Math 340–101 Fall 2015

Homework #2

1. Consider the LP:

$$\max 6x_1 + 7x_2, \quad \text{s.t. } x_1 \le 5, \quad x_2 \le 8,$$

$$x_1 + x_2 \le 10, \quad x_1, x_2 \ge 0.$$

   (a) Solve this using the simplex method, starting with x_2 entering the basis after the first dictionary (i.e., x_1, x_2 will be non-basic in the first dictionary, and you should hold x_1 fixed at 0 and increase x_2).

   (b) Solve this using the simplex method, starting with x_1 entering the basis after the first dictionary.

2. Consider a matrix game, A. Let $\mathbf{x}$ be a stochastic vector—a vector of non-negative components whose sum is 1—such that

$$\mathbf{x}^{\mathrm{T}} A \ge [v \ v \ \dots \ v], \tag{1}$$

for some number v i.e., each entry of $\mathbf{x}^{\mathrm{T}} A$ is at least v. [For example, if

$$A = \begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix},$$

we know that the value of "Alice announces a mixed strategy" equals 1/3; by choosing $\mathbf{x}^{\mathrm{T}} = [1/2 \ 1/2]$ (for no particular reason) we have

$$\mathbf{x}^{\mathrm{T}} A = [1/4 \ 1/2] \ge [v \ v]$$

where $v = 0.1$ (or $v = 0.2$ or v can be anything $\le 1/4$.] Explain why:

   (a) the value of "Alice announces a mixed strategy" is at least v;

   (b) if $\mathbf{y}$ is another stochastic vector, then explain why

$$\mathbf{x}^{\mathrm{T}} A \mathbf{y} \ge v;$$

(c) similarly, if $\mathbf{y}$ is a stochastic vector such that

$$A\mathbf{y} \leq \begin{bmatrix} w \\ \vdots \\ w \end{bmatrix}, \tag{2}$$

explain why the value of "Betty announces a mixed strategy" is at most w, and why for any stochastic $\mathbf{x}$ we have

$$\mathbf{x}^{\mathrm{T}} A\mathbf{y} \leq w.$$

(d) Show that if $\mathbf{x}$ and $\mathbf{y}$ are stochastic vectors such that Equations 1 and 2 hold, then $v \leq w$.

(e) If it turns out that for a matrix A we have

$$[0.5\ 0.5]A = [1.2\ 1.4\ 1.1] \quad \text{and} \quad A \begin{bmatrix} 0.1 \\ 0.3 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 1.1 \\ 1.3 \end{bmatrix},$$

what can you say about the value of the mixed strategy games for A?

(f) Is it possible that for some matrix A we have

$$[0.6\ 0.4]A = [1.2\ 1.4\ 1.1] \quad \text{and} \quad A \begin{bmatrix} 0.1 \\ 0.3 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 0.9 \\ 0.8 \end{bmatrix}?$$

3. Use the simplex method to find the value of "Alice announces a mixed strategy" and Alice's optimal mixed strategy for the matrix game

$$A = \begin{bmatrix} 1 & 3 \\ 8 & 1 \end{bmatrix}$$

as discussed in class, i.e., maximizing v subject to $[x_1\ 1-x_1]A \geq [v\ v]$, $1 - x_1 \geq 0$ and $x_1, v \geq 0$.

Find the value of "Betty announces a mixed strategy" and Betty's optimal mixed strategy for the above matrix game by the methods used in Homework #1. Do you see Betty's optimal strategy somewhere in the final dictionary of the simplex method above?