

Homework #4

1. (a) Apply the perturbation method (here we use $\epsilon_1 = \epsilon$, $\epsilon_2 = \epsilon^2$, $\epsilon_3 = \epsilon^3$, as in the textbook) to the LP:

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2, \quad \text{subject to} \\ & x_1 \leq 2 \\ & x_2 \leq 3 \\ & x_1 + x_2 \leq 5 \\ & x_1, x_2 \geq 0 \end{aligned}$$

taking x_2 to enter the basis on your first pivot; specifically add ϵ_1 to the first inequality (writing $x_1 \leq 2 + \epsilon_1$), ϵ_2 to the second, and ϵ_3 to the third, with “ $1 \gg \epsilon_1 \gg \epsilon_2 \gg \epsilon_3$ ”. Find the maximum and draw a picture of feasible region, and indicate what your simplex steps look like in the picture.

- (b) Do the same thing with ϵ_1 added to the third inequality and ϵ_3 added to the first (e.g., writing $x_1 \leq 2 + \epsilon_3$).
- (c) How does the simplex method differ from part (a) to part (b)? Does it make sense that two perturbations of the above LP can give different dictionaries, even when we ignore the ϵ 's in the dictionaries?
2. Say that you have n -data points $(t_1, y_1), \dots, (t_n, y_n)$, and you want to find the a, b that minimize

$$L^1\text{-Error}(a, b) = \sum_{i=1}^n |y_i - a - bt_i|.$$

This is often called L^1 -regression of the data to the curve $y = a + bx$. Describe this as a linear program. Assume that $a, b \geq 0$ in the optimum a, b ; show that the simplex method implies that one can find an a, b for which $y_i = a + bt_i$ for at least one value of i . (If you like, you can read about this L^1 -regression in the textbook by Vanderbei on the course webpage, Section 12.4.)