# Logic: Resolution Proofs; Datalog

### CPSC 322 - Logic 5

#### Textbook §5.2; 12.2

Logic: Resolution Proofs; Datalog

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## Lecture Overview





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### Proofs

- A proof is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure,  $KB \vdash g$  means g can be derived from knowledge base KB.
- Recall  $KB \models g$  means g is true in all models of KB.

#### Definition (soundness)

A proof procedure is sound if  $KB \vdash g$  implies  $KB \models g$ .

### Definition (completeness)

A proof procedure is complete if  $KB \models g$  implies  $KB \vdash g$ .

Recap

### $KB\vdash g$ if $g\subseteq C$ at the end of this procedure:

 $C := \{\};$ repeat
select clause " $h \leftarrow b_1 \land \ldots \land b_m$ " in KB such that  $b_i \in C$  for all i, and  $h \notin C$ ;  $C := C \cup \{h\}$ until no more clauses can be selected.

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### Soundness of bottom-up proof procedure

### If $KB \vdash g$ then $KB \models g$ .

- Suppose there is a g such that  $KB \vdash g$  and  $KB \not\models g$ .
- Let h be the first atom added to C that's not true in every model of KB.
- Suppose h isn't *true* in model I of KB.
- There must be a clause in *KB* of form

$$h \leftarrow b_1 \land \ldots \land b_m$$

Each  $b_i$  is true in I. h is false in I. So this clause is false in I.

• Therefore *I* isn't a model of *KB*. Contradiction: thus no such *g* exists.

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### Minimal Model

We can use proof procedure to find a model of KB.

- First, observe that the C generated at the end of the bottom-up algorithm is a fixed point.
  - further applications of our rule of derivation will not change C.

#### Definition (minimal model)

Let the minimal model I be the interpretation in which every element of the fixed point C is true and every other atom is false.

#### Claim: I is a model of KB. Proof:

- Assume that I is not a model of KB. Then there must exist some clause h ← b<sub>1</sub> ∧ ... ∧ b<sub>m</sub> in KB (having zero or more b<sub>i</sub>'s) which is false in I.
- This can only occur when h is false and each  $b_i$  is true in I.
- If each  $b_i$  belonged to C, we would have added h to C as well.
- Since C is a fixed point, no such I can exist.

### Completeness

#### If $KB \models g$ then $KB \vdash g$ .

- Suppose  $KB \models g$ . Then g is true in all models of KB.
- Thus g is true in the minimal model.
- Thus g is generated by the bottom up algorithm.
- Thus  $KB \vdash g$ .

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### Lecture Overview





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### Top-down Ground Proof Procedure

Idea: search backward from a query to determine if it is a logical consequence of KB. An answer clause is of the form:

$$yes \leftarrow a_1 \land a_2 \land \ldots \land a_m$$

The SLD Resolution of this answer clause on atom  $a_i$  with the clause:

$$a_i \leftarrow b_1 \land \ldots \land b_p$$

is the answer clause

$$yes \leftarrow a_1 \land \cdots \land a_{i-1} \land b_1 \land \cdots \land b_p \land a_{i+1} \land \cdots \land a_m.$$

### Derivations

- An answer is an answer clause with m = 0. That is, it is the answer clause yes ← .
- A derivation of query " $?q_1 \land \ldots \land q_k$ " from KB is a sequence of answer clauses  $\gamma_0, \gamma_1, \ldots, \gamma_n$  such that
  - $\gamma_0$  is the answer clause  $yes \leftarrow q_1 \land \ldots \land q_k$ ,
  - $\gamma_i$  is obtained by resolving  $\gamma_{i-1}$  with a clause in KB, and
  - $\gamma_n$  is an answer.

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To solve the query ?q_1 \land \ldots \land q_k:
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$$ac := "yes \leftarrow q_1 \land \ldots \land q_k"$$

repeat

select atom  $a_i$  from the body of ac; choose clause C from KB with  $a_i$  as head; replace  $a_i$  in the body of ac by the body of Cuntil ac is an answer.

Recall:

- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives. select
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

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$$\begin{array}{lll} a \leftarrow b \wedge c. & a \leftarrow e \wedge f. & b \leftarrow f \wedge k. \\ c \leftarrow e. & d \leftarrow k. & e. \\ f \leftarrow j \wedge e. & f \leftarrow c. & j \leftarrow c. \end{array}$$

Query: ?a

Recap

$$\begin{array}{lll} \gamma_0: & yes \leftarrow a & & \gamma_4: & yes \leftarrow e \\ \gamma_1: & yes \leftarrow e \wedge f & & \gamma_5: & yes \leftarrow \\ \gamma_2: & yes \leftarrow f & & \\ \gamma_3: & yes \leftarrow c & & \end{array}$$

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### Example: failing derivation

#### Query: ?a

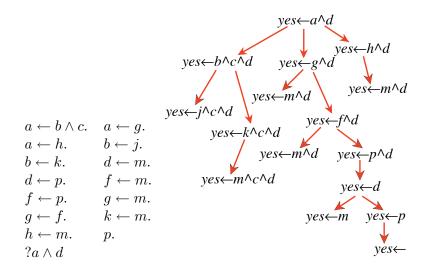
 $\begin{array}{ll} \gamma_0: & yes \leftarrow a \\ \gamma_1: & yes \leftarrow b \wedge c \\ \gamma_2: & yes \leftarrow f \wedge k \wedge c \\ \gamma_3: & yes \leftarrow c \wedge k \wedge c \end{array}$ 

 $\begin{array}{ll} \gamma_4: & yes \leftarrow e \wedge k \wedge c \\ \gamma_5: & yes \leftarrow k \wedge c \end{array}$ 

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### Search Graph



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