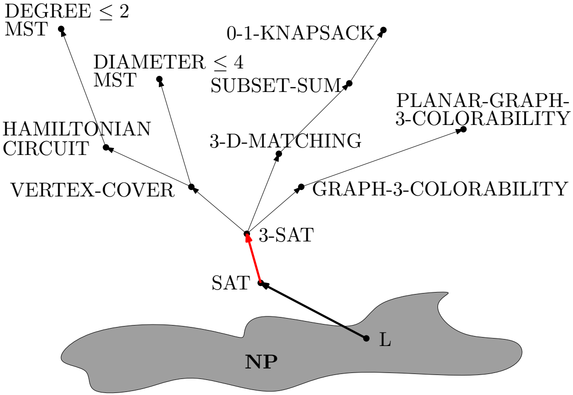
Overview of Reductions



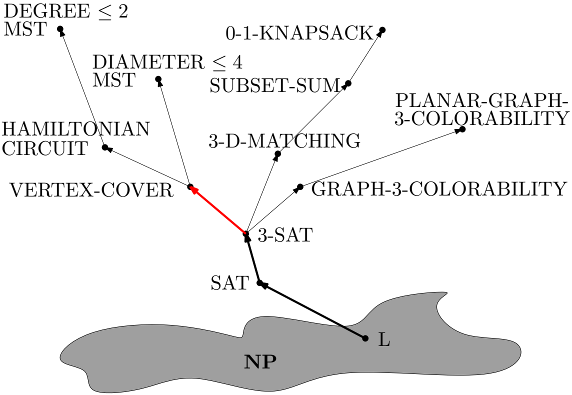
Overview of Reductions



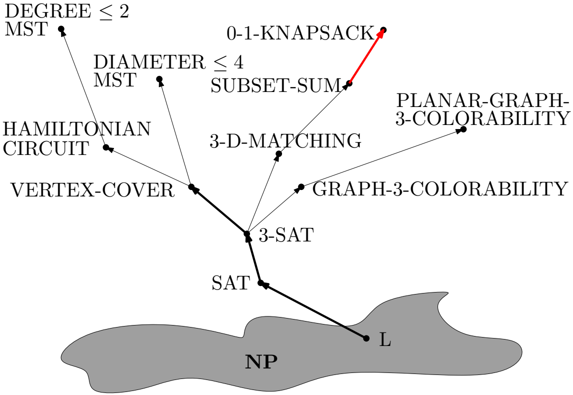
Overview of Reductions



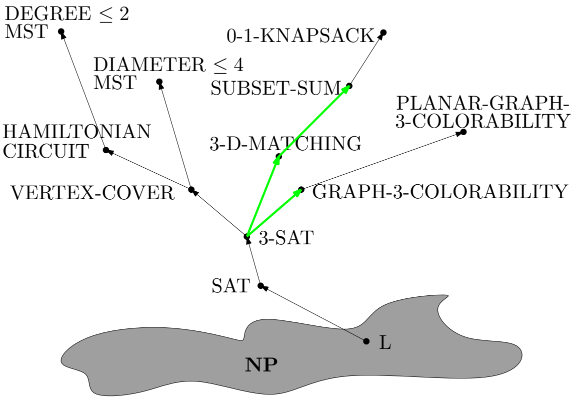
Overview of Reductions



Overview of Reductions



Overview of Reductions



Some more examples

- 3-SAT $\leq_P$ 3-D-MATCHING
- 3-D-MATCHING $\leq_P$ SUBSET-SUM

Today...

Coping with **NP**-hardness

- approximate solutions
- heuristics (empirically effective algorithms)
- restriction (exploiting structure of certain input classes)

Approximations?

- VERTEX-COVER
  - 2-approximation via maximal matching
  - best approx known
- KNAPSACK
  - $(1 + \epsilon)$-approximation (via dynamic programming)
  - *polynomial-time approximation scheme*
- COLORING
  - approximation factor n^{ϵ}, for some fixed ϵ, cannot be achieved, unless **P**$=$**NP**