

Homework #6

1. Consider an LP maximize $\vec{c} \cdot \vec{x}$ subject to $A\vec{x} \leq \vec{b}$ and $\vec{x} \geq 0$, where
 - (a) n , the number of decision variables, is much larger than m , the number of slack variables, say $n = m^4$ (or $n = m^5$, $n = m^6$, etc.), and
 - (b) the matrix A is *sparse* meaning that only 5% of the entries of A are non-zero (you may assume for simplicity that 95% of the entries in each row and in each column are zero).

Argue that even if you naively invert the $m \times m$ matrix A_B each time (taking time m^3), the revised simplex method will still be faster than the ordinary simplex method, at least once the dictionaries have mostly non-zero coefficients. Roughly quantify this in terms of FLOPs (floating-point operations).

2. Consider an LP as in the previous problem, but now you don't assume that A is sparse (but you do assume that $n = m^4$). However, assume that in some dictionary you can identify some $n/3$ variables which will serve as the only entering variables for the next 20 pivots. Can the revised simplex method save you some time for the next 20 pivots? Roughly quantify this in terms of FLOPs (floating-point operations). What if $n/3$ is replaced with $n/30$ or \sqrt{n} ?