Logic: Resolution Proofs; Datalog

CPSC 322 - Logic 5

Textbook §5.2; 12.2

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Lecture Overview





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Proofs

- A proof is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB.
- Recall $KB \models g$ means g is true in all models of KB.

Definition (soundness)

A proof procedure is sound if $KB \vdash g$ implies $KB \models g$.

Definition (completeness)

A proof procedure is complete if $KB \models g$ implies $KB \vdash g$.

Recap

$KB\vdash g$ if $g\subseteq C$ at the end of this procedure:

 $C := \{\};$ repeat
select clause " $h \leftarrow b_1 \land \ldots \land b_m$ " in KB such that $b_i \in C$ for all i, and $h \notin C$; $C := C \cup \{h\}$ until no more clauses can be selected.

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Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Let h be the first atom added to C that's not true in every model of KB.
- Suppose h isn't *true* in model I of KB.
- There must be a clause in *KB* of form

$$h \leftarrow b_1 \land \ldots \land b_m$$

Each b_i is true in I. h is false in I. So this clause is false in I.

• Therefore *I* isn't a model of *KB*. Contradiction: thus no such *g* exists.

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Minimal Model

We can use proof procedure to find a model of KB.

- First, observe that the C generated at the end of the bottom-up algorithm is a fixed point.
 - further applications of our rule of derivation will not change C.

Definition (minimal model)

Let the minimal model I be the interpretation in which every element of the fixed point C is true and every other atom is false.

Claim: I is a model of KB. Proof:

- Assume that I is not a model of KB. Then there must exist some clause h ← b₁ ∧ ... ∧ b_m in KB (having zero or more b_i's) which is false in I.
- This can only occur when h is false and each b_i is true in I.
- If each b_i belonged to C, we would have added h to C as well.
- Since C is a fixed point, no such I can exist.

Completeness

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB.
- Thus g is true in the minimal model.
- Thus g is generated by the bottom up algorithm.
- Thus $KB \vdash g$.

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Top-down Ground Proof Procedure

Idea: search backward from a query to determine if it is a logical consequence of KB. An answer clause is of the form:

$$yes \leftarrow a_1 \land a_2 \land \ldots \land a_m$$

The SLD Resolution of this answer clause on atom a_i with the clause:

$$a_i \leftarrow b_1 \land \ldots \land b_p$$

is the answer clause

$$yes \leftarrow a_1 \land \cdots \land a_{i-1} \land b_1 \land \cdots \land b_p \land a_{i+1} \land \cdots \land a_m.$$

Derivations

- An answer is an answer clause with m = 0. That is, it is the answer clause yes ← .
- A derivation of query " $?q_1 \land \ldots \land q_k$ " from KB is a sequence of answer clauses $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that
 - γ_0 is the answer clause $yes \leftarrow q_1 \land \ldots \land q_k$,
 - γ_i is obtained by resolving γ_{i-1} with a clause in KB, and
 - γ_n is an answer.

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To solve the query ?q_1 \land \ldots \land q_k:
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$$ac := "yes \leftarrow q_1 \land \ldots \land q_k"$$

repeat

select atom a_i from the body of ac; choose clause C from KB with a_i as head; replace a_i in the body of ac by the body of Cuntil ac is an answer.

Recall:

- Don't-care nondeterminism If one selection doesn't lead to a solution, there is no point trying other alternatives. select
- Don't-know nondeterminism If one choice doesn't lead to a solution, other choices may. choose

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$$\begin{array}{lll} a \leftarrow b \wedge c. & a \leftarrow e \wedge f. & b \leftarrow f \wedge k. \\ c \leftarrow e. & d \leftarrow k. & e. \\ f \leftarrow j \wedge e. & f \leftarrow c. & j \leftarrow c. \end{array}$$

Query: ?a

Recap

$$\begin{array}{lll} \gamma_0: & yes \leftarrow a & & \gamma_4: & yes \leftarrow e \\ \gamma_1: & yes \leftarrow e \wedge f & & \gamma_5: & yes \leftarrow \\ \gamma_2: & yes \leftarrow f & & \\ \gamma_3: & yes \leftarrow c & & \end{array}$$

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Example: failing derivation

Query: ?a

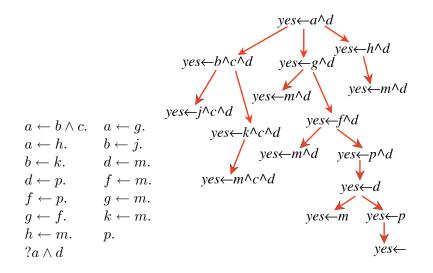
 $\begin{array}{ll} \gamma_0: & yes \leftarrow a \\ \gamma_1: & yes \leftarrow b \wedge c \\ \gamma_2: & yes \leftarrow f \wedge k \wedge c \\ \gamma_3: & yes \leftarrow c \wedge k \wedge c \end{array}$

 $\begin{array}{ll} \gamma_4: & yes \leftarrow e \wedge k \wedge c \\ \gamma_5: & yes \leftarrow k \wedge c \end{array}$

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