# HOMEWORK 3 SOLUTIONS, MATH 340, FALL 2015

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(1)

$$\begin{array}{rcrrr} x_3 & = & 1 & -x_1 & -x_2 \\ x_4 & = & -1 & -2x_1 & +x_2 \\ x_5 & = & -4 & +3x_1 & +2x_2 \\ \hline z & = & & 3x_1 & +x_2 \end{array}$$

This is not feasible; we use the two-phase method. Introduce $x_0$ and maximize $w = -x_0$:

$$\begin{array}{rcrrrr} x_3 & = & 1 & -x_1 & -x_2 & +x_0 \\ x_4 & = & -1 & -2x_1 & +x_2 & +x_0 \\ x_5 & = & -4 & +3x_1 & +2x_2 & +x_0 \\ \hline w & = & & & & -x_0 \end{array}$$

The most negative variable is $x_5$, so $x_0$ enters and $x_5$ leaves.

$$\begin{array}{rcrrrr} x_0 & = & 4 & -3x_1 & -2x_2 & +x_5 \\ x_3 & = & 5 & -4x_1 & -3x_2 & +x_5 \\ x_4 & = & 3 & -5x_1 & -x_2 & +x_5 \\ \hline w & = & -4 & +3x_1 & +2x_2 & -x_5 \end{array}$$

$x_1$ enters and $x_4$ leaves:

$$\begin{array}{rcrrrr} x_0 & = & 11/5 & -(7/5)x_2 & +(3/5)x_4 & +(2/5)x_5 \\ x_1 & = & 3/5 & -(1/5)x_2 & -(1/5)x_4 & +(1/5)x_5 \\ x_3 & = & 13/5 & -(11/5)x_2 & +(4/5)x_4 & +(1/5)x_5 \\ \hline w & = & -11/5 & +(7/5)x_2 & -(3/5)x_4 & -(2/5)x_5 \end{array}$$

$x_2$ enters and $x_3$ leaves

$$\begin{array}{rcrrrr} x_0 & = & 6/11 & +(7/11)x_3 & +(1/11)x_4 & +(3/11)x_5 \\ x_1 & = & 4/11 & +(1/11)x_3 & -(3/11)x_4 & +(2/11)x_5 \\ x_2 & = & 13/11 & -(5/11)x_3 & +(4/11)x_4 & +(1/11)x_5 \\ \hline w & = & -6/11 & -(7/11)x_3 & -(1/11)x_4 & -(3/11)x_5 \end{array}$$

The maximum value of $w = -6/11$. Hence the minimum value of $x_0 = 6/11$. Since this is non-zero, we conclude that the original LP problem is not feasible.

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(2) Again, we introduce slacks and then $x_0$ to the right-hand-sides.

$$\begin{array}{rcrrr} x_2 & = & 7 & -x_1 & +x_0 \\ x_3 & = & -1 & +x_1 & +x_0 \\ x_4 & = & -4 & +x_1 & +x_0 \\ \hline w & = & & & -x_0 \end{array}$$

(You could omit the $x_0$ from the $x_2$ line; the dictionaries would be slightly different.) We pivot $x_0$ in and $x_4$, which has the most negative constant, leaves.

$$\begin{array}{rcrrr} x_2 & = & 11 & -2x_1 & +x_4 \\ x_3 & = & 3 & & +x_4 \\ x_0 & = & 4 & -x_1 & +x_4 \\ \hline w & = & -4 & +x_1 & -x_4 \end{array}$$

So $x_1$ enters and $x_0$ leaves.

$$\begin{array}{rcrrr} x_2 & = & 3 & +2x_0 & -x_4 \\ x_3 & = & 3 & & +x_4 \\ x_1 & = & 4 & -x_0 & +x_4 \\ \hline w & = & & -x_0 & \end{array}$$

So $x_0$ is eliminated and we bring in the old objective:

$$\begin{array}{rcrr} x_2 & = & 3 & -x_4 \\ x_3 & = & 3 & +x_4 \\ x_1 & = & 4 & +x_4 \\ \hline z & = & 12 & +3x_4 \end{array}$$

So $x_4$ enters and $x_2$ leaves and we get:

$$\begin{array}{rcrr} x_4 & = & 3 & -x_2 \\ x_3 & = & 6 & -x_2 \\ x_1 & = & 7 & -x_2 \\ \hline z & = & 21 & -3x_2 \end{array}$$

This is a final dictionary, so we are done.

Had we pivoted $x_3$ instead of $x_0$ into the first dictionary, we would have obtained

$$\begin{array}{rcrrr} x_2 & = & 8 & -2x_1 & +x_3 \\ x_0 & = & 1 & -x_1 & +x_3 \\ x_4 & = & -3 & & +x_3 \\ \hline z & = & -1 & +x_1 & -x_3 \end{array}$$

In this case we get a negative constant in the $x_4$ row.

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