Bayesian Action-Graph Games: A New Representation for Games of Incomplete Information

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Introduction

Game Theory

Systems with multiple, self-interested agents

•Complete information: game is common knowledge among the players
  •Much research on computation of solution concepts e.g. Nash equilibria

•Games of incomplete information (Bayesian games):
  •Proposed by Harsanyi (1967)
  •Players are uncertain about game being played
  •Each player receive private information (type)
  •Many applications in economics: e.g. auctions

Bayesian Game

•Set of players: N = {1, 2, ..., n}

•Each player i's action set A_i

•Set of type profiles $\Theta = \prod_i \Theta_i$

•Type distribution $P(\Theta)$

•Player i's utility function $u_i : A \times \Theta \to \mathbb{R}$

Bayes-Nash Equilibrium

Natural extension of Nash equilibrium to Bayesian games

•Mixed strategy σ_i

probability of playing action a_i given type θ_i is $\sigma_i(a_i \mid \theta_i)$

•Expected utility of i given θ_i is

$$u_i(\sigma|\theta_i) = \sum_{\theta_{-i}} P(\theta_{-i}|\theta_i) \sum_a u_i(a,\theta) \prod_j \sigma_j(a_j|\theta_j)$$

•Mixed strategy profile σ is Bayes-Nash equilibrium if for all i, for all θ_i, for all a_i,

$$u_i(\sigma|\theta_i) \geq u_i(\sigma^{\theta_i \to a_i}|\theta_i)$$

Obstacles to efficient computation

•Representation
  •The straightforward Bayesian Normal Form requires exponential space in number of players

•Lack of practical algorithms
  •Can be reduced to finding a Nash equilibrium in a complete-information game
  •But this transformation causes a further exponential blowup in size

Compact Representations

Most games of interest have highly-structured utility functions

•Compact representations for complete-information games
  •Graphical games (Kearns et al. 2001)
  •Action-graph games (Jiang et al. 2010)

•Dynamic games
  •Multi-agent influence diagrams (Koller & Milch 2001)
  •Temporal action-graph games (Jiang et al. 2009)

Our Contributions

Bayesian Action-Graph Games (BAGGs)
•Can represent arbitrary Bayesian games
•Compactly express games with *structure*
  •symmetry
  •action- and type- specific utility independence
  •probabilistic independence of type distribution

•Efficient computation of Bayes-Nash equilibria
  •adapt existing algorithms for Nash equilibria
  •exponential speedup
  •http://agg.cs.ubc.ca

Bayesian Action-Graph Games

Representation

•Represent type distribution P as a Bayesian network
  •Containing at least n random variables

  representing $\theta_1, ..., \theta_n$

•Represent utility functions on an *action graph*: directed graph on set of action nodes $\mathcal{A}$

  player i, given θ_i, chooses an action from type-action set $A_{i,\theta_i} \subset \mathcal{A}$

  for each action node α, action count: number of players that have chosen α

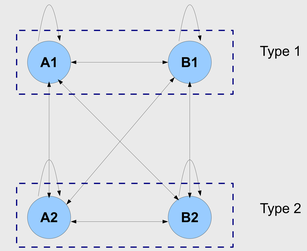
  utility depends only on action node chosen and the action counts of its neighbors

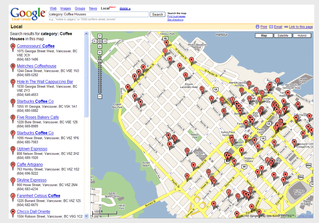
Theorem: if constant in-degrees, representation size is *polynomial* in n, $\mathcal{A}$, Θ_i

•Extension: function nodes
  •represents some function of its neighbors' action counts
  •e.g. counting function node: sum

Simple Example

•Symmetric Bayesian game, n players, 2 types, 2 actions per type





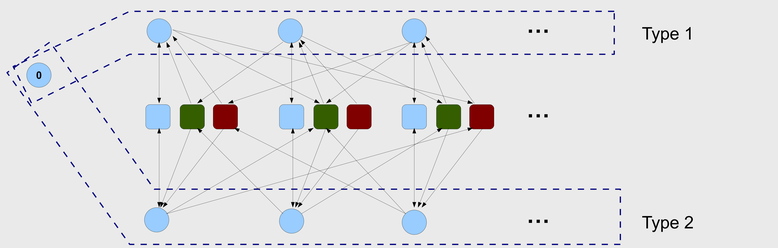
Example: Coffee Shop Game

•Each player chooses a location (in an r by k grid) to open a coffee shop, or decide not to enter.

•Utility of player i choosing a location depends on:
  •her type,
  •# of players choosing same block
  •# of players choosing surrounding blocks
  •# of players choosing any other block

Coffee Shop BAGG

•For each location
  •one counting function node for # of players choosing this block
  •one for # of players choosing surrounding blocks
  •one for # of players choosing other locations



Computing a Bayes-Nash Equilibrium

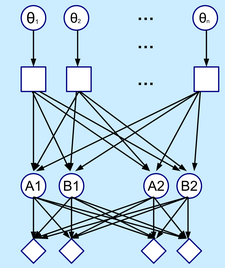
•Reduce to complete-information game (agent form)
  •one player for each type
  •set of actions for player (i, θ_i): type-action set A_{i,θ_i}
  •Nash equilibria correspond to Bayes-Nash of BAGG
  •do not need to represent explicitly: the BAGG serves as a compact representation

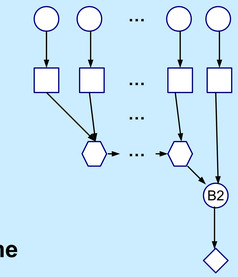
•Adapt state-of-the-art algorithms for Nash equilibrium
  •Global Newton Method (Govindan & Wilson 2001)
  •Simplicial Subdivision (van der Laan et al. 1987)

•A key subtask: computing expected utility (EU) of agent form given a mixed strategy profile
  •equiv. to computing EU of the BAGG
  •formulate as Bayesian network (BN) inference problem
  •further exploit causal independence by creating intermediate variables

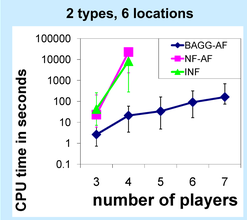
Theorem: if transformed BN has constant treewidth, EU can be computed in polynomial time

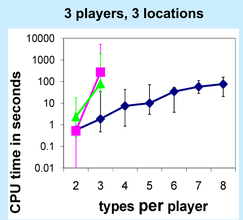
Theorem: for independent type distributions, EU can be computed in time polynomial in the size of the BAGG

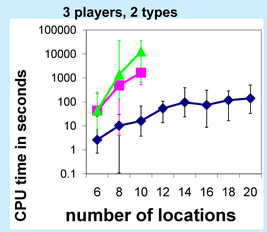




Experiments: Coffee Shop Game







Resources

•Paper online:
http://www.cs.ubc.ca/~jiang

•Software: http://agg.cs.ubc.ca

Bibliography

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