HOMEWORK 1 SOLUTIONS, MATH 340, FALL 2015

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Exercise 9.3

If A' is A with its first two rows exchanged, then A' represents the same game as A, except that Alice's first two pure strategies are exchanged.

Hence all the values of the games are the same, and all of Betty's optimum strategies are the same; all of Alices optimum strategies for games with A' are the same as those for A, except that the first two components (in her pure and mixed strategies, corresponding to her first two rows) are exchanged.

Exercise 9.4

(1) and (2): (There are many ways to doing this problem.) Multiplying by 2 is equivalent to dividing the currency by 2. Hence all the strategies are the same, and each game value for the new matrix game is twice that of the corresponding original value.

(3) There is no simple relationship between the matrix games A and $-A$; for an example, see items (1) and (2) of the next exercise.

Exercise 9.7(1)

Alice announces a pure strategy: since the minimum entry of row 1 is 1, and the minimum entry of row 2 is 2, Alice chooses row 2, which gives a value of 2 to this game.

Betty announces a pure strategy: since the maximum entry of column 1 is 2, and the maximum entry of row 2 is 5, Betty chooses column 1, which gives a value of 2 to this game.

Hence the duality gap is $2 - 2 = 0$. Since the pure strategy games have the same value, the values of the mixed games are also 2, and optimal mixed strategies are just the pure strategies above.

Exercise 9.7(2)

Alice announces a pure strategy: similarly to the above reasoning (see also the previous problem set), Alice chooses row 1, with a value of -3.

Betty announces a pure strategy: Betty chooses column 2, with a value of -3.

Hence the duality gap is $-3-(-3)=0$. Again, since the pure strategy games have the same value, the values of the mixed games are also -3, and optimal mixed strategies are just the pure strategies above.

Exercise 9.7(3)

Reasoning as above, the value of Alice announces a pure strategy is 2, and of Betty announces a pure strategy is 3. Hence the duality gap is $3-2=1$, and since this is positive we know that A is irreducible; and we see that Alice's optimal strategy $[p_1\ p_2]$ is given by

$$[p_1\ p_2]\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} = [v\ v], \quad p_1+p_2=1,$$

so

$$p_1+5p_2=v=3p_1+2p_2$$

so $3p_2=2p_1$, so we get $p_1=3/5$ and $p_2=2/5$ and $v=13/5$. Similarly Betty's optimal strategy is $[q_1\ q_2]$ where

$$\begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} v \\ v \end{bmatrix}, \quad q_1+q_2=1,$$

and we find that $q_1=1/5$, $q_2=4/5$, $v=13/5$.

Exercise 9.7(4)

Alice announces a pure strategy has value 0, since she chooses the first row, and Betty announces a pure strategy has value 0, since she chooses the first colomn. Then the duality gap is zero, and the above pure strategies are also optimal mixed strategies.

Exercise 9.8

In class we have seen that the value of this game is $1/3$. Therefore, to get a fair game, i.e., a game of value 0, we have Alice pay a fee of $1/3$ to Betty.

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