

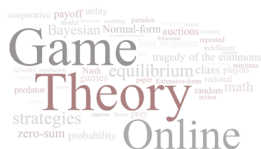
Game Theory Week I

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Agenda

1. Lecture + Discussion about Week 1's Material
2. Quick Latex Tutorial
3. Homework 1

Defining Games - The Normal Form



- Finite, n -person **normal form** game: $\langle N, A, u \rangle$:
 - **Players:** $N = \{1, \dots, n\}$ is a finite set of n , indexed by i
 - **Action set** for player i A_i
 - $a = (a_1, \dots, a_n) \in A = A_1 \times \dots \times A_n$ is an **action profile**
 - **Utility function or Payoff function** for player i : $u_i : A \mapsto \mathbb{R}$
 - $u = (u_1, \dots, u_n)$, is a **profile of utility functions**
- Writing a 2-player game as a **matrix**:
 - “row” player is player 1, “column” player is player 2
 - rows correspond to actions $a_1 \in A_1$, columns correspond to actions $a_2 \in A_2$
 - cells listing utility or payoff values for each player: the row player first, then the column

Best Response



- If you knew what everyone else was going to do, it would be easy to pick your own action
- Let $a_{-i} = \langle a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_n \rangle$.
 - now $a = (a_{-i}, a_i)$

Definition (Best response)

$a_i^* \in BR(a_{-i})$ iff $\forall a_i \in A_i, u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$.

Nash Equilibrium



- Really, no agent knows what the others will do.
- What can we say about which actions will occur?

- Idea: look for **stable** action profiles.

Definition (Nash Equilibrium)

$a = \langle a_1, \dots, a_n \rangle$ is a (“pure strategy”) **Nash equilibrium** iff
 $\forall i, a_i \in BR(a_{-i})$.

Nash Equilibria of Example Games



	C	D
C	-1, -1	-4, 0
D	0, -4	-3, -3

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

	B	F
B	2, 1	0, 0
F	0, 0	1, 2

	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

	<i>L</i>	<i>M</i>	<i>R</i>
<i>T</i>	59, 58	46, 83	85, 61
<i>B</i>	38, 29	70, 52	37, 23

Domination



- Let s_i and s'_i be two strategies for player i , and let S_{-i} be the set of all possible strategy profiles for the other players
 - What's a “strategy”?
 - For now, just choosing an action (“pure strategy”)

Definition

s_i **strictly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$

Definition

s_i **very weakly dominates** s'_i if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$

Pareto Optimality

- When one outcome o is at least as good for every agent as another outcome o' , and there is some agent who strictly prefers o to o' :
 - it seems reasonable to say that o is better than o'
 - we say that o **Pareto-dominates** o' .

Definition (Pareto Optimality)

An outcome o^* is **Pareto-optimal** if there is no other outcome that Pareto-dominates it.



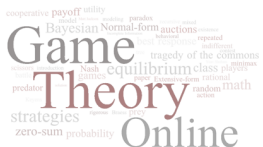
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- can a game have more than one Pareto-optimal outcome?



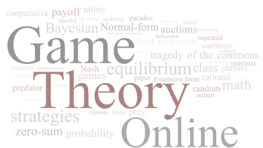
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Definition (Pareto Optimality)

An outcome o^* is **Pareto-optimal** if there is no other outcome that Pareto-dominates it.

- can a game have more than one Pareto-optimal outcome?
- does every game have at least one Pareto-optimal outcome?

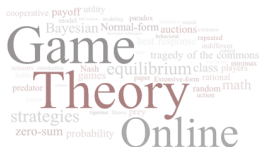


Pareto Optimal Outcomes in Example Games

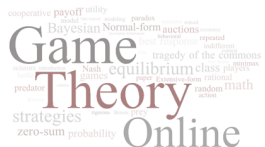
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Pareto Optimal Outcomes in Example Games



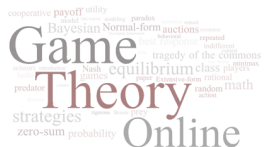
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Pareto Optimal Outcomes in Example Games



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The paradox of *Prisoner's dilemma*:

the (DS) Nash equilibrium is the only non-Pareto-optimal outcome!

Beginner's L^AT_EX Tutorial

Imran Rashid

lots borrowed from Marius

October 2, 2007

Why Use Latex?

- ▶ Great for typesetting math
- ▶ automated placement of figures & tables
- ▶ automatic generation of references to tables, figures, bibliographies
- ▶ free and universal
- ▶ separate content from layout
- ▶ Can create documents, slides, etc.
- ▶ Pretend to be a theory student

The Good, The Bad, ...

😊	😞
tools(?)	compile, debug, view, edit
write logically latex will take care of layout	not WYSIWYG
$\sqrt{\frac{t\beta^x}{\lambda x: \sum_{n=1}^{x^8} \log(\rho \otimes x)}}$	
automated content	
extremely powerful	steep learning curve
	collaborators may not know latex

Latex Commands

Two basic forms:

1. `\SomeCommand{AnArgument}`

2. `\begin{SomeEnvironment}`

...

`\end{SomeEnvironment}`

Math Mode

By default, LaTeX is in “text” mode. Have to switch to math to use math mode:

- ▶ Use `$... $` in the middle of a text-block
- ▶ Use `\ [... \]` to insert a block of math
- ▶ Use `\begin{align} ... \end{align}` to have aligned equations

Lists

```
\begin{itemize}  
  item ...  
  item ...  
\end{itemize}
```

can use enumerate instead of itemize

Tables

```
\begin{table}
\centering
\begin{tabular}{|c|r}
Height & & Weight \\
\hline
5.4 & & 160 \\
6.1 & & 234 \\
\end{tabular}
\caption{Some text that is a caption for the table}
\label{tableLabel}
\end{table}
```

Height	Weight
5.4	160
6.1	234

Table: Some text that is a caption for the table

Referencing Tables and Figures in the text

1. Use `\label{aLabelName}` in your figures and tables
2. In the text, reference them with `\ref{aLabelName}`
3. run latex twice
4. Reorganize your figures as much as you want – numbering will always be correct.

See BibTeX

Miscellaneous Tips

1. Symbols need to be in math mode — use the $\$...\$$.
2. Don't mess with spacing too much — try to let latex do it for you.
3. Format your source code.
4. Don't freak out if you have 100 errors — you probably forgot a `\end{}` or a $\$$.
5. Compile often; if there are a lot of problems, try to narrow it down piecemeal.
6. Use the other grad students

Useful Tools

- ▶ **MikTeX** — (<http://miktex.org/>) latex distribution + package manager
- ▶ **TeXnicCenter** — (http://texniccenter.sourceforge.net/front_content.php) IDE for windows
- ▶ **AucTeX** — (<http://www.gnu.org/software/auctex/>) mode for latex authoring in emacs (from Marius)
- ▶ **TeXShop** — (<http://www.uoregon.edu/~koch/texshop/>) IDE for latex on Mac (from Krzysztof et. al)
- ▶ **Kile** — (<http://kile.sourceforge.net/>) IDE for linux
- ▶ **JabRef** — (<http://jabref.sourceforge.net/>) for managing your bibliographies (from Julie)
- ▶ Many others out there ... consult your local tex guru