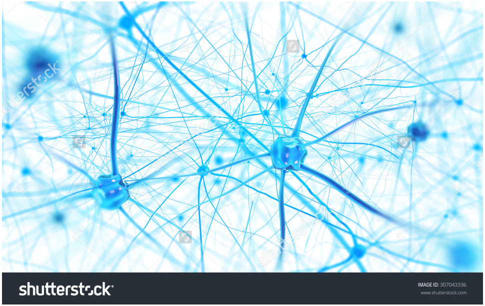
# Feedforward Neural Nets and Backpropagation

Julie Nutini

*University of British Columbia*




MLRG

September $28^{\text{th}}$, 2016

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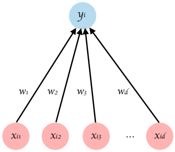
- Supervised Learning:
  - Learn features $z_i$ that are good for supervised learning.

Linear Model



Change of Basis

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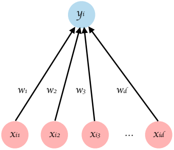


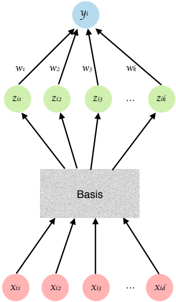

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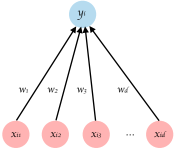


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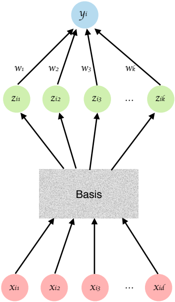


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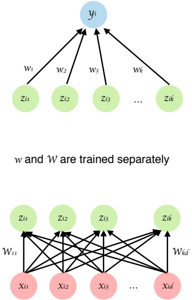

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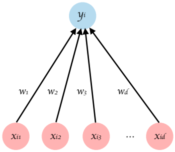


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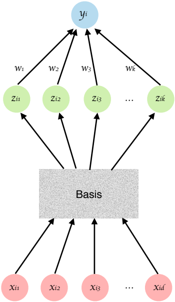


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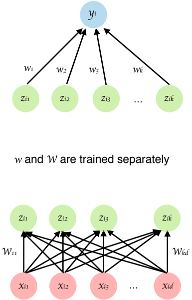


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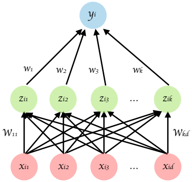

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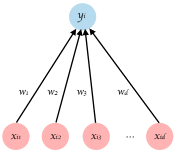


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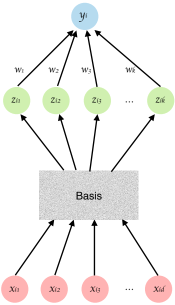


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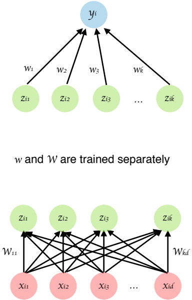


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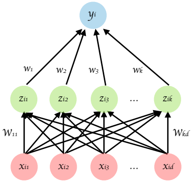
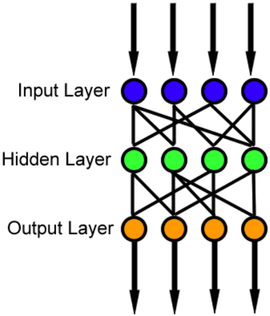



→ These are all examples of Feedforward Neural Networks.

# Feed-forward Neural Network

- Information always moves one direction.
  - No loops.
  - Never goes backwards.
  - Forms a directed acyclic graph.



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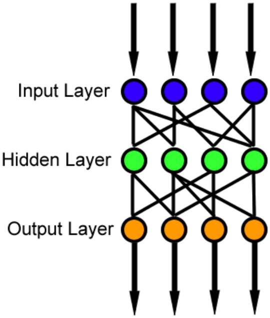


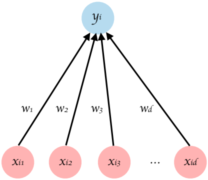

- Each node receives input only from immediately preceding layer.
- Simplest type of artificial neural network.

# Neural Networks - Historical Background

- **1943**: McCulloch and Pitts proposed first computational model of neuron
- **1949**: Hebb proposed the first learning rule
- **1958**: Rosenblatt's work on perceptrons
- **1969**: Minsky and Papert's paper exposed limits of theory
- **1970s**: Decade of dormancy for neural networks
- **1980-90s**: Neural network return (self-organization, back-propagation algorithms, etc.)

# Model of Single Neuron

- McCulloch and Pitts (1943): "integrate and fire" model (no hidden layers)

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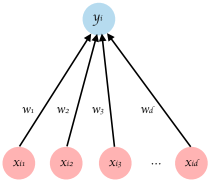


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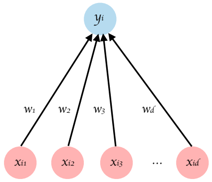


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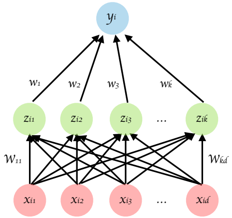
- Use $\hat{y}_i$ in some loss function:

$$\frac{1}{2}(y_i - \hat{y}_i)^2$$

- Consider more than one neuron:



# Incorporating Hidden Layers

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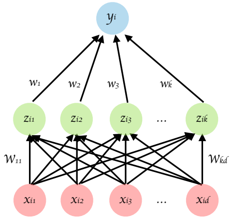


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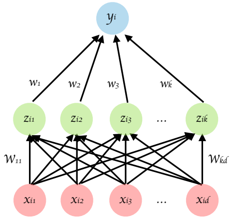


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- This function $h$ is often called the activation function.
  - Each unit/node applies an activation function.

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$$\underset{w \in \mathbb{R}^k, W \in \mathbb{R}^{k \times d}}{\operatorname{argmin}} \quad \underbrace{\frac{1}{2}\sum_{i=1}^{n}(y_i - w^T z_i)^2}_{\text{linear regression with } z_i \text{ as features}} \quad = \frac{1}{2}\sum_{i=1}^{n}(y_i - w^T h(Wx_i))^2$$

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- But this is just a linear model:

$$w^T(Wx_i) = (W^T w)^T x_i = \tilde{w}^T x_i$$

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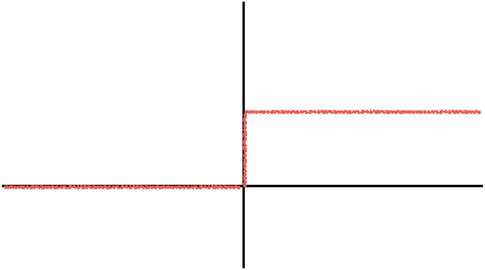

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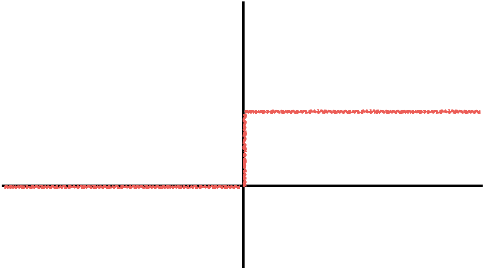

- This yields a binary $z_i = h(Wx_i)$.
- $Wx_i$ has a concept encoded by each of its $2^k$ possible signs.

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- Perceptrons only capable of learning linearly separable patterns.

# Learning Algorithm for Perceptrons

- The perceptron learning rule is given as follows:
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- An upper bound exists on number of times weights adjusted in training.
- Can also use gradient descent if function is differentiable.

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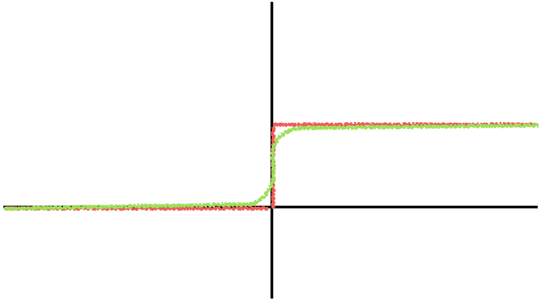

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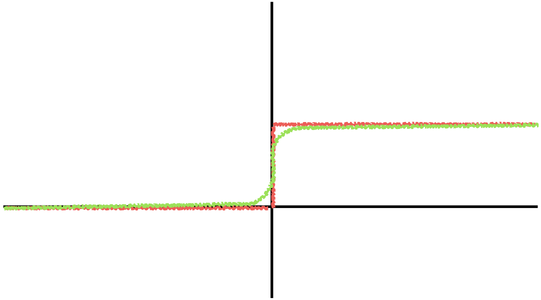

- Applying the sigmoid function element-wise,

$$z_{ic} = \frac{1}{1 + e^{-W_c^T x_i}}.$$

- This is called a multi-layer perceptron or neural network.

- A “typical” neuron.
- Neuron has many dendrites.
  - Each dendrite “takes” input.
- Neuron has a single axon.
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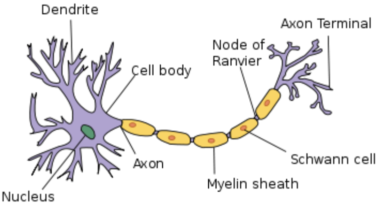

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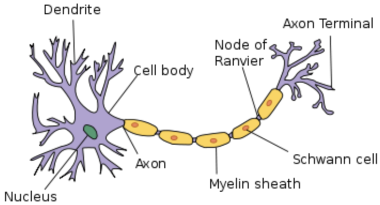
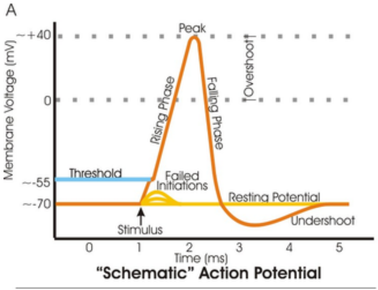



- With the “right” input to dendrites:
  - Action potential along axon.



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- First neuron:
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  - Nucleus: computes $W_c^T x_i$
  - Axon: sends binary signal $\frac{1}{1+e^{-W_c^T x_i}}$
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- Second neuron:
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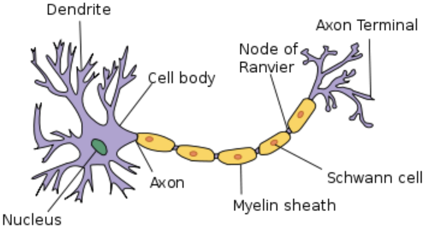

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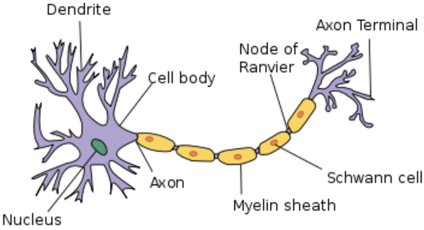


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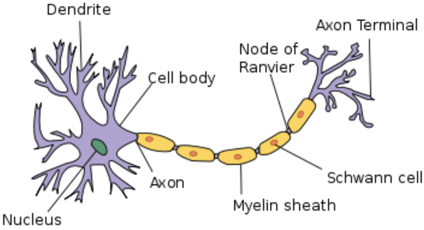


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- Human brain has approximately $10^{11}$ neurons.

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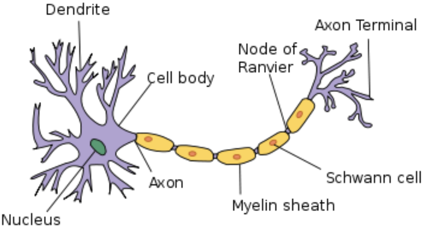


- Describes a neural network with one hidden layer (2 neurons).
- Human brain has approximately $10^{11}$ neurons.
- Each neuron connected to approximately $10^4$ neurons.

# Why Neural Network?

- First neuron:
  - Each dendrite: $x_{i1}, x_{i1}, \ldots, x_{id}$
  - Nucleus: computes $W_c^T x_i$
  - Axon: sends binary signal $\frac{1}{1+e^{-W_c^T x_i}}$
- Axon terminal: neurotransmitter, synapse
- Second neuron:
  - Each dendrite receives: $z_{i1}, z_{i2}, \ldots, z_{ik}$
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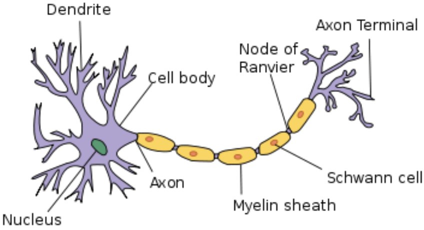


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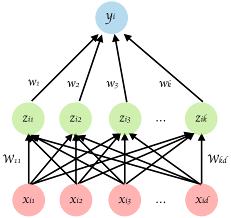

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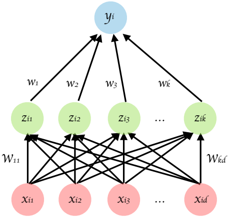


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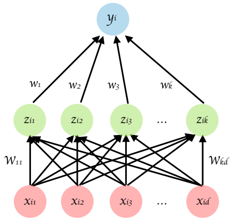


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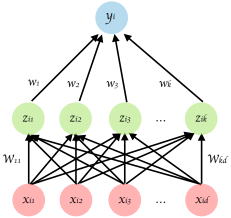


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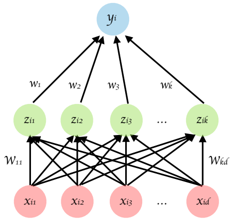


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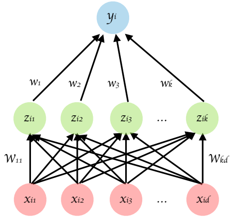


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  - "Algorithmic learnability" of those parameters?

- With squared loss, our objective function is:

$$\underset{w \in \mathbb{R}^k, W \in \mathbb{R}^{k \times d}}{\text{argmin}} \frac{1}{2} \sum_{i=1}^{n} (y_i - w^T h(W x_i))^2$$

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- To adjust weights, use non-linear optimization method gradient descent.
  $\rightarrow$ Backpropagation can only be applied to differentiable activation functions.

repeat

**until** some stopping criterion is satisfied

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**for each** weight $w_{i,j}$ in the network **do**

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