Bidding Clubs in First-Price Auctions
Extended Abstract

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Abstract

We introduce a class of mechanisms, called *bidding clubs*, that allow agents to coordinate their bidding in auctions. Bidding clubs invite a set of agents to join, and each invited agent freely chooses whether to accept the invitation or whether to participate independently in the auction. Clubs first conduct a "pre-auction"; depending on the outcome of the pre-auction some subset of the members of the club bid in the primary auction in a prescribed way. We model this setting as a Bayesian game, including agents' choices of whether or not to accept a bidding club's invitation. We examine the specific case of bidding clubs for first-price auctions, showing the existence of a Bayes-Nash equilibrium where agents choose to participate in bidding clubs when invited and truthfully declare their valuations to the coordinator. Furthermore, we show that the existence of bidding clubs benefits all agents, including those who do not belong to a bidding club.

Introduction

Economic and game-theoretic models have had significant impact on recent work in AI. Of particular interest has been work on economic mechanism design dealing with protocols for non-cooperative environments, which has not only applied the existing theory to computational settings, but has also extended it in various ways (Boutilier, Shoham, & Wellman 1997; Tennenholtz 1999). Work in AI has revisited the assumptions underlying optimal mechanism design (e.g. (Monderer & Tennenholtz 2000)), and considered computational issues in the design of such mechanisms (e.g. (Sandholm *et al.* 2001)). Much of the game-theoretic multiagent work in AI differs from related work in economics by approaching problems from agents' perspectives rather than from the perspective of the seller or mechanism designer. There is a body of work in AI that concerns agent behavior in various economic settings where the choice of mechanism is out of the agents' control, but where the mechanism is sufficiently elaborate to permit some form of strategic manipulation. Greenwald introduced the use of shopbots (Greenwald 1999) as a (relatively non-strategic) way for buyers to profit from competition between sellers on the internet; Parkes and Ungar studied proxy bidding (Parkes & Ungar 2000). In the recent Trading Agent Competition (see, e.g., (Stone & Greenwald 2000)) many AI researchers constructed competitive agents to operate in a rich economic setting; although strategic considerations were essential in this competition, the complexity of the setting defied theoretical analysis and forced agent behavior to rely on heuristics.

In this work we continue in the AI tradition of taking an agent's perspective, but tackle a fundamental economic mechanism which is simple enough to permit a theoretical approach. Auctions are the most well-studied and basic economic mechanisms, and have received a great deal of attention as a general approach for resource allocation in non-cooperative environments. We present a class of systems to assist sets of bidders, *bidding clubs.* The idea is similar to the idea behind "buyer clubs" on the internet (e.g., www.mobshop.com): to aggregate the market power of individual bidders. Buyer clubs work when buyers' interests are perfectly aligned; the more buyers join in a purchase the lower the price for everyone. In auctions held on the internet it is relatively easy for multiple agents to cooperate, hiding behind a single auction participant. Intuitively, these bidders should be able to gain by causing others to lower their bids in the case of a first-price auction or by possibly removing the second-highest bidder in the case of a second-price auction. However, the situation in auctions is not as simple as in buyer clubs, because while bidders can gain by sharing information, the competitive nature of auctions means that bidders' interests are not aligned. Thus there is a complex strategic relationship among bidders in a bidding club, and bidding club rules must be designed accordingly. This work comes under the umbrella of *collusion* in auctions, a negative term reflecting a seller-oriented perspective. We adopt a more neutral stance towards such bidder activities and thus use the term *bidding clubs* rather than the terms *bidding rings* and *cartels* that have been used in the past. Of course, the technical development is not impacted by such subtle differences in moral attitude.

There are four classical auction types: first-price, second-price, Dutch and English. Since first-price and Dutch are strategically equivalent, the latter may be omitted for our purposes without loss of generality. Agents cooperation in second-price auctions (and in English auctions, which are equivalent under standard economic assumptions) is well studied, most notably in (Graham & Marshall 1987). Cooperation among agents in the framework of first-price auctions has received much less attention. This is possibly explained by the fact that since second-price auctions give rise to dominant strategies, it is possible to study collusion in many settings related to these auctions without performing strategic equilibrium analysis. The key exception to the scarcity of formal work on first-price auctions is a very influential paper by McAfee and McMillan (McAfee & McMillan 1992). Several sections of their paper are directly applicable to our work, including the discussion of enforcement and the argument for independent private values as a model of agents' valuations as well as parts of their model. However, the setting introduced in their work assumes that a fixed number of agents participate in the auction and that all agents are part of a single cartel that coordinates its behavior in the auction. The authors show optimal collusion protocols for "weak" cartels (in which transfers between agents are not permitted: all bidders bid the reserve price, using the auctioneer's random tie-breaking rule to select a winner) and for "strong" cartels (the cartel holds a pre-auction, the winner of which bids the reserve price in the main auction while all other bidders sit out; the winner distributes some of his gains to other cartel members through side payments). A small part of the paper deals with the case where in addition to the single cartel there are also additional agents. However, results are shown only for two cases: (1) when non-cartel members bid without taking the existence of a cartel into account and (2) when each agent i has valuation $v_i \in \{0, 1\}$.

An earlier paper (Leyton-Brown, Shoham, & Tennenholtz 2000) anticipated some of our results, considering bidding clubs for five different economic mechanisms. This earlier paper considered bidding clubs for first-price auctions, second-price auctions, parallel second-price auctions with substitutable goods, second-price auctions with complementary goods, and general mechanisms where agents' valuations are drawn from a finite set. However, this earlier paper was not developed in the context of a general game-theoretic model. For example, it relied upon the assumption that only a single bidding club exists and that bidders who were not invited to join the club behave as though they are not aware of the possibility that a bidding club might exist. This makes the analysis carried out in that earlier work restrictive and limited from a game-theoretic perspective. Our current paper is a substantial extension and generalization of that earlier work, concentrating on the case of first-price auctions.

Distinguishing Features of our Model

Our goal in this work is to study cooperation between self-interested bidders in a rich model that captures many of the characteristics of auctions on the internet. This leads to many differences between our model and models proposed in (McAfee & McMillan 1992) and (Graham & Marshall 1987). We argue that a model of internet auctions with bidding clubs should include the following features:

1. The number of bidders is stochastic.
2. A bidding club may contain any subset of the bidders in the auction (e.g., it is not restricted to contain all bidders)
3. No limit to the number of bidding clubs in any auction.
4. All agents behave strategically, taking into account the possibility that other agents may collude.

The first feature above is crucial. In many real-world internet auctions, bidders are not aware of the number of other agents in the economic environment. A bidding club that drops one or more bidders is thus undetectable to other bidders in such an auction. An economic environment with a fixed number of bidders would not model this uncertainty, as the number of involved bidders would be common knowledge among all bidders regardless of the number of bids received in the auction. For this reason, we consider economic environments where the number of bidders is chosen at random, drawing on a model of auctions with stochastic numbers of participants from a second paper by McAfee and McMillan which is unrelated to collusion (McAfee & McMillan 1987); we also refer to equilibrium analysis of this model from (Harstad, Kagel, & Levin 1990).

To make bidding clubs a reasonable model of collusion in internet auctions, we restrict our protocols as follows:

1. Bidders must be free to decline an invitation to join a bidding club without (direct) penalty. In this way we include the choice to collude as one of agents' strategic decisions, rather than assuming that agents will collude.
2. Bidding club coordinators must make money on expectation, and must never lose money. This ensures that third-parties have incentive to run bidding club coordinators.
3. The bidding club protocol must give rise to an equilibrium where all invited agents choose to participate, even when the bidding club operates in a single auction as opposed to a sequence of auctions. Thus agents can not be induced to collude in a given auction by the threat of being denied future opportunities to collude. This restriction is necessary for modelling internet auction settings in which the pool of participants varies substantially from one auction to the next, and where many bidders are interested in participating only in single auctions.

Overview

The first part of our paper does not directly concern bidding clubs. First, we consider different variations on the first-price auction mechanism. We begin with classical first-price auctions, in which the number of bidders is common knowledge, and then consider first-price auctions where the number of bidders is drawn from a known distribution. Combining results from both auction types, we present first-price auctions with participation revelation: auctions in which the number of bidders is stochastic, but the auctioneer announces the number of participants before taking bids. This is the auction mechanism upon which we will base our bidding club protocol for first-price auctions.

The second part of our paper is concerned explicitly with bidding clubs, using material from the first part to present a general model of bidding clubs and then a bidding club protocol for first-price auctions. First, we describe an economic environment with the following novel features:

- A finite set of bidding clubs is selected from an infinite set of potential bidding clubs.
- A finite set of agents is selected to participate in the auction, from an infinite set of potential agents. Some agents

are associated with bidding clubs, and the whole procedure is carried out in such a way that no agent can gain information about the total number of agents in the economic environment from the fact of his own selection.

- The space of agent types is expanded to include both an agent's valuation, and the number of agents present in that agent's bidding club (equal to one if the agent does not belong to a bidding club).

We introduce notation to describe each agent's beliefs about the number of agents in the economic environment, conditioned on that agent's private information. Next, we examine bidding club protocols for first-price auctions. We begin with two assumptions about the distribution of agent valuations: the first related to continuity of the distribution, and the second to monotonicity of equilibrium bids. We then give a bidding club protocol for first-price auctions with participation revelation, and present our main technical results:

- It is an equilibrium for agents to accept invitations to join bidding clubs when invited and disclose their true valuations to their coordinator, and for singleton agents to bid as they would in an auction with a stochastic number of participants in an economic environment without bidding clubs, in which the distribution over the number of participants is the same as in the bidding clubs setting.
- In equilibrium each agent is better off as a result of his own club (that is, his expected payoff is higher than would have been the case if his club never existed, but other clubs—if any—still did exist).
- In equilibrium each club increases all non-members' expected payoffs, as compared to equilibrium in the case where all club members participated in the auction as singleton bidders, but all other clubs—if any—still existed.
- In equilibrium each agent's expected payoff is identical to the case in which no clubs exist. If clubs are willing to make money (or break even) only on expectation, they could distribute some of their *ex ante* expected profits among the club members, ensuring that all bidders gain.

First-Price Auctions

Let $\mathcal{T}$ be the set of possible agent types. The type $\tau_i \in \mathcal{T}$ of agent i is the tuple $(v_i, s_i) \in V \times \mathcal{S}$. v_i denotes an agent's valuation: his maximal willingness to pay for the good offered by the center. We assume that v_i represents a purely private valuation for the good, and that v_i is selected independently from the other v_j's of other agents from a known distribution, F, having density function f. By s_i we denote agent i's signal: his private information about the number of agents in the auction. In this section we will consider the simple case where $\mathcal{S} = \{\varnothing\}$: it is common knowledge that all agents receive the null signal, and hence gain no additional information about the number of agents. Recall that the economic environment itself is always common knowledge, and so agents always have some information about the number of agents even when they receive the null signal.

Classical first-price auctions

In a classical first-price auction, the economic environment consists of n agents. Each participant submits a bid in a sealed envelope; the agent with the highest bid wins the good and pays the amount of his bid, and all other participants pay nothing. In the case of a tie, the winner of the auction is selected uniformly at random from the bidders who tied for the highest bid. The equilibrium analysis of first-price auctions is quite standard:

Proposition 1 *If valuations are selected independently and uniformly from $[0,1]$ then it is a symmetric equilibrium for each agent i to follow the strategy $b(v_i) = \frac{n-1}{n} v_i$.*

Using classical equilibrium analysis it is possible to show how classical first-price auctions can be generalized to an arbitrary continuous distribution F.

Proposition 2 *If valuations are selected from a continuous distribution F then it is a symmetric equilibrium for each agent i to follow the strategy $b(v_i) = v_i - F(v_i)^{-(n-1)} \int_0^{v_i} F(u)^{n-1} du$.*

In both cases, observe that although n is a free variable, n is not a parameter of the strategy; the same is true of the distribution F. Agents deduce this information from their full knowledge of the economic environment. It is useful, however, to have notation specifying the amount of the equilibrium bid as a function of both v and n. We write $b^e(v_i, n) = v_i - F(v_i)^{-(n-1)} \int_0^{v_i} F(u)^{n-1} du$.

First-price auctions with a stochastic number of bidders

It is also possible to model an economic environment in which the number of agents is not a constant, but is instead chosen stochastically from a known probability distribution P; by p_j we denote the probability that there are exactly j agents. An equilibrium for this setting was demonstrated in (Harstad, Kagel, & Levin 1990):

Proposition 3 *If valuations are selected from a continuous distribution F and the number of bidders is selected from the distribution P then it is a symmetric equilibrium for each agent i to follow the strategy $b(v_i) = \sum_{j=2}^{\infty} p_j b^e(v_i, j)$.*

Observe that $b^e(v_i, j)$ is the amount of the equilibrium bid for a bidder with valuation v_i in a setting with j bidders as described above. P is deduced from the economic environment. We overload our previous notation for the equilibrium bid, this time as a function of the agent's valuation and the probability distribution P. Thus we write $b^e(v_i, P) = \sum_{j=2}^{\infty} p_j b^e(v_i, j)$.

First-price auctions with participation revelation

In an economic environment with a stochastic number of bidders, the auctioneer may choose to reveal the number of participants to all bidders, for example by introducing a two-phase mechanism with revelation of the number of participants between the stages. Specifically, we define a first-price auction with participation revelation as follows:

1. Agents indicate their intention to bid in the auction.

2. The auctioneer announces n, the number of agents who registered in the first phase.
3. Agents submit bids to the auctioneer. The auctioneer only accepts bids from agents who registered in the first phase.
4. The agent who submitted the highest bid is awarded the good for the amount of his bid; all other agents pay 0.

Unsurprisingly, it is an equilibrium for bidders to bid as in a classical first-price auction:

Proposition 4 *There exists an equilibrium of the first-price auction with participation revelation where every agent i indicates the intention to participate and bids $b^e(v_i, n)$.*

In our discussion of bidding clubs we will be concerned with first-price auctions with information revelation, but we will show an equilibrium in which the number of agents registering in the first phase is smaller than the total number of agents participating in the auction, because some bidders with low valuations drop out as part of a collusive agreement. The auctioneer's declaration acts as a signal about the total number of bidders, but individual agents will still be uncertain about the total number of opponents they face.

Auction Model for Bidding Clubs

We now give a formal description of the economic environment in which bidding clubs operate, define the bidding club mechanism for bidding club members, and define symmetric Bayes-Nash equilibria. Because our aim is not to model a situation where agents' *decision* to collude is exogenous—as this would gloss over the question of whether the collusion is stable—we include the collusive protocol as part of the model and show that it is individually rational *ex post* (i.e., after agents have observed their valuations) for agents to choose to collude. However, we do consider exogenous the selection of the sets of agents who are *invited* to collude.

The Economic Environment

We construct an economic environment E consisting of a set of agents who have non-negative valuations for a good at auction, the distinguished agent 0 and a set of bidding club coordinators who may invite agents to participate in a bidding club. Intuitively, the number of agents in each bidding club is independent of the number of agents in every other bidding club, because we construct an environment where an agent's belief update after observing the number of agents in his bidding club does not result in any change in the distribution over the number of *other* agents in the auction.

Coordinators Coordinators are not free to choose their own strategies; rather, they act as part of the mechanism for a subset of the agents in the economic environment. We denote the probability that an auction will involve n_c potential coordinators as $\gamma_C(n_c)$. γ_C may be any distribution satisfying $\gamma_C(0) = \gamma_C(1) = 0$: at least two potential coordinators will be associated with each auction. We assume that the name of each potential coordinator is selected from the uniform distribution on $[0, 1]$.

Agents The probability that n agents will be associated with a potential coordinator is denoted $\gamma_A(n)$. γ_A may be any distribution satisfying $\gamma_A(0) = 0$ and $\gamma_A(1) < 1$. If only one (actual) agent is associated with a potential coordinator, the potential coordinator will not be actualized and hence the agent will not belong to a bidding club. In this way we model agents who participate directly in the auction without being associated with a coordinator. If more than one agent is associated with a potential coordinator, the coordinator *is* actualized and all the agents receive an invitation to participate in the bidding club. As before, we assume that the name of each agent associated with a potential coordinator is selected from the uniform distribution on $[0, 1]$. The key consequence of our technical construction of coordinator and agent names is that an agent's knowledge of the coordinator with whom he is associated does not give him additional information about what other agents have been selected. Any other technique for providing this property may also be used; e.g., other constructions draw coordinator and agent names from finite sets.

Signals Each agent receives a signal informing him of the number of agents in his bidding club; we denote this signal s_i. Of course, if this number is 1 then there is no coordinator for the agent to deal with, and he will simply participate in the main auction. Note also that agents are neither aware of the number of potential coordinators for their auction nor the number of actualized potential coordinators, though they are aware of both distributions.

Beliefs Each agent has beliefs about the number of agents in the economic environment. Not all agents have the same beliefs—agents who have been signaled that they belong to a bidding club will expect a larger number of agents than singleton agents. We denote by $p_m^{n,k}$ the (true) probability that there will be a total of m agents in the auction, given that n potential coordinators were selected and that there are k agents associated with one of the potential coordinators; we denote the whole distribution $P^{n,k}$. Because the numbers of agents in each bidding club are independent, every agent in the whole auction has the same beliefs about the number of other agents in the economic environment discounting those agents in his own bidding club. Hence agent i's beliefs are described by the distribution P^{n,s_i}.

The Auction Mechanism

Bidding clubs, in combination with a main auction (along the lines of (Monderer & Tennenholtz 2000)), induce this auction mechanism for their members:

1. A set of bidders is invited to join the bidding club.
2. Each agent i sends a message μ_i to the bidding club coordinator. This may be the null message, which indicates that the agent will not participate in the coordination and will instead participate freely in the main auction. Otherwise, agent i agrees to be bound by the bidding club rules, and μ_i is agent i's declared valuation for the good. Of course, i can lie about his valuation.
3. Based on commonly-known rules, and on the information all the members supply, the coordinator selects a subset

of the agents to bid in the main auction. The coordinator may bid on behalf of these agents (e.g., using their ID's on the auction web site) or it may instruct agents on how to bid. In either case we assume that the coordinator can force agents to bid as desired, for example by imposing a charge on agents who do not behave as directed.

4. If a bidder represented by the coordinator wins the main auction, he is made to pay the amount required by the auction mechanism to the auctioneer, and may be required to make an additional payment to the coordinator.

Any number of coordinators may participate in an auction. However, we assume that all coordinators follow the same protocol, which is common knowledge. Singleton bidders submit messages directly to the auctioneer in the main auction, pay the amount of their message if they win.

The Bayesian Game

Given our economic environment and auction mechanism, a well-defined Bayesian game will be specified by every tuple of primary auction type, bidding club rules and distributions over agent types, numbers of agents and numbers of bidding clubs. A strategy $b_i : \mathcal{T} \rightarrow \mathcal{M}$ for agent i is a mapping from his type τ_i to a message μ_i. This may be the null message, which indicates non-participation in the auction. Σ denotes the set of possible strategies, i.e., the set of functions from types to messages in $\mathcal{M}$. Each agent's type is that agent's private information, but the whole setting is common knowledge. For notational simplicity we only define symmetric equilibria, where all agents bid the same function of their type, as this is sufficient for our purposes in this paper. By $L_i(\tau_i, b_i, b^{j-1})$ we denote agent i's *ex post* expected utility given that his type is τ_i, he follows the strategy b_i, all other agents use the strategy b, and there are a total of j agents. Let A be the set of participants in the auction, where $|A| = n$. The strategy profile $b^n \in \Sigma^n$ is a symmetric Bayes-Nash equilibrium if and only if $\forall i \in A, \forall \tau_i \in \mathcal{T}, b \in \text{argmax}_{b_i \in \Sigma} \sum_{j=2}^{\infty} p_j^{\tau_i} L_i(\tau_i, b_i, b^{j-1})$.

Bidding Clubs for First-Price Auctions

Assumptions

Our results hold for a broad class of distributions of agent valuations—all those for which the following two assumptions are true. First, we assume that F is continuous and atomless. Before giving our second assumption, we define $P_{x \geq i} = \sum_{x=i}^{\infty} p_x$, and define the relation "$<$", corresponding to a notion of stochastic dominance: $P < P'$ iff $\exists l (\forall i < l, P_{x\geq i} = P'_{x\geq i}$ and $\forall i \geq l, P_{x\geq i} < P'_{x\geq i})$. Our second assumption is that $(P < P')$ implies that $\forall v$, $b^e(v, P) < b^e(v, P')$. Intuitively, we assume that every agent's symmetric equilibrium bid in a setting with a stochastic number of participants drawn from P' is strictly greater than that agent's symmetric equilibrium bid in a setting with a stochastic number of participants drawn from P, whenever P' stochastically dominates P.

First-Price Auction Bidding Club Protocol

What follows is the first-price auction bidding club protocol for a coordinator who has invited k agents:

1. Each agent i sends a message μ_i to the coordinator.
2. If at least one agent declines participation then the coordinator registers in the main auction for every agent who accepted the invitation to the bidding club. For each bidder i, the coordinator submits a bid of $b^e(\mu_i, P^{n,k})$, where n is the number of bidders announced by the auctioneer.
3. If all agents accepted the invitation, the coordinator drops all bidders except the bidder with the highest reported valuation, who we will denote as bidder h. For h the coordinator will place a bid of $b^e(\mu_h, P^{n,1})$ in the main auction.
4. If bidder h wins in the main auction, he is made to pay $b^e(\mu_h, P^{n,1})$ to the center and $b^e(\mu_h, P^{n,k}) - b^e(\mu_h, P^{n,1})$ to the coordinator.

Theorem 1 *It is an equilibrium for all bidding club members to choose to participate and to truthfully declare their valuations to their respective bidding club coordinators, and for all non-bidding club members to participate in the main auction with a bid of* $b^e(v, P^{n,1})$.

Remark. Despite the fact that this is the central theorem of this paper, it is difficult to summarize here as it makes use of two general lemmas and consists of a lengthy case analysis. In particular, in one of these lemmas we identify a particular class of auction mechanisms that are asymmetric in the sense that every agent is subject to the same allocation rule but to a potentially different payment rule, and furthermore that agents may receive different signals. We show that truthful bidding is an equilibrium for this class of mechanisms. Under the equilibrium demonstrated in the theorem the coordinator makes money on expectation and never loses money. The equilibrium also gives rise to an economically efficient allocation: i.e., the good is allocated to the agent with the globally highest valuation.

Do bidding clubs cause agents to gain?

We can show that bidders are better off being invited to a bidding club than being sent to the auction as singleton bidders. Intuitively, an agent gains by not having to consider the possibility that other bidders who would otherwise have belonged to his club might themselves be bidding clubs.

Theorem 2 *An agent i has higher expected utility in a bidding club of size k bidding as described in theorem 1 than he does if the bidding club does not exist and k additional agents (including i) participate directly in the main auction as singleton bidders, again bidding as in theorem 1.*

We can also show that singleton bidders and members of other bidding clubs benefit from the existence of each bidding club in the same sense. Intuitively, other bidders gain from not having to consider the possibility that additional bidders might represent bidding clubs. Paradoxically, other bidders' gain from the existence of a given bidding club is greater than the gain of that club's members.

Corollary 1 *In the equilibrium described in theorem 1, singleton bidders and members of other bidding clubs have higher expected utility when other agents participate in a given bidding club of size $k \geq 2$, as compared to a case where k additional agents participate directly in the main auction as singleton bidders.*

Finally, we can show that agents are indifferent between participating in the equilibrium from theorem 1 in a bidding club of size k (thus, where the number of agents is distributed according to $P^{n,k}$) and participating in an economic environment with a stochastic number of bidders distributed according to $P^{n,k}$, but with no coordinators.

Theorem 3 *For all $\tau_i \in \mathcal{T}$, for all $k \geq 1$, for all $n \geq 2$, agent i obtains the same expected utility by:*

1. *participating in a bidding club of size k in the bidding club economic environment and following the equilibrium from theorem 1;*
2. *participating in a first-price auction with participation revelation in an economic environment with a stochastic number of bidders distributed according to $P^{n,k}$ where all bidders receive the null signal and no coordinators exist.*

This theorem shows that an agent would be as happy in a world without bidding clubs as he is in our economic environment. The difference between the two worlds is that in the latter bidding club coordinators make a positive profit on expectation, and indeed never lose money. That is, in the bidding club economic environment some expected profit is shifted from the auctioneer to the bidding club coordinator(s) without affecting the bidders' expected utility. We observe that it would be easy for coordinators to redistribute some of these gains to bidders along the lines of the second-price auction protocol proposed by (Graham & Marshall 1987): coordinators make a payment to every bidder who accepts the invitation to join, where the amount of this payment is less than or equal to the *ex ante* expected difference that the bidder makes to the coordinator's profit. With this modification coordinators would be budget balanced only on expectation (violating our earlier requirement that coordinators *never* lose money), but agents would strictly prefer the bidding club economic environment to the economic environment in which coordinators do not exist.

Disrupting Bidding Clubs

There are two things an auctioneer can do to disrupt bidding clubs in a first-price auction. First, she can permit "false-name bidding" (see, e.g., (Yokoo, Sakurai, & Matsubara 2000)). Our auction model has assumed that each agent may place only a single bid in the auction, and that the center has a way of uniquely identifying agents. (For example, the auctioneer might make it impossible for bidders to place bids claiming to originate from different agents by keying user accounts to credit card billing addresses in combination with a reputation ranking.) Second, she can refrain from publicly disclosing the winner of the auction. If the auctioneer does either of these things, a given bidder i has incentive to deviate from the equilibrium in theorem 1 by accepting the invitation to join the bidding club but placing a very low bid with the coordinator and at the same time directly submitting a competitive bid in the main auction. Agent i will gain when all other agents bid truthfully because the bidding club will drop all but one of its members (lowering the number of participants announced by the auctioneer) and will also require its high bidder to bid less than he would if he were not bound to the coordination protocol. If false-name bidding is impossible and the winner of the auction is publicly disclosed then the bidding club coordinator can detect an agent who has deviated in this way and impose a punitive fine, making the deviation unprofitable.

Conclusion

We have presented a formal model of bidding clubs which departs in many ways from models traditionally used in the study of collusion; most importantly, all agents behave strategically based on correct information about the economic environment, including the possibility that other agents will collude. Other features of our setting include a stochastic number of agents and a stochastic number of bidding clubs in each auction. Agents' strategy space is expanded so that the decision of whether or not to join a bidding club is part of an agent's choice of strategy. Bidding clubs never lose money, and gain on expectation. We have showed a bidding club protocol for first-price auctions that leads to a (globally) efficient allocation in equilibrium, and which does not make use of side-payments; we also showed that this protocol can benefit agents in three different senses.

References

Boutilier, C.; Shoham, Y.; and Wellman, M. 1997. Economic principles of multi-agent systems (editorial). *Artificial Intelligence* 94.

Graham, D., and Marshall, R. 1987. Collusive bidder behavior at single-object second-price and english auctions. *Journal of Political Economy* 95:579–599.

Greenwald, A. 1999. Shopbots and Pricebots. In *IJCAI-99*, 506–511.

Harstad, R.; Kagel, J.; and Levin, D. 1990. Equilibrium bid functions for auctions with an uncertain number of bidders. *Economic Letters* 33(1):35–40.

Leyton-Brown, K.; Shoham, Y.; and Tennenholtz, M. 2000. Bidding clubs: institutionalized collusion in auctions. In *ACM Conference on Electronic Commerce*.

McAfee, R., and McMillan, J. 1987. Auctions with a stochastic number of bidders. *Journal of Economic Theory* 43:1–19.

McAfee, R., and McMillan, J. 1992. Bidding rings. *The American Economic Theory* 82:579–599.

Monderer, D., and Tennenholtz, M. 2000. Optimal Auctions Revisited. *Artificial Intelligence* 120(1):29–42.

Parkes, D., and Ungar, L. 2000. Preventing Strategic Manipulation in Iterative Auctions: Proxy-Agents and Price-Adjustment. In *AAAI-00*, 82–89.

Sandholm, T.; Suri, S.; Gilpin, A.; and Levine, D. 2001. Cabob: A fast optimal algorithm for combinatorial auctions. In *IJCAI-01*, 1102–1108.

Stone, P., and Greenwald, A. 2000. The first international trading agent competition: autonomous bidding agents. *Journal of Electronic Commerce Research* 1–36.

Tennenholtz, M. 1999. Electronic commerce: From game-theoretic and economic models to working protocols. In *IJCAI-99*.

Yokoo, M.; Sakurai, Y.; and Matsubara, S. 2000. Robsut combinatorial auction protocol against false-name bids. In *Proceedings of the 17th national conference on Artificial Intelligence*, 110–115.