CSPs: Representation and Search

CPSC 322 – CSPs 2

Textbook §4.3 – 4.5

Lecture Overview

Variables

- We define the state of the world as an assignment of values to a set of variables
  - variable: a synonym for feature
  - we denote variables using capital letters
  - each variable V has a domain $dom(V)$ of possible values
- Variables can be of several main kinds:
  - Boolean: $|dom(V)| = 2$
  - Finite: the domain contains a finite number of values
  - Infinite but Discrete: the domain is countably infinite
  - Continuous: e.g., real numbers between 0 and 1
- We'll call the set of states that are induced by a set of variables the set of possible worlds

Constraints

Constraints are restrictions on the values that one or more variables can take

- Unary constraint: restriction involving a single variable
  - of course, we could also achieve the same thing by using a smaller domain in the first place
- k-ary constraint: restriction involving the domains of k different variables
  - it turns out that k-ary constraints can always be represented as binary constraints, so we'll often talk about this case
- Constraints can be specified by
  - giving a list of valid domain values for each variable participating in the constraint
  - giving a function that returns true when given values for each variable which satisfy the constraint
- A possible world satisfies a set of constraints if the set of variables involved in each constraint take values that are consistent with that constraint

Lecture Overview

Constraint Satisfaction Problems: Definition

Definition

A constraint satisfaction problem consists of:

- a set of variables
- a domain for each variable
- a set of constraints

Definition

A model of a CSP is an assignment of values to variables that satisfies all of the constraints.

Constraint Satisfaction Problems: Variants

We may want to solve the following problems with a CSP:

- determine whether or not a model exists
- find a model
- find all of the models
- count the number of models
- find the best model, given some measure of model quality
  - this is now an optimization problem
- determine whether some property of the variables holds in all models

CSPs: Game Plan

It turns out that even the simplest problem of determining whether or not a model exists in a general CSP with finite domains is $\mathcal{NP}$-hard

- we can't hope to find an efficient algorithm.

However, we can try to:

- find algorithms that are fast on "typical" cases
- identify special cases for which algorithms are efficient (polynomial)
- find approximation algorithms that can find good solutions quickly, even they may offer no theoretical guarantees
- develop parallel or distributed algorithms so that additional hardware can be used

Lecture Overview

1. Recap
2. CSPs
3. Search

CSPs as Search Problems

In order to think about how to solve CSPs, let's map CSPs into search problems.

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- nodes: assignments of values to a subset of the variables
- neighbours of a node: nodes in which values are assigned to one additional variable
- start node: the empty assignment (no variables assigned values)
- leaf node: a node which assigns a value to each variable
- goal node: leaf node which satisfies all of the constraints

Note: the path to a goal node is not important

CSPs as Search Problems

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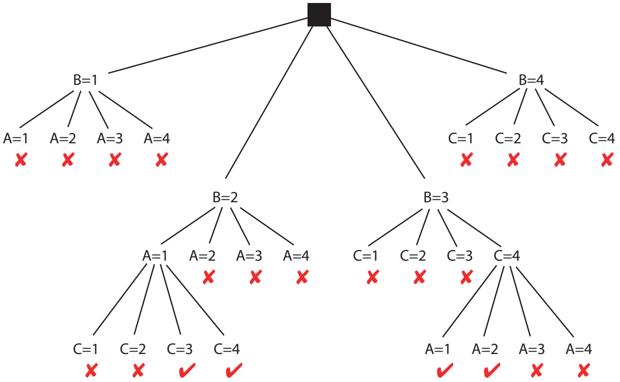
  - there are no costs, so there's no role for a heuristic function
  - the tree is always finite and has no cycles, so DFS is better than BFS
- How can we prune the DFS search tree?
  - once we reach a node that violates one or more constraints, we know that a solution cannot exist below that point
  - thus we should backtrack rather than continuing to search
  - this can yield us exponential savings over unpruned DFS, though it's still exponential

Example

Problem:

- Variables: A, B, C
- Domains: $\{1, 2, 3, 4\}$
- Constraints: $A < B,\ B < C$

Example



Note: the algorithm's efficiency depends on the order in which variables are expanded