Reasoning under Uncertainty(3): Marginal and Conditional Independence

CPSC 322 – Uncertainty 3
(Giuseppe Carenini)

Textbook – 10.2

What is Co-op?

- Integration of academic studies with relevant, paid, and productive work experience.

- Co-op students gain skills and experience which prepare them for the future job market and give them improved employment opportunities upon graduation.

CPSC Co-op Info Session

Industry work experience
Networking
Money

Think Co-op!

- Date: Nov 8 (Thur)
- Time: 12:30 – 1:30 pm
- Location: DMP 301

Prizes to be won!

Lecture Overview

- Recap with Example
  - Joint Distribution
  - Marginalization
  - Conditional Probability
  - Chain Rule
  - Bayes' Rule
- Marginal Independence
- Conditional Independence

Recap Joint Distribution

- 3 binary random variables: **P(H,S,F)**
  - **H dom(H)={h, ¬h}** will have, will not have heart-attack
  - **S dom(S)={s, ¬s}** smokes, does not smoke
  - **F dom(F)={f, ¬f}** high fat diet, low fat diet

Recap Joint Distribution

- 3 binary random variables: **P(H,S,F)**
  - **H** **dom(H)={h, ¬h}** will have, will not have heart-attack
  - **S** **dom(S)={s, ¬s}** smokes, does not smoke
  - **F** **dom(F)={f, ¬f}** high fat diet, low fat diet

	f		**¬ f**	
	s	**¬ s**	**s**	**¬ s**
h	.015	.007	.005	.003
¬ h	.21	.51	.07	.18

Recap Marginalization

	f: s	f: ¬ s	¬ f: s	¬ f: ¬ s
h	.015	.007	.005	.003
¬ h	.021	.51	.07	.18

$$P(H,S) = \sum_{x \in dom(F)} P(H,S,F=x)$$

P(H,S)?

P(H)?

P(S)?

Recap Conditional Probability

P(H,S)	**s**	**¬ s**	P(H)
h	.02	.01	.03
¬ h	.28	.69	.97
P(S)	.30	.70	

$$P(S \mid H) = \frac{P(S, H)}{P(H)}$$

P(H|S)	**s**	**¬ s**
h		
¬ h		

P(S|H)

Recap Chain Rule

$$P(H, S, F) =$$

Recap Bayes Theorem

$$P(S \mid H) = \frac{P(S, H)}{P(H)}$$

$$P(H \mid S) = \frac{P(H, S)}{P(S)}$$

$$P(S \mid H) = \frac{P(H \mid S) P(S)}{P(H)}$$

Lecture Overview

- Recap with Example
- **Marginal Independence**
- Conditional Independence

Marginal Independence

DEF. Random variable **X** is marginal independent of random variable **Y** if, for all $x_i \in dom(X)$, $y_k \in dom(Y)$,

$$P(X = x_i \mid Y = y_k) = P(X = x_i)$$

That is, your knowledge of **Y**'s value doesn't affect your belief in the value of **X**

True in our example? E.g. Is Smoking conditional Independent of Heart-attack?

Marginal Independence

True in our example? Is Smoking conditional Independent of Heart-attack?

P(H,S)	**s**	**¬ s**	P(H)
h	.02	.01	.03
¬ h	.28	.69	.97
P(S)	.30	.70	

P(S|H)	**s**	**¬ s**
h	.666	.334
¬ h	.29	.71

Marginal Independence

Toss two fair coins. Two random variables

- **T1   dom(T1)={h, t}**
- **T2 dom(T2)={h, t}**

P(T1,T2)

	h	**t**
h		
t		

P(T1)

P(T2)

P(T2|T1)

	h	**t**
h		
t		

Marginal Independence

Consequence…

P(X,Y)=

Other Intuitive Examples

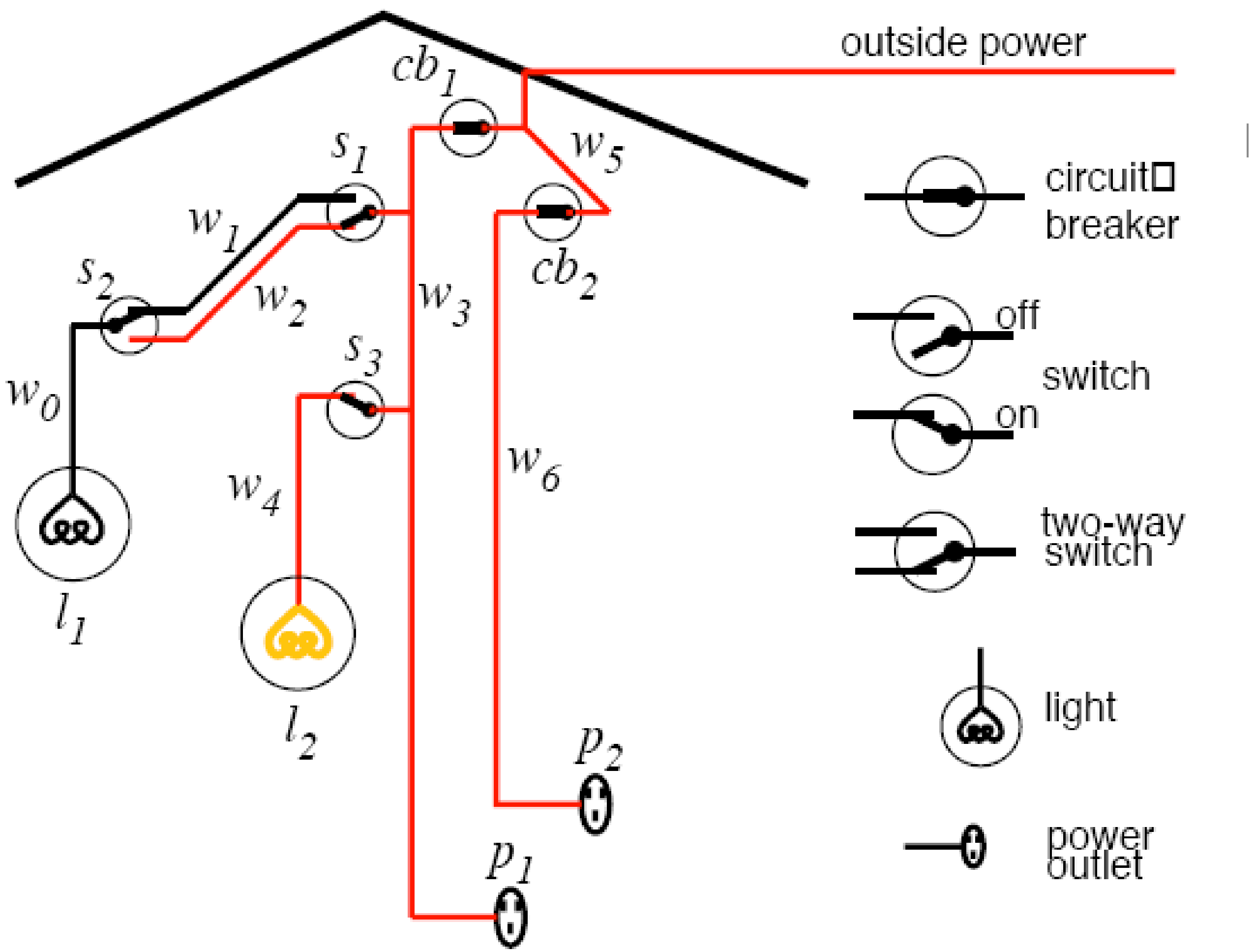
- **The probability that the Canucks will win the Stanley Cup is independent on ….**

is dependent on….

- **(Diagnostic assistant) whether light l_1 is lit is not independent of the position of switch s_2**

Diagnostic assistant example

- (Diagnostic assistant) whether light l_1 is lit is **not** independent of the position of switch s_2



Lecture Overview

- Recap with Example
- Marginal Independence
- Conditional Independence

Conditional Independence

Sometimes, two variables might not be marginally independent. However, they *become* independent after we observe some third variable

DEF. Random variable **X** is conditionally independent of random variable **Y** given random variable **Z** if, for all $x_i \in dom(X)$, $y_k \in dom(Y)$, $z_m \in dom(Z)$

$$P(X = x_i \mid Y = y_k, Z = z_m) = P(X = x_i \mid Z = z_m)$$

That is, knowledge of **Y**'s value doesn't affect your belief in the value of **X**, given a value of **Z**

Conditional Independence Example (1)

- Giuseppe separately phones two students, Alice and Bob.
- To each, he tells the same number, $n_g \in \{1, .. ,10\}$
- Due to the noise in the phone, Alice and Bob each imperfectly (and independently) draw a conclusion about what number Kevin said.
- Let the numbers Alice and Bob think they heard be n_a and n_b respectively.
- Are n_a and n_b marginally independent?

No, we'd expect (e.g.) $P(n_a = 1 \mid n_b = 1) > P(n_a = 1)$

Conditional Independence Example (1 cont')

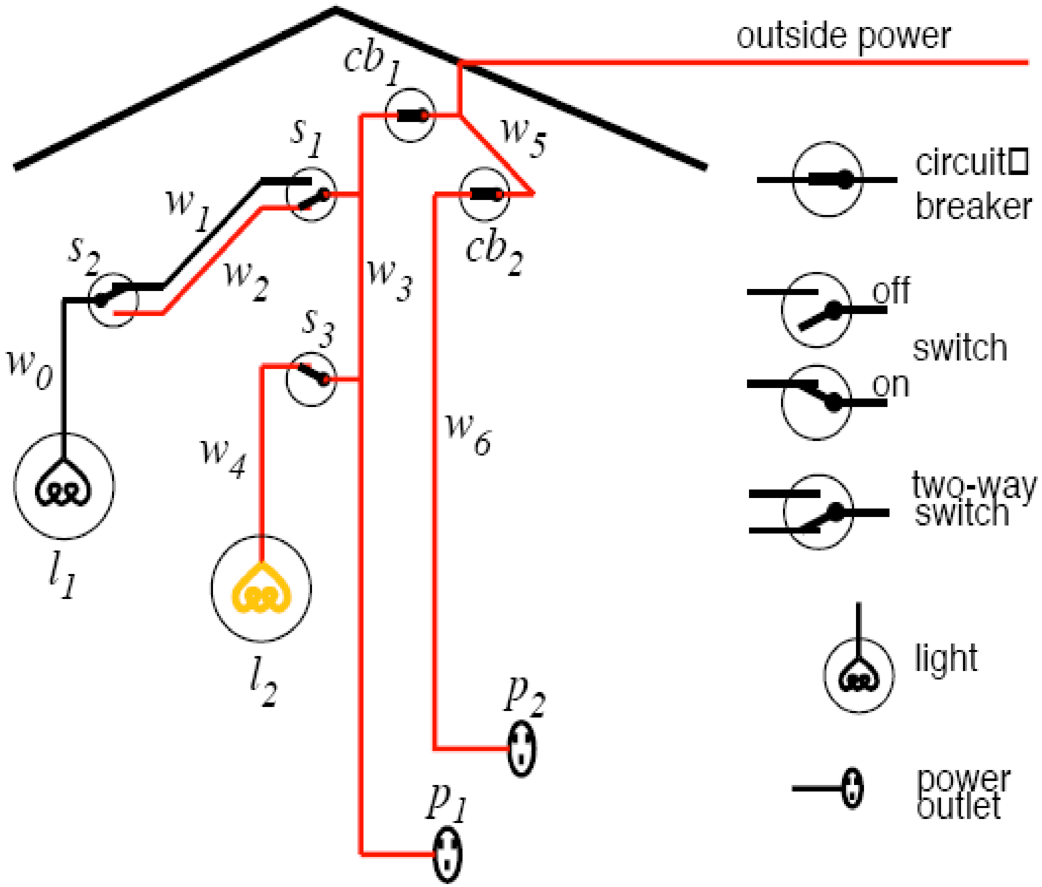
Why are n_a and n_b conditionally independent given n_g?

Because if we know the number that Giuseppe actually said, the variable n_a is irrelevant

e.g. $P(n_a = 1 \mid n_b = 1, n_g = 2) = P(n_a = 1 \mid n_g = 2)$

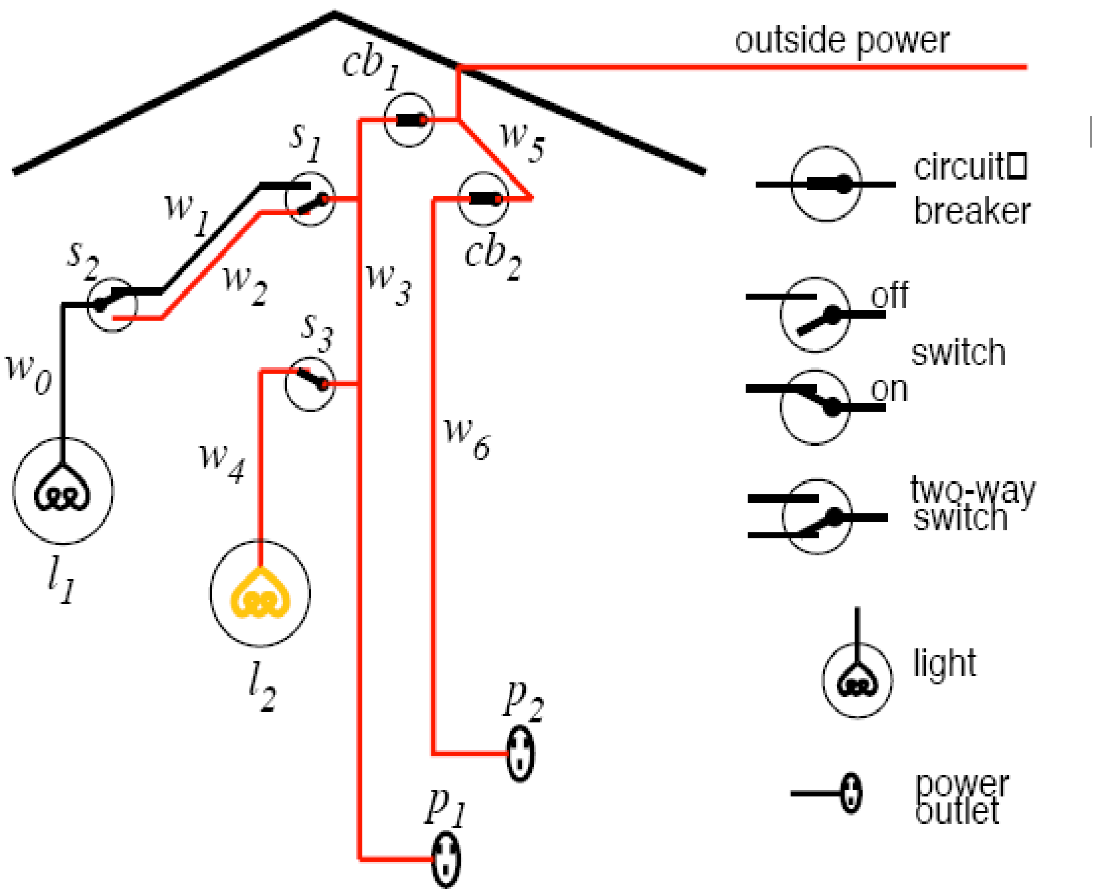
Conditional Independence Example 2

- Whether light l1 is lit is independent of the position of light switch s2 given whether there is/isn't power in wire w0.



Conditional Independence Example 3

- Every other variable may be independent of whether light l1 is lit given whether there is power in wire w0 and the status of light l1 (if it's ok, or if not, how it's broken).



Next Class

- Belief Networks…….