Fair Resource Block and Power Allocation for Femtocell Networks: A Game Theory Perspective

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Abstract

One of the important issues in building the femtocell networks in existing macrocell networks is how to balance the spatial reuse of bandwidth and the two-tier interference, so that the capacity of the hybrid network can be maximized with proper fairness consideration. In this paper, we aim to analyze this issue in orthogonal frequency-division multiple access (OFDMA) cellular systems from the perspective of game theory. First, we will propose the system model where both macrocells and femtocells are presented. Then, we will introduce the potential game model and submodular game model in OFDMA cellular systems to provide a solution on resource block allocation, power allocation, and the network fairness.

1 Introduction

Femtocells are promising to extend the coverage of macrocells to the indoor environment [1]. The macrocell networks mainly refer to the wireless communication system in urban area supported by big antennas on tall buildings. That is the wireless service we use everyday. Before femtocell emerges, macrocell networks are just called cellular networks, whose coverage radius could be several kilometers. The macrocell networks can provide relative good access quality for the outdoor mobile subscribers, however sometimes it is not the case for the indoor users because of the absorptions and reflections of steel and concrete [2]. Femtocell is in fact a small base station with similar size as a wireless router. It can be installed by any person and has very small transmission power, so usually it just has 10 meters coverage radius. The popularity of smartphones makes more and more wireless data are requested by the indoor users, so the femtocell is quite promising in the near future.

Wireless communication requires a band of frequency to carry the wireless signal. The interference happens on a receiver when the receiver can be reached by another wireless signal transmitters who is working on the same frequency band with its transmitter partner. As the result, on the perspective of macrocell base station, a frequency band can only be used to serve one user at any given time. So another reason people would like to use femtocells is that, with the small coverage radius, the frequency band can be reused in different femtocells, and the frequency spectrum's efficiency can be further explored.

From another perspective, the orthogonal frequency-division multiple access (OFDMA) [3] has been included in the 4G standards [4]. OFDMA can allocate the resource in a flexible way because it allocates the frequency spectrum resource not only in the frequency domain, but also in the time domain, as illustrated by Figure 1. In the figure, each cell is called a resource block (RB),

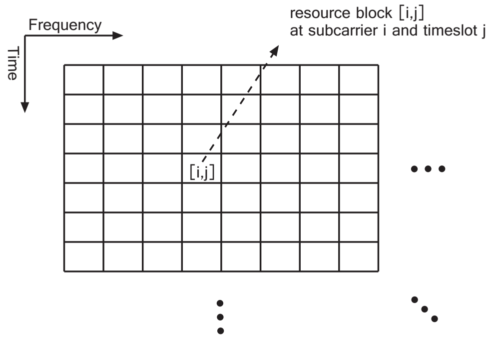

Figure 1: Example of resource blocks in OFDMA cellular systems

and it can be interpreted as allocating a narrow band of frequency spectrum to a user in a short period. So at a given time, the frequency resource is always finite. As well, femtocells can expand the capacity for current macrocell cellular systems.

Even though femtocell has its advantages, the macrocell base station should be maintained for the outdoor users at the same time. As we introduced above, when the same bandwidth is utilized by many transmitters, the users' devices in the overlapping areas would experience the interference. In this situation, the higher transmission power used by the interference source, the stronger interference it can cause. When the interference is strong enough, the user cannot decode any information at all. On the other hand, the higher power is good for its own receiver. A femtocell base station can be deployed by any people and may be set up among other femtocell base stations, so one important topics for the researchers in communication area is how to allocated the RB for the femtocell base stations closed to each other, and how to optimize their transmission power [5].

A centralized control solution to this problem introduces extra information and control data in the networks, so it becomes inefficient when the number of fellcell base stations grows larger. As the result, people want to optimize the system in a distributed fashion. Usually, the femtocell base stations want to have more RBs and use higher transmission power, so that they can serve its users better. From this perspective, the femtocell base stations can be taken as self-interested agents. Meanwhile, each base station's behavior affects others' performance. For example, if all femtocell base stations simply take all RBs and use their highest transmission power, the strong interference between them may make the system unusable. According to the femtocell's requirements and its features, game theoretic approach had been noticed by the researchers in communication areas. That is mainly because we need the behaviors of femtocell based stations be a stable and efficient state, even though these femtocell base stations are self-interested and non-centralized controled. In fact, that is a Nash equilibrium in game theory. Such a state or states are important because communication engineers can predict the network capacity and to know if the system can meet the potential requirements.

This paper is organized as follows. Section 2 will introduce the system model; Section 3 will give the game formulations and analysis based on the system model; Finally, Section 4 concludes this paper.

2 System Model

In this section, we present our system model. We first introodcue the current OFDMA cellular network system shown in Figure 2. Each hexagon in the figure

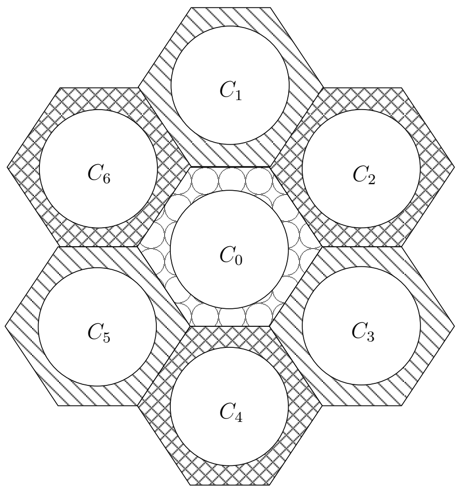

Figure 2: Illustration of resource allocation in OFDMA system

presents a macrocell, the base station of the macrocell is at the hexagon's center point. Different textures in the figure presents different segments of frequency spectrum[1]. To reduce the interference at the boundaries of the hexagons, the macrocell base stations in adjacent cells will allocate different frequency spectrum to their boundary users. For example, the macrocells C_1 and C_2 are adjacent, so they will allocated different frequency segments to their users in

[1]A frequency segment may contain bunches of frequency bands. Namely, a frequency band is much narrower than a frequency segment.

the shaded ares in the hexagons. Meanwhile, to reuse the frequency as efficiently as possible, all the frequency will be reused totally in the central areas of each cell (presented by the circles in these hexagons), and the frequency segments for the boundary users will be reused if two macrocells are far away (presented by the same texture in hexagons' boundaries, such as C_1, C_3, and C_5). This frequency segments' allocation scheme is popular in 4G networks such as long term evolution (LTE), so the interference caused by other macrocell base stations in a circular area can be kept very small. In this paper we just ignore it, and for a particular macrocell, we focus on the model shown in Figure 3.

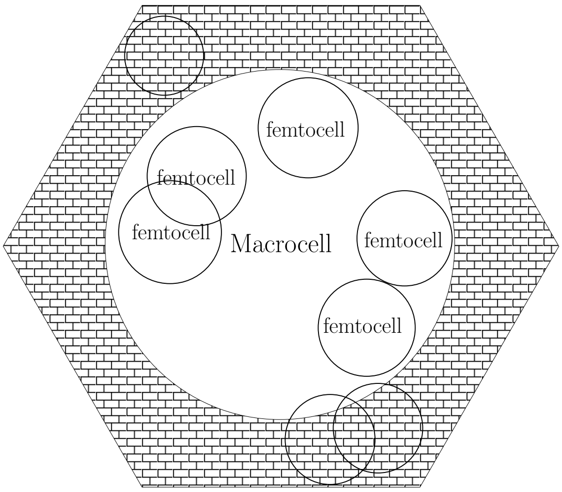

Figure 3: Co-existence of the Macrocell and Femtocells

In the figure we can see many femtocells coexist with the macrocell, and we call this kind of cellular system the two-tier cellular networks. We consider the femtocells in the circular area only. In the circle, both the macrocell base station and the femtocell base stations are using the entire bandwidth (frequency spectrum) to serve their users. As we introduced above, all the femtocell base stations are self-interested, so how to allocated the RBs and their transmission power is the question we want to address[2].

In our model, we assume there are F femtocells in the macrocell central area, and the macrocell and all its femtocells inside, are OFDMA systems operated on the entire frequency band $\mathcal{W}$. We consider the granularity of the frequency band allocation is ΔW. The number of RBs at any given time is just the number of frequency bands given by $N = \mathcal{W}/\Delta W$ (assume it is an integer), so the frequency band is as same as the RB in this paper. Assume for every femtocell i

[2]When we focus on a particular macrocell, the transmission power of the macrocell base station is a constant, and the macrocell base station always uses all RBs. This is also the case in practical system. What we are considering is how to allocate the RBs and optimize the transmission power for the femtocell base stations in the same macrocell.

($i \in \{1, 2, \ldots, F\}$), we have an $N \times 1$ vector $\mathbf{a}_i$, where each element $a_i^w \in \{0, 1\}$ denotes whether the femtocell base station will use the RB w (when it is 1) or not (when it is 0) in its coverage. At any given time, we also have an $N \times 1$ vector $\mathbf{p}_i$ for every femtocell base station i, where each element p_i^w presents the transmission power on the frequency band w. Similarly, another $N \times 1$ vector for the macrocell base station is presented by $\mathbf{q}$. One obvious constraint on the power vectors is that $\forall\, i \in \{1, 2, \ldots, F\}$, $\mathbf{a}_i^T \mathbf{p}_i \le p_{max}$, and $\mathbf{1}^T \mathbf{q}_i \le q_{max}$, where p_{max} and q_{max} are the maximum possible transmission power on the femtocell and macrocell base stations, respectively.

We consider the downlink interference only. In our two-tier system, there are two types of interference to the femtocell users and there is one type of interference to the macrocell users. In particular, the femtocell users experience the interference from both the macrocell base station and other femtocell base stations. The macrocell users experience the interference from all the femtocell base station inside its macrocell coverage. Since the femtocell has relative small coverage, we take the interference on the users in the same femtocell as a constant, this assumption holds for the interference from both the macrocell and other femtocells nearby. Since the channel gain is frequency related, for the frequency band w, we introduce following variables: the channel gain from femtocell j to femtocell i is $g_{j,i}^w$, the channel gain from femtocell i to itself's users is presented by $g_{i,i}^w$, and the channel gain from macrocell base station to femtocell base station i is depicted by $g_{b,i}^w$. For the macrocell users, recall that the number of femtocell base stations could be very large in a building. If we assume the femtocell base stations are deployed uniformly, the interference on a macrocell user caused by the femtocells is independent from its position in the macrocell. Hence, from the perspective of femtocells, interference they caused to the macrocell base station is as same as the interference they caused to the macrocell users. Therefore, for the frequency band w, we assume that femtocell i to the macrocell base station b is $g_{i,b}^w$, and the channel gain from macrocell base station to the macrocell user is $g_{b,b}^w$. The signal to noise and interference ratio (SINR) of the femtocell user in femtocell i on the RB w is given by

$$r_i^w = \frac{a_i^w p_i^w g_{i,i}^w}{a_i^w q^w g_{b,i}^w + \sum_{j=1, j\neq i}^{F} a_i^w a_j^w p_j^w g_{j,i}^w + \sigma^2}, \tag{1}$$

where σ is the deviation of zero-mean Gaussian noise. Recall we are working on the resource allocation with the issues of interference, and the femtocell base station has extremely low working power. Therefore we can assume the system is totally *interference constrained.* On the other hand, we do not consider the SINR on RB w for the users in femtocell i when its base station does not use RB w (means the case that we consider a_i^w has the value 1 only). So we have

$$r_i^w = \frac{p_i^w g_{i,i}^w}{q^w g_{b,i}^w + \sum_{j=1, j\neq i}^{F} a_j^w p_j^w g_{j,i}^w}. \tag{2}$$

By Shannon Theory [6], the i^{th} femtocell service capacity C^i is given by

$$C^i = \sum_{w=1}^{N} \Delta W log_2 \left(1 + r_i^w\right), \tag{3}$$

3 Game Theoretic Formulation and Analysis

We consider the joint RB and power allocation problem as the original problem. In this section, we will introduce the game formulation we need to solve it. Before presenting the game formulation, we want to give the basic idea first. Different from existing literatures, we will first formulate the problem in an *exact potential game* G_{ep}, where for each femtocell (self-interested agent) i, both $\mathbf{a}_i$ and $\mathbf{p}_i$ are involved in its strategy space. Then, from the considerations of the proportional fairness of the interference and the tractability of G_{ep}, we further formulate the power control as sub-problem to be a *submodular game* G_{sm}, where only $\mathbf{p}_i$ is in the strategy space for each femtocell i. The solution to G_{sm} can be used in G_{ep} to reduce its complexity, the original normal form game G_{ep} could be solved efficiently.

3.1 G_{ep} Game Formulation

We try to model the original problem in the normal form game. Every femtocell base station i ($i \in \{1, 2, \dots, F\}$) is taken as an agent, and they compose a player set $\mathcal{F}$. Every player i has its strategy space defined by $S_i = \mathcal{A}_i \times \mathcal{P}_i$, where $\mathbf{a}_i \in \mathcal{A}_i$ and $\mathbf{p}_i \in \mathcal{P}_i$, so G_{ep} has the strategy space $\mathcal{S} = \prod_{i=1}^{F} S_i$. Now, as the most important part, we want to consider all kinds of the interference related with femtocell i, and as well as the femtocell i's capacity. The proposed utility function for femtocell i is given by

$$\begin{aligned} u_i\left(s_i, s_{-i}\right) = \sum_{w=1}^{N} a_i^w \Bigg(& \Delta W log_2 \left(1 + p_i^w g_{i,i}^w\right) - \alpha p_i^w g_{i,b}^w - \beta q^w g_{b,i}^w \\ & -\gamma \sum_{j=1, j\neq i}^{F} a_j^w p_i^w g_{i,j}^w - \delta \sum_{j=1, j\neq i}^{F} a_j^w p_j^w g_{j,i}^w \Bigg) \end{aligned} \tag{4}$$

where the constants α, β, γ, and δ are positive real numbers in our system, which represent the weights of the interference from a femtocell to macrocell, the interference from macrocell to a femtocell i, the interference from femtocell i to other femtocells, and the interference from other femtocells to femocell i, respectively. The last term is the capacity of the i^{th} femtocell in using the RB w and gives the motivation for femtocell i to increase its power. The rationality behind the utility definition can be interpreted by two parts, 1) higher interference on it definitely decreases its utility, 2) for the same data transmission capacity, the femtocell base station should keep the interference it caused to other as small as possible.

Even though we defined the normal form game above, because of the tight coupling between the $\mathbf{a}_i$ and $\mathbf{p}_i$ for all femtocells, it is too complex to find the Nash equilibrium directly. Now, we introduce the exact potential game [7] [8].

Definition 1 (Exact Potential Game) *The exact potential game is such a kind of normal form game: There exists an potential function* $P{:}S \mapsto R$, *such that* $P(s_i, s_{-i}) - P(s_i', s_{-i}) = u_i(s_i, s_{-i}) - u_i(s_i', s_{-i}), \forall\ S \in \mathcal{S}, \forall\ i \in \mathcal{F}$.

For our normal form game given above, we give the potential function simply as follows to satisfy the requirement of a potential game

$$P(s_i, s_{-i}) = \sum_{i=1}^{F} u_i(s_i, s_{-i}). \tag{5}$$

One of the properties of the exact potential game that we are interested in is that the solution S which maximizes $P(S)$ is the unique Nash equilibrium for the original normal form game. Some existing work to solve this kind of question just try to find the $\arg\max_{S\in\mathcal{S}} P(S)$ as the solution [8], however the solution to the maximization problem defined on the potential functions have the fairness issues in the interference control. Also, the searching space of the optimal value S is till within the dimension $\prod_{i=1}^{F}(\mathcal{A}_i \times \mathcal{P}_i)$, so it is too complex to solved directly, and that is the reason we need the following section.

3.2 G_{sm} Game Formulation

We further consider the fairness issues and try to reduce the complexity based on the game G_{ep}. The fairness is important because just pursuing the overall equilibrium strategy in G_ep may sacrifice some femtocells' service capacity. That means even though each femtocell base station is using the best response to other femtocells' strategies, the service capacities in different femtocells may be different severely. In this paper, we consider the proportional fairness [9].

Definition 2 (Proportional Fairness) *In network data rate allocation context, an allocation vector* $\mathbf{x} \in \mathbb{R}^n$ *satisfies the proportional fairness if and only if the following inequality holds true for any possible allocation vector* $\mathbf{y} \in \mathbb{R}^n$.

$$\sum_{i=1}^{n} \frac{y_i - x_i}{x_i} \leq 0. \tag{6}$$

Meanwhile, if we define an operator $\nabla J_{\mathbf{x}} \cdot (\mathbf{y} - \mathbf{x})$ on $\mathbf{x}$ and $\mathbf{y}$ to represent the formula in the definition, then it is easy to show that the proportional fairness solution $\mathbf{x}$ is the maximizer of the function $J(\mathbf{x})$

$$J(\mathbf{x}) = \sum_{i=1}^{n} \ln(x_i), \tag{7}$$

where the $\nabla J_{\mathbf{x}}$ is given by the derivative of J at $\mathbf{x}$.

Note the definition of proportional fairness we presented above does not directly related with data rate, even though it is mainly used for the data rate allocation issues from the existing literatures. In fact, the proportional fairness can be defined over any limited resources. We introduce the definition carefully because the data rate (or capacity) for each femtocell i depends on the RB allocation matrix $\mathbf{a}_i$ and power allocation vector $\mathbf{p}_i$ both. By analysis, if we want to calculate the vector of femtocells capacities with proportional fairness, that would require us to find a matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_F]$ and a matrix $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_F]$ as the solution to maximize the problem given below

$$J(\mathbf{A}, \mathbf{P}) = \sum_{i=1}^{F} \ln \left(\sum_{w=1}^{N} \Delta W log_2 \left(1 + \frac{p_i^w g_{i,i}^w}{q^w g_{b,i}^w + \sum_{j=1, j \neq i}^{F} a_j^w p_j^w g_{j,i}^w} \right) \right), \quad (8)$$

which is still too complex.

We notice the fact that the interference is only related with a particular RB between femtocells, and power allocation is for the particular RB as well. We propose apply the proportional fairness on the signal to interference ratio for each RB w over the femtocells, so we can simplify the problem by $a_i^w = 1, \forall\, i \in \{1, 2, \ldots, F\}$. Now we are required to find the vector $\hat{\mathbf{p}}^w = [\hat{p}_1^w, \hat{p}_2^w, \ldots, \hat{p}_F^w]$ to maximize

$$J(\hat{\mathbf{p}}^w) = \sum_{i=1}^{F} \ln \left(\frac{\hat{p}_i^w g_{i,i}^w}{q^w g_{b,i}^w + \sum_{j=1, j \neq i}^{F} \hat{p}_j^w g_{j,i}^w} \right) \quad (9)$$

with the constrain that $0 \leq p_i^w \leq p_{max}$. Recall that p_{max} is the identical upper bound for all femtocell base stations. Formula (9) can be rewritten as $J(\hat{\mathbf{p}}^w) = \sum_{i=1}^{F} \left(\ln \left(\hat{p}_i^w g_{i,i}^w \right) - \ln \left(q^w g_{b,i}^w + \sum_{j=1, j \neq i}^{F} \hat{p}_j^w g_{j,i}^w \right) \right)$. Note $q^w g_{b,i}^w$ is a constant, the formula above is in fact a summation of concave functions, which is also concave. So it has only one $\hat{\mathbf{p}}^w$ as its solution. Note there are N frequency bands, so after solving the similar question for all frequency band $w \in \{1, 2, \ldots, N\}$, we would have a matrix $\hat{\mathbb{P}} = \left[\hat{\mathbf{p}}^1, \hat{\mathbf{p}}^2, \ldots, \hat{\mathbf{p}}^N \right]^T$, which gives us every $\hat{p}_i^w$ for $\forall\, i\, \forall\, w$.

Two other propositions can be made here, and before that we need to give the third definition about submodular game [10] [11].

Definition 3 (Submodular Game) *We say a normal form game is submodular if and only if all of the three conditions hold for this game: 1) The action set for each player is compact; 2) The payoff function for each player must be continuous; 3) The Hessian matrix of the payoff function for each player should be negative-semidefinite.*

Some important properties of submodular games are 1) Nash equilibrium exists in games with increasing best responses; 2) Pure strategy Nash equilibrium exists; 3) The equilibrium is stable if it is the unique equilibrium in the game [11].

Proposition 1: *We consider a game for each w: a) femtocells are the players, b) each player i has strategy p_i^w within interval $[0, p_{max}]$, c) the utility function for player i is ln $\left(\frac{\hat{p}_i^w g_{i,i}^w}{q^w g_{b,i}^w + \sum_{j=1, j \neq i}^{F} \hat{p}_j^w g_{j,i}^w} \right)$, then the game is a submodular game. Present the game regarding to RB w as G_{sm}^w, then the $\hat{\mathbf{p}}^w$, which maximizes* *the summation over all players' utilities, is a Nash equilibrium profile for these players envolved in G^w_{sm}.*

Proof Sketch: It is obvious that G^w_{sm} satisfies the first and second conditions of submodular game. It satisfies the third condition as well because the utility function is concave so the Hessian is negative-semidefinite for each player. And the conclusion that $\hat{\mathbf{p}}^w$ is a Nash equilibrium profile is drawn from the property of submodular games. ■

Proposition 1 actually tells us that the solution vector $\hat{\mathbf{p}}^w$ that maximizes the Formula (9) is a stable strategy profile for all femtocells. For $\forall\ i$, $\hat{p}^w_i$ in $\hat{\mathbf{p}}^w$ represents the *best response* power that femtocell i should use to transmit it signal on RB w when other femtocells $-i$ select their transmission power $\hat{\mathbf{p}}^w_{-i}$ to transmit their signals on RB w. Moreover, according to properties of the submodular game, such a stable strategy profile is unique and globally stable.

Proposition 2: *Scaling each row vector in the matrix $\hat{\mathbb{P}}$ will not violate the proportional fairness we proposed above.*

Proof Sketch: Recall the femtocell users are usually in the building, where many femtocell base stations are deployed nearby. Hence in practice, $\sum_{j=1,j\neq i}^{F} p^w_j g^w_{j,i}$ is much greater than $q^w g^w_{b,i}$. Furthermore, scaling the vector $\hat{\mathbf{p}}^w$ is actually multiplying a constant to the nominator and denominator of the fraction for each summation term in (9), so the proportional fairness still holds after scaling the vector $\hat{\mathbf{p}}^w$. ■

According to Proposition 2, we can scale the row vectors in the matrix $\hat{\mathbb{P}}$ without violating the proportional fairness over signal to noise ratio. We tried to solve the maximization problem presented by Formula (9) regarding to the RB w only, so we have the constrain $0 \leq p^w_i \leq p_{max}$. However, when the matrix $\hat{\mathbb{P}}$ has been solved by solving all its row vectors, we need to consider the constrain $\sum_{w=1}^{M} p^w_i \leq p_{max}$. This constrain simply means for every femtocell, the summation of its transmission power over all RBs cannot be greater than the highest possible power that the femtocell base station can use. Therefore, we need to find an $N \times 1$ vector $\mathbf{v}$ in $\mathbb{R}^N$, with all element greater than or equal to zero, so that for $\forall\ i \in \{1, 2, \ldots, F\}$, $\sum_{w=1}^{w=N} v_w \hat{p}^w_i \leq p_{max}$ holds. Denote $\mathbb{P}$ as the matrix after scaling, then every row vector $\mathbf{p}^w$ satisfies the proportional fairness, and every column vector $\mathbf{p}_i$ satisfies $\sum_{w=1}^{M} p^w_i \leq p_{max}$.

Now $\mathbb{P}$ can help us to solve the problem given by Formula (5). Since the power allocation problem has been solved, each femtocell i know its $\mathbf{b}_i$, so femtocell i can optimize the RB assignment vector $\mathbf{a}_i$ to its own users, which is finally given by $\mathbf{a}_i = \arg\max u\,(s_i, s_{-i})$.

4 Conclusions

In this paper, we considered the issues about RB allocation, power allocation, and interference fairness together for femtocells in a two-tier OFDMA cellular system. The first two issues can be modeled in a exact potential game. Although the equilibrium does exist in potential games, our problem has high complexity be to solved because of the tight coupled dimensions in the strategy space. So we proposed to model the second and third issues together as a set of submodular games, and each submodular game is for each frequency band (or RB) w. Based on the fact that the interference happens on the same frequency band only, we proved that the proportional fair power allocation over all femtocell base stations for each RB w, is just the Nash equilibrium strategy profile for the submodular game regarding to w, which is also globally stable. As the result, the equilibrium strategy profile can be ready solved by the minimization problem transformed by proportional fairness power allocation problem.

Each Nash equilibrium strategy profile for frequency band w is in fact a row vector in the power allocation matrix, which describes the power for each RB in each femtocell. After these submodular games for these frequency bands are solved, the power allocation matrix will be taken back to the exact potential game. Then the RB allocation vector for each femtocell over all frequency bands can be readily solved, so the entire solution of the ordinal problem is achieved.

This solution is also globally stable because the utility functions in all submodular game are with the same form and concave, by the submodular game's property, the Nash equilibrium is unique and globally stable. We can further conclude the power allocation matrix is unique and stable, so when each femtocell picks its RB allocation vector to maximize its utility, the achieved RB allocation vector is the stable solution as well.

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