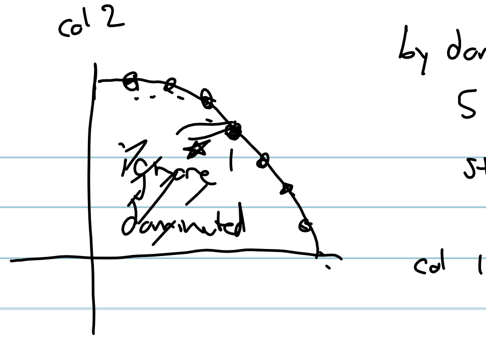
Math 340, Dec 2

Poker Game 2^{52} strategies

Alice's pure strategies payouts $\in \mathbb{R}^2$



by domination 53 strategies

If Alice plays b of 52 cards

Betty Folds:

$$\frac{b}{52}(+1) + \frac{52-b}{52}(-1)$$

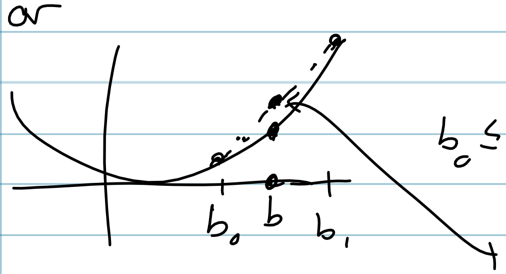
$$\text{Col 2} = 2 \cdot \frac{b}{52} - 1 \quad \text{linear}$$

Col 1 : long mess :

$$-b^2 \begin{pmatrix}\text{pos}\\ \text{const}\end{pmatrix} + \text{linear}(b)$$

$f(b)$ convex ($f'' \geq 0$)

or



$b_0 \le b \le b_1$

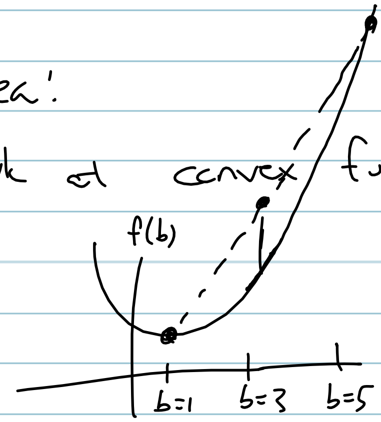
$$f(b) \le \begin{cases} \left(\frac{b_1 - b}{b_1 - b_0}\right) f(b_0) \\ + \left(\frac{b - b_0}{b_1 - b_0}\right) f(b_1) \end{cases}$$

$$\begin{cases} \text{i.e. } \omega_b f(b_0) \\ + (1 - \omega_b) f(b_1) \end{cases}$$

Idea:

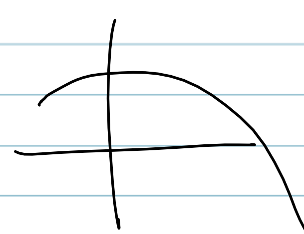
Look at convex functions

(simpler)



Concave functions

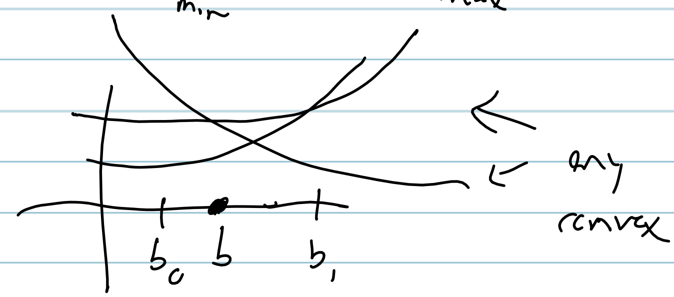
(concave down)



together, with weighted average, dominate

$$\begin{bmatrix} f_1(b) & f_2(b) \end{bmatrix}$$

with $b_{min} \le b \le b_{max}$



$$f(b) \le \frac{2}{3} f(b_0) + \frac{1}{3} f(b_1)$$

(say)

So if Betty

$$\begin{array}{c|cc|l} & \text{col } 1 & \text{col } 2 & \\ b_{min} \to 0 & & & \\ b \to 1 & f_1(b) & f_2(b) & \leftarrow b \\ \vdots & & & \\ b_{max} \to 52 & & & \end{array}$$

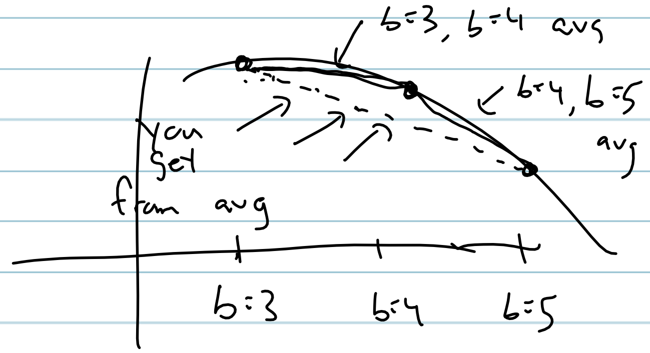
f_1, f_2 convex, then

$$\begin{bmatrix} f_1(b_{min}) & f_2(b_{min}) \end{bmatrix}$$

and

$$\begin{bmatrix} f_1(b_{max}) & f_2(b_{max}) \end{bmatrix}$$

Concave Down is same idea



combo $b=3$, $b=5$

$$\begin{array}{l} \text{best} \\ \text{concave} \\ \text{in } col_1(b) \end{array} \begin{bmatrix} \vdots & \cdot \\ \hline \\ \vdots & \end{bmatrix}, \quad 2 \cdot \frac{b}{52} - 1 = col_2(b) \text{ linear}$$

So better to play

$b = b_{min}$, $b = b_{max}$

rather than any other intermediate values of b

$$\begin{array}{l} b_{min} \\ \\ \text{dominated} \\ \\ b_{max} \end{array} \begin{bmatrix} - & - \\ & \\ & \\ & \\ - & - \end{bmatrix} \to \begin{array}{l} b_{min} \\ b_{max} \end{array} \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

Old final exams --

- final 2014 ←
- 2010 ←
- 2009 ←

1	1
4	4
9	9
16	16
25	25

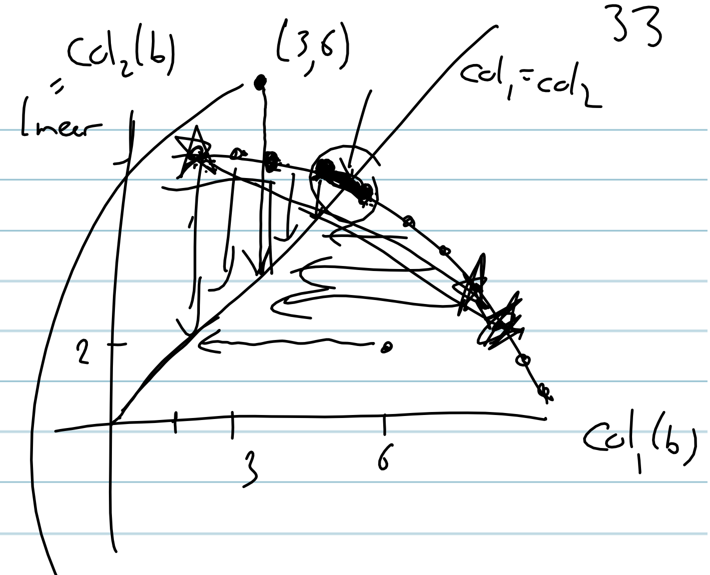
Final 2014:

5 (g)

Find the value of mixed strat for A

$$A = \begin{array}{c|c} & \text{Betty} \\ \hline \text{Alice} & \begin{bmatrix} -1 & -4 & -9 & -16 & -25 & -36 & -49 \\ -49 & -36 & -25 & -16 & -9 & -4 & -1 \end{bmatrix} \end{array}$$

Payout to Alice



Betty given (3,6) chooses 3

Claim: $\boxed{col_1(b) = col_2(b)}$

b integer :) , otherwise nearest points...

$$-A^T = \begin{array}{c|c|c} & \text{Alice} & \\ \hline \text{Betty} & \begin{pmatrix} 1 & 49 \\ 4 & 36 \\ 9 & 25 \\ 16 & 16 \\ 25 & 9 \\ 36 & 4 \\ 49 & 1 \end{pmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ \cdot \\ \cdot \\ \cdot \\ 7 \end{matrix} \end{array}$$

Payout to Betty

convex

$b \mapsto b^2$ $\qquad b \mapsto (8-b)^2$

Poker:

$$\begin{array}{l|c} \text{Alice Bluffs (Bets R/B)} \to 1 & \begin{bmatrix} 0 & 1/2 \\ 1 & 0 \end{bmatrix} \\ \text{Alice folds} \to 2 & \end{array}$$

- R b=1 ___
- P b=2 ___
- S b=3 ___

$$(9 \quad 25) \quad \text{vs} \quad \frac{2}{3}\,(1 \quad 49)$$

$$\frac{1}{3}\,[49 \quad 1]$$

$$\begin{bmatrix} \frac{51}{3} & \frac{99}{3} \end{bmatrix}$$

$$[17 \quad 33]$$

extremes best

e.g.

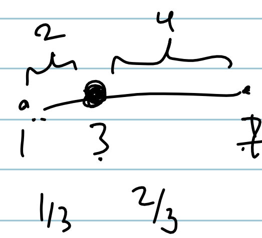
Betty ?

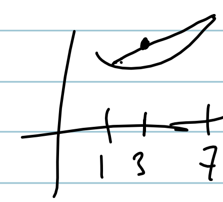
$(9 \quad 25)$

row 3

$$\le \frac{2}{3}\,(1 \quad 49) \text{ row 1}$$

$$+ \frac{1}{3}\,[49 \quad 1] \text{ row 7}$$





Dec 2014 Final:

1. Mixed strat, value of

$$\begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$$

2×2: gap = Betty Pure − Alice Pure

if gap = 0, values all games are Alice Pure = Betty Pure

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{Pure Game} = 3$$

$$\begin{array}{ll} & \\ \text{Betty} & \begin{array}{l} \text{row 1} \\ \text{row 7} \end{array} \begin{bmatrix} 1 & 49 \\ 49 & 1 \end{bmatrix} \text{ gap} \end{array}$$

$=$

$$[x, \; 1-x] \begin{bmatrix} 1 & 49 \\ 49 & 1 \end{bmatrix} = [v \quad v]$$

$$v = \frac{1+49}{2}$$

(Not valid in 3×3, 3×20
4×4, 5×16, ...)

$$\begin{bmatrix} x_1 & 1-x_1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} v & v \end{bmatrix}$$

$$1 \cdot x_1 + 5(1-x_1) = v = 4x_1 + 2(1-x_1)$$

Prob 2: Two-phase method

Prob 3: Complementary Slackness)

$$\begin{array}{l} \to \\ \to \end{array}\begin{bmatrix} 1 & 4 \\ 5 & 2 \end{bmatrix}$$

Betty Pure : $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ or $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$

Alice: 5 4

Betty Pure: value = 4

Alice Pure: value = 2

gap = 4 - 2 ≠ 0

Mixed games we balance ...