Temporal Action-Graph Games:
A New Representation for Dynamic Games

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Game Theory In One Slide ☺

- A game:
  - an interaction between two or more **self-interested agents**
  - each agent independently chooses a **strategy**
  - each agent derives utility from the resulting **strategy profile**

- Strategies:
  - **simultaneous-move games**: choosing from a set of actions
  - **dynamic games**: choosing actions at multiple points in time; conditioned on observations
  - can randomize over actions

- Reasoning about games:
  - Often involves computation of solution concepts e.g. **Nash equilibrium**

Representations of Games

	no utility structure
Simultaneous-move	normal form
Temporal	extensive form

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Temporal	extensive form	Multi-agent influence diagrams (MAIDs) [Koller & Milch 2001]

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	no utility structure	strict utility independence	context-specific indep., anonymity
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Overview

- AGGs
- TAGGs
- Computation
- Experiments

AGGs

- played on a set of action nodes
- each agent chooses an action node from a subset of action nodes
- for each action node, an action count is tallied

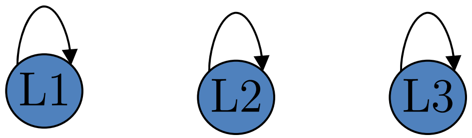
- utility dependence expressed by the action graph
  - utility of an agent depends only on
    - action chosen by the agent
    - action counts on the neighbors of the chosen action (configuration)

- representation is compact when action graph has small in-degrees

Example: simultaneous-move tollbooth game

- tollbooth with 3 lanes
  - 5 cars arrive
  - cars choose lanes simultaneously
  - utility depends on number of cars choosing same lane

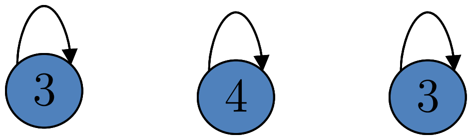
- context-specific independence (CSI): different independencies under diff context (player's own action)
- anonymity: utility depends on the numbers of agents taking certain actions, not their identities



Example: Dynamic Tollbooth Game

- tollbooth with 3 lanes
  - 20 cars arrive in 4 waves of 5 cars each
  - in each wave, cars choose lanes simultaneously
  - driver can observe number of cars in each lane
  - utility depends on number of cars choosing same lane, either before him or at the same time

- Extending AGGs to multiple time steps:
  - Action counts accumulate over time
  - Need to be able to specify agents' observations
  - Model uncertainty using chance variables



Defining TAGGs

- A TAGG is a tuple $(N, T, \mathcal{A}, \mathcal{X}, \mathcal{D}, \mathcal{U})$
  - N: set of players
  - T: duration
  - set of actions $\mathcal{A}$
  - set of chance variables $\mathcal{X}$
  - set of decisions $\mathcal{D}$
  - set of utility functions $\mathcal{U}$

TAGGs

- A decision D
  - Action set: a subset of $\mathcal{A}$
  - Observation set O[D]: of actions, decisions and chance vars
  - Set of payoff times pt(D)

- Utility function U_A^τ
  - One for each action at each time step
  - Set of parents
  - Utility depends only on its parents' instantiation at time τ
  - Evaluated at payoff times of decisions

Strategies

- A behavior strategy at decision D is a mapping from an instantiation of O[D] at time t(D)-1 to a distribution over its action set.

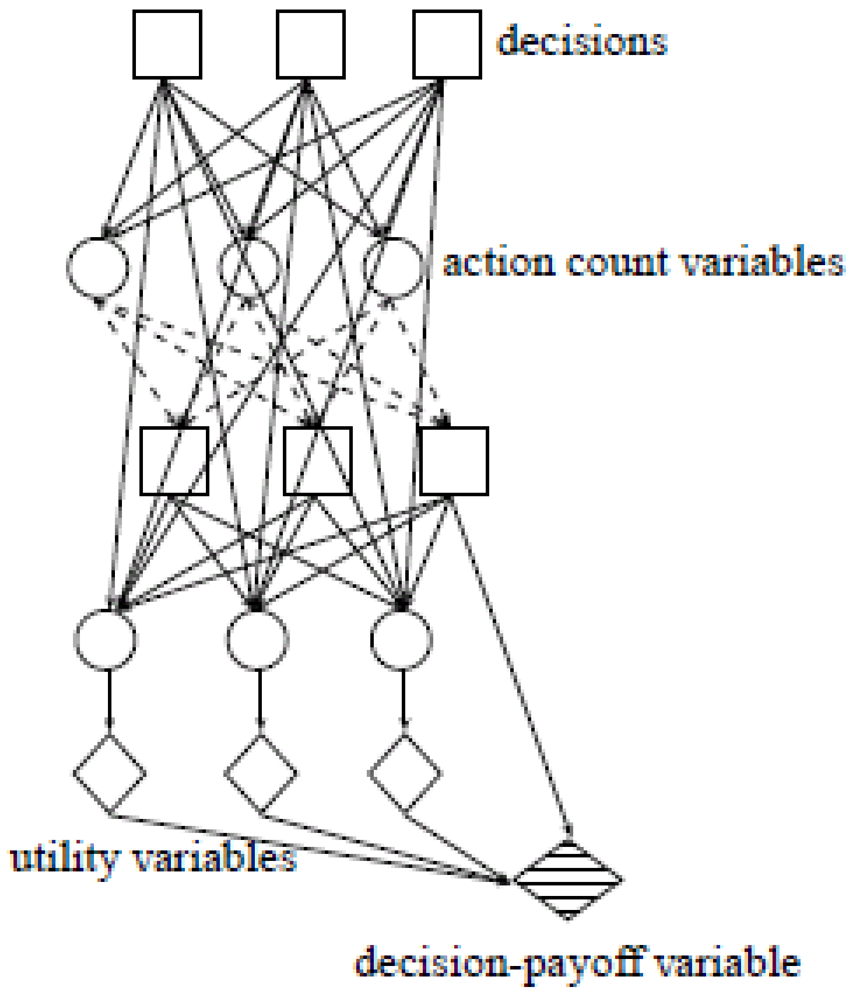
- A behavior strategy for player *i* is a tuple consisting of a behavior strategy for each of her decisions

- Behavior strategy profile: tuple of behavior strategies for all players

Induced Bayes Net

- Induced BN of a TAGG given a strategy profile
  - Formally describes how the TAGG is played out
  - Decisions, chance variables and utilities correspond to random variables in the BN
  - Action counts are time-dependent: we have a separate action count variable for each action at each time step
  - Decision-payoff variable u_D^τ utility of decision D received at payoff time τ
  - Expected utility of a player is the sum of the expected values of her decisions' decision-payoff variables.
  - Can similarly define induced MAID of a TAGG

Induced BN / MAID: tollbooth example



TAGGs and MAIDs

- Any TAGG is utility-equivalent to its induced MAID
  - However, induced MAID (and BN) has large in-degree; exponentially larger representation size
- For the other direction, any MAID can be efficiently encoded as a TAGG with same space complexity

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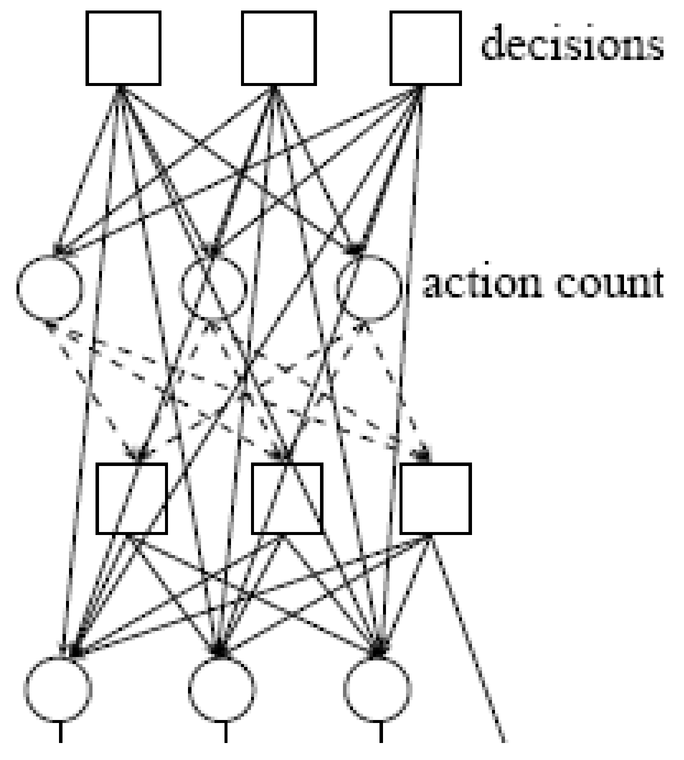
Computing Expected Utility

- Computing expected utility of a player, given a behavior strategy profile
  - An essential step in many game-theoretic computations
- Can be cast as inference problem on the induced BN: compute expected values of u_D^T
  - Can apply standard BN inference algorithm
  - TAGGs have additional structure; can be exploited to speedup computation

Exploiting anonymity

- Induced BN has large in-degre for action-count variables
  - Their CPDs are counting functi with causal independence structure [Heckerman&Breese 199
  - Can reduce in-degree by transforming the BN
    - Create nodes representing intermediate counts



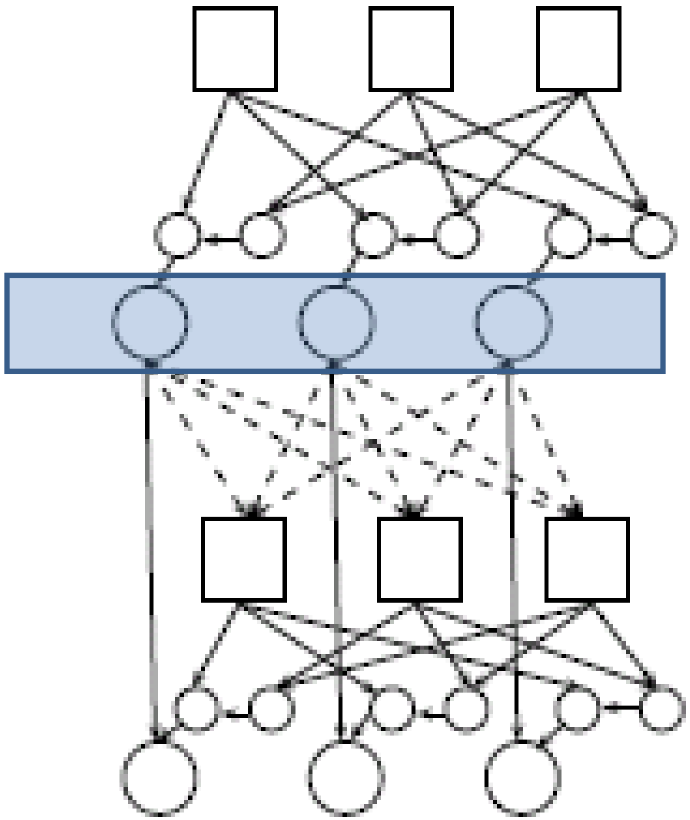
Exploiting Temporal Structure

- A network satisfies the Markov property if parents of variables at time t are at t or t-1.
  - Parts of the transformed BN (action counts) satisfy MP
  - Can transform it to one satisfying MP by duplicating variables

- Adapt the interface algorithm for Dynamic BNs
  - interface: set of variables in time t that have children in time t+1
    - d-separates "past" from "future"
  - algorithm eliminates variables in the temporal order; keeping distribution over interface variables

Tollbooth example

- interface at t: the action count variables at time t



Computation, Ctd

- Further exploiting the structure of transformed BN within the same time step
- We can exploit CSI for further speedup
- Our algorithm computes EU in poly time if
  - the number of interface variables at each time are bounded
  - inference over the chance variables can be done efficiently
- Our methods an be applied to speedup computation of Nash equilibria

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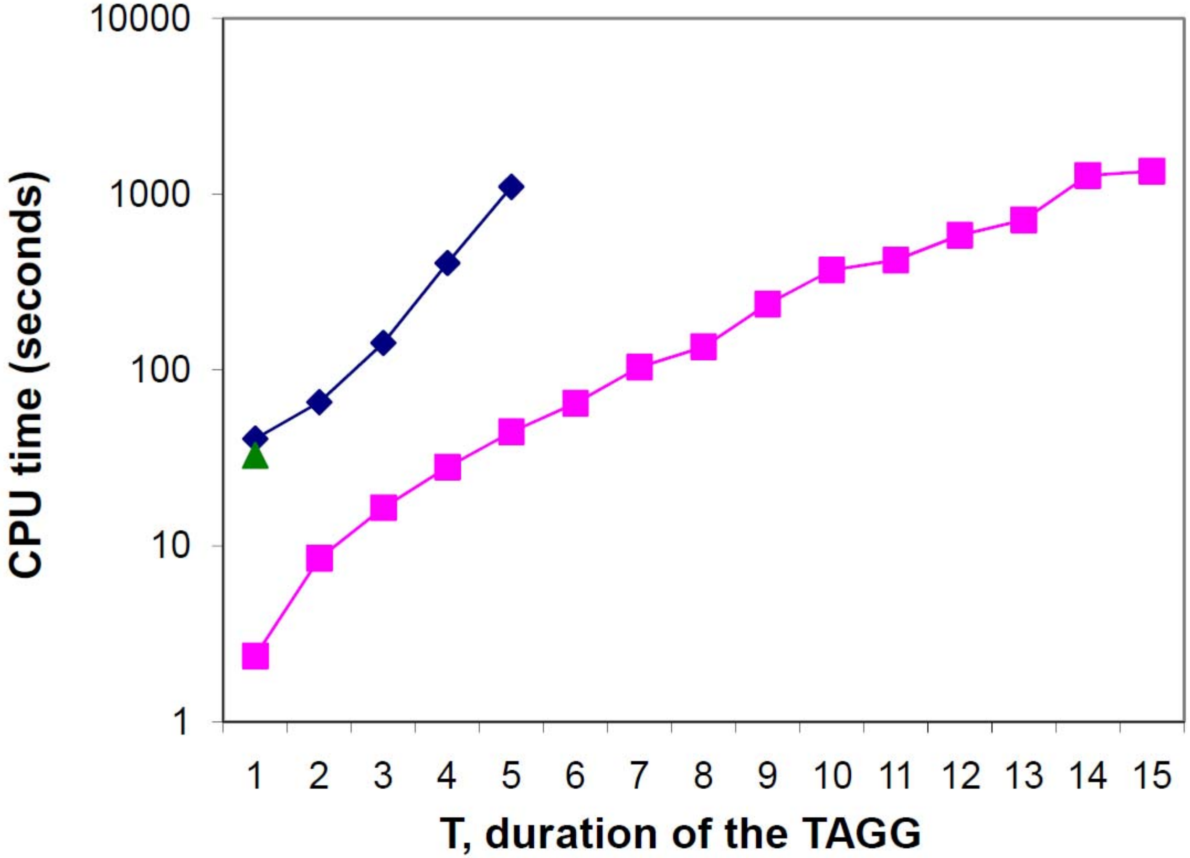
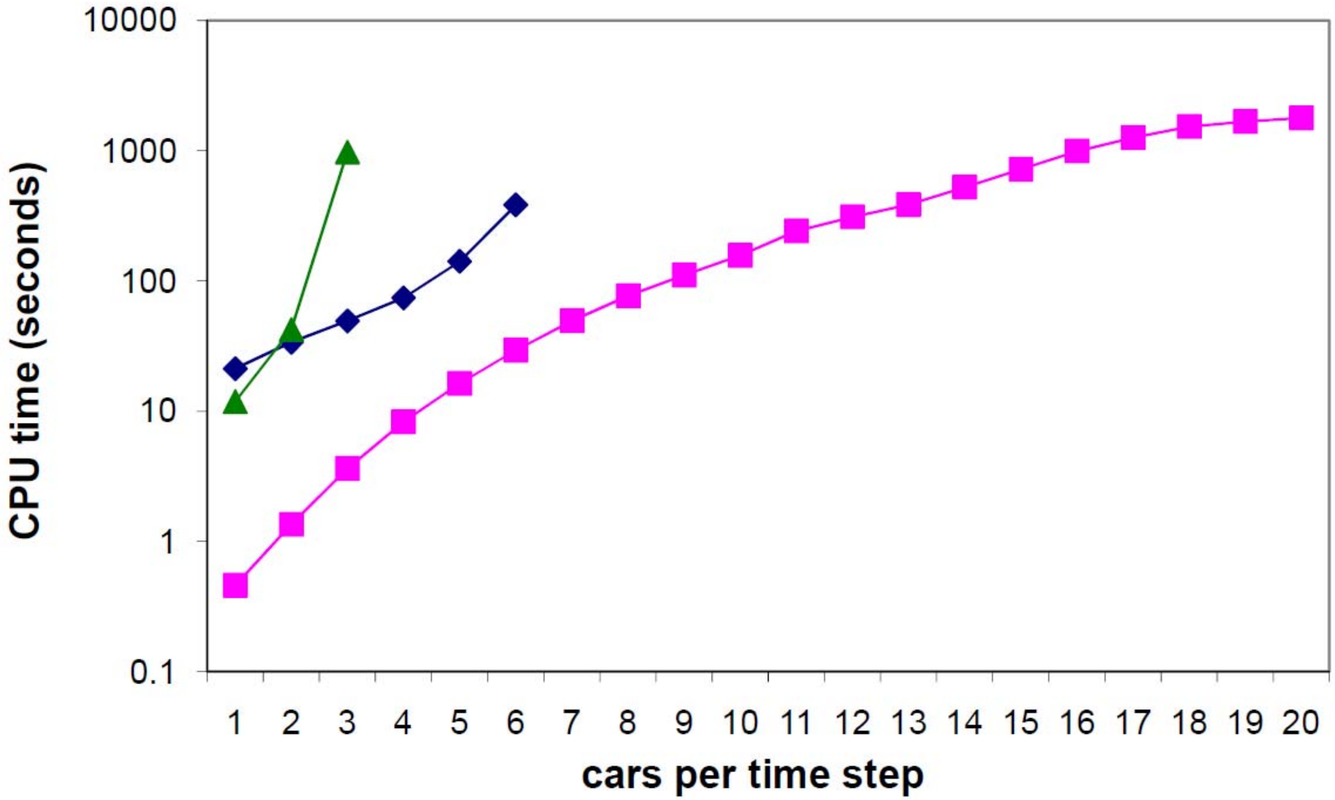
Experiments

- Compute EU for tollbooth games (3 lanes)
- Approach 1: standard clique tree algorithm on induced BN
- Approach 2: same clique tree algorithm on transformed BN
- Approach 3: our algorithm

Experiments: Tollbooth (5 cars per time step, varying T)



Experiments: Tollbooth
(T=3, varying # cars per time step)



Conclusions

- Temporal Action-Graph Games (TAGGs)
  - novel compact representation for dynamic games
  - extends AGGs to dynamic setting
  - compactly express wider variety of utility structure, including CSI and anonymity
  - exploit such structure for efficient computation
    - expected utility
    - Nash equilibrium