Reasoning Under Uncertainty: Belief Network Inference

CPSC 322 – Uncertainty 5

Textbook §10.4

Lecture Overview

1. Recap
2. Belief Network Examples

Components of a belief network

Definition (belief network)

A belief network consists of:

- a directed acyclic graph with nodes labeled with random variables
- a domain for each random variable
- a set of conditional probability tables for each variable given its parents (including prior probabilities for nodes with no parents).

Belief network summary

- A belief network is a directed acyclic graph (DAG) where nodes are random variables.
  - A belief network is automatically acyclic by construction.
- The parents of a node n are those variables on which n directly depends.
- A belief network is a graphical representation of dependence and independence:
  - A variable is conditionally independent of its non-descendants given its parents.

Relating BNs to the joint

Belief networks are compact representations of the joint.

To encode the joint as a BN:

1. Totally order the variables of interest: $X_1, \ldots, X_n$
2. Write down the chain rule decomposition of the joint, using this ordering: $P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | X_{i-1}, \ldots, X_1)$
3. For every variable X_i, find the smallest set $pX_i \subseteq \{X_1, \ldots, X_{i-1}\}$ such that $P(X_i | X_{i-1}, \ldots, X_1) = P(X_i | pX_i)$.
   - If $pX_i \neq \{X_1, \ldots, X_{i-1}\}$, X_i is conditionally independent of some of its ancestors given pX_i.
4. Now we can write $P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i | pX_i)$
5. Construct the BN:
   - Nodes are variables
   - Incoming edges to each variable X_i from each variable in pX_i
   - Conditional probability table for variable X_i: $P(X_i | pX_i)$

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Example: Fire Diagnosis

Suppose you want to diagnose whether there is a fire in a building

- you receive a noisy report about whether everyone is leaving the building.
- if everyone *is* leaving, this may have been caused by a fire alarm.
- if there is a fire alarm, it may have been caused by a fire or by tampering
- if there is a fire, there may be smoke

Example: Fire Diagnosis

First you choose the variables. In this case, all are boolean:

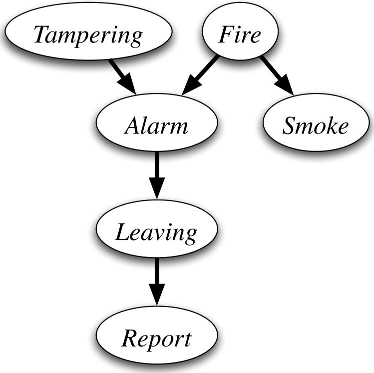
- Tampering is true when the alarm has been tampered with
- Fire is true when there is a fire
- Alarm is true when there is an alarm
- Smoke is true when there is smoke
- Leaving is true if there are lots of people leaving the building
- Report is true if the sensor reports that people are leaving the building

Example: Fire Diagnosis

- Next, you order the variables: $Fire$; $Tampering$; $Alarm$; $Smoke$; $Leaving$; $Report$.
- Now evaluate which variables are conditionally independent given their parents:
  - $Fire$ is independent of $Tampering$ (learning that one is true would not change your beliefs about the probability of the other)
  - $Alarm$ depends on both $Fire$ and $Tampering$: it could be caused by either or both.
  - $Smoke$ is caused by $Fire$, and so is independent of $Tampering$ and $Alarm$ given whether there is a $Fire$
  - $Leaving$ is caused by $Alarm$, and thus is independent of the other variables given $Alarm$.
  - $Report$ is caused by $Leaving$, and thus is independent of the other variables given $Leaving$.

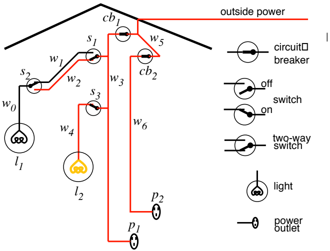
Example: Fire Diagnosis

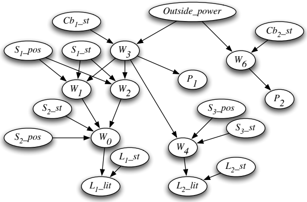
This corresponds to the following belief network:



Of course, we're not done until we also come up with conditional probability tables for each node in the graph.

Example: Circuit Diagnosis





The belief network also specifies:

- The domain of the variables: $W_0, \ldots, W_6 \in \{live, dead\}$ S_1_pos, S_2_pos, and S_3_pos have domain $\{up, down\}$ S_1_st has $\{ok, upside\_down, short, intermittent, broken\}$.
- Conditional probabilities, including:
  $P(W_1 = live | s_1\_pos = up \land S_1\_st = ok \land W_3 = live)$
  $P(W_1 = live | s_1\_pos = up \land S_1\_st = ok \land W_3 = dead)$
  $P(S_1\_pos = up)$
  $P(S_1\_st = upside\_down)$

Example: Circuit Diagnosis

The power network can be used in a number of ways:

- Conditioning on the status of the switches and circuit breakers, whether there is outside power and the position of the switches, you can simulate the lighting.
- Given values for the switches, the outside power, and whether the lights are lit, you can determine the posterior probability that each switch or circuit breaker is ok or not.
- Given some switch positions and some outputs and some intermediate values, you can determine the probability of any other variable in the network.

Example: Liver Diagnosis

Source: Onisko et al., 1999