Formula Sheet for Math 340, Section 101, Fall 2015

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This is a list of formulas that you will be given with your final exam. You are responsible to know what these formulas mean.

$$\text{DualityGap} = (\text{ValueBettyAnnouncesPure}) - (\text{ValueAliceAnnouncesPure})$$

If A is a 2×2 matrix, then either (1) the duality gap is zero, or (2) Alice and Betty have mixed strategies where the values are balanced, e.g.,

$$[x_1 \quad 1 - x_1]A = [v \ v]$$

for Alice.

LP standard form: maximize $\vec{c} \cdot \vec{x}$, subject to $A\vec{x} \le \vec{b}$, $\vec{x} \ge \vec{0}$.

Unbounded LP: A variable enters, but nothing leaves.

2-phase method: (1) introduce x_0 on right, (2) pivot x_0 into the basis for a feasible dictionary, and try to maximize $w = -x_0$, (3) if w reaches 0, pivot x_0 out of dictionary and eliminate all x_0; e.g.,

$$\begin{array}{rl} x_4 = & -7 + \cdots + x_0 \\ x_9 = & -8 + \cdots + x_0 \end{array} \qquad x_0 \text{ enters, } x_9 \text{ leaves}$$

The formulas for simplex method dictionaries (in standard form) is

$$\begin{aligned} \vec{x}_B &= A_B^{-1}\vec{b} - A_B^{-1}A_N\vec{x}_N \\ z &= \vec{c}_B^{\mathrm{T}}A_B^{-1}\vec{b} + (\vec{c}_N^{\mathrm{T}} - \vec{c}_B^{\mathrm{T}}A_B^{-1}A_N)\vec{x}_N \end{aligned}$$

In the computation above, we compute $\vec{c}_B^{\mathrm{T}}A_B^{-1}A_N$ by first computing $\vec{c}_B^{\mathrm{T}}A_B^{-1}$, and then multiplying the result (a row vector) times A_N; it would be more expensive to first compute $A_B^{-1}A_N$.

For the A_B^{-1} of the $i-1$-th and i-th dictionaries we have

$$A_{B_i}^{-1} = E_i^{-1}A_{B_{i-1}}^{-1}$$

where E_i is an eta matrix, equal to the identity except in one column. This formula can be applied recursively to get

$$A_{B_{i+k}}^{-1} = E_{i+k}^{-1}E_{i+k-1}^{-1} \cdots E_i^{-1}A_{B_{i-1}}^{-1}.$$

Let the b-th row in a matrix game be $\vec{f}(b)$. If $\vec{f}$ is a convex function (i.e., concave up), then Alice has an optimal strategy that is some combination of the smallest and largest values of b (i.e., the top and bottom rows). If $\vec{f}$ is concave down, then Alice has an optimal strategy this is some combination of two adjacent rows. (These combinations can be 100% of one row in certain cases.)