Homework #5

Consider the matrix game

$$\begin{bmatrix} 11 & 9 \\ 8 & 12 \end{bmatrix}$$

and the resulting linear program for Alice's maximal strategy: maximize v
subject to $v, x_1, x_2 \geq 0$ and

$$v \leq 11x_1 + 8x_2, \quad v \leq 9x_1 + 12x_2, \quad x_1 + x_2 \leq 1$$

(here, for reasons of symmetry, we will leave x_2 as a decision variable—instead
of writing it as $1 - x_1$—and write $x_1 + x_2 \leq 1$ as a constraint).

1. In this linear program we wrote $x_1 + x_2 \leq 1$, when the true constraint
   is $x_1 + x_2 = 1$; is enough just to write $x_1 + x_2 \leq 1$ in view of the above
   matrix? Explain.

2. Use complementary slackness to evaluate:

   (a) $x_1 = 1/2$, $x_2 = 1/2$, $v = 19/2$;

   (b) $x_1 = 1/3$, $x_2 = 2/3$, $v = 9$;

   (c) $x_1 = 2/3$, $x_2 = 1/3$, $v = 10$;

   (d) $x_1 = 1$, $x_2 = 0$, $v = 9$;

   (e) $x_1 = x_2 = v = 0$.

3. Write down the dual of this LP. Can you interpret this dual in terms
   of an LP for Betty's optimal mixed strategy?