Math 340 Nov 30

Today finish up

$2^{52} \times 2$ Poker game

- dominance
- convexity / concavity

=

Linear programming

=

Old poker:

Alice & Betty put $1 in middle

A draws red/black 50% / 50% } Alice looks at card

A: folds or bets $1

B: folds or calls $1

If both in then { red wins for A, black " " B }

$$
\begin{array}{ll|cc|l}
 & & \text{Calls} & \text{Folds} & \leftarrow \text{Betty} \\
\hline
 & \text{Bet R, Bet B} & 0 & 1/2 & \\
\text{Alice} & \text{Bet R, F B} & 1 & 0 & \\
 & \text{F R, B B} & & & \\
 & \text{F B, F B} & & & \\
\end{array}
$$

$\Leftarrow$ expected or average payout to Alize

weren't so important:

Bet R . B $\geq$ Folds R, . B

Alice

Dominance

More sophisticated

Same except:

Alice draws

High Card, 2nd Card, . . . , $\underbrace{\text{52 Highest Card}}_{\text{Lowest}}$

Old Game:

Alice Red $\rightarrow$ Alize wins 100%

" Black $\rightarrow$ " loses 100%

New Game

Alize Highest Card (Ace Spades) wins $\frac{51}{51}$

- - - 2nd Highest " - - $50/51$

$49/51$

$\vdots$

Alice Lowest - - - wins $\frac{0}{51}$

Claim: If Alize bets on b cards:

① Alize bets on b highest cards

i.e. Alice bets on any ace, any 5, folds on rest

better: Alize bet on any ace or king, fold on rest

So if Alice bets on b cards, folds on $52-b$ cards she bets b highest

What does a row look like?

$$\begin{array}{c|cc} & \text{Betty Call} & \text{Betty Fold} \\ \text{Alice bets on } b \text{ of 52 cards} & \left[\begin{array}{c} \vdots \\ \\ \vdots \end{array}\right. & \left.\begin{array}{c} \vdots \\ \frac{b}{52}(+1) + \frac{52-b}{52}(-1) \\ \vdots \end{array}\right] \end{array}$$

$\underbrace{A\,S_p, A \ldots}_{\text{Alice } b \text{ cards}}$. 3, 3, 3, 3, 2, 2, 2, 2

$$\begin{array}{l|cc} & \text{Call} & \text{Fold} \\ \text{Bets on 52} & & \\ \text{-- 51} & & \\ \text{-- -- .} & & \\ \text{--} & & \\ \text{Bets 2 card} & & \\ \text{Bets 1 card} & & \\ \end{array}$$

52 ~~53~~ poss strategies

? → Bets 0 card dominated by

Choose all ~~pairs~~ of 2 elements (rows) in ~~53~~ 52 and find the value...

$$A\begin{cases}\text{Bets 47} \\ \text{Bets 3}\end{cases}\begin{array}{cc} \text{Call} & \text{Fold} \\ \left[\quad\right. & \left.\quad\right]\end{array}$$

doable with computer

Alice bets top $\underline{b}$ cards, Betty Calls:

$$\frac{52-b}{52}(-1) \leftarrow \text{Alice fold}$$

$$+\ \frac{b}{52}\Big(\qquad\qquad\Big) \text{ Alice bets}$$

Alice wins $\frac{51}{51}$ top card (card 1)

$\frac{50}{51}$ 2nd card

$\vdots$

$\frac{51-b+1}{51}$ b^{th} card

Alice bets top $\underline{b}$ cards. Betty folds (if Alice bets) average payout is

$$\frac{b}{52}\cdot 1 + \frac{52-b}{52}(-1)$$

$$= \frac{2b-52}{52}$$

(nice function of b, linear function of b)

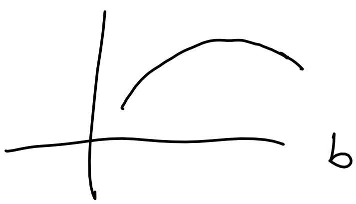
$$\frac{b-52}{52} + \frac{b}{52}\left(4 f(b) - 2\right)$$

$$= \frac{b-52}{52} + \frac{b}{52}\left(\frac{4}{2}\left(\frac{102-b+1}{51}\right) - 2\right)$$

$$= \text{linear}(b) + (-b^2)\,\frac{1}{52}\cdot\frac{4}{2}\cdot\frac{1}{51}$$

$$= \text{linear}(b) + c \cdot b^2$$

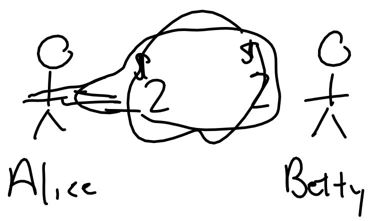
$c < 0$.



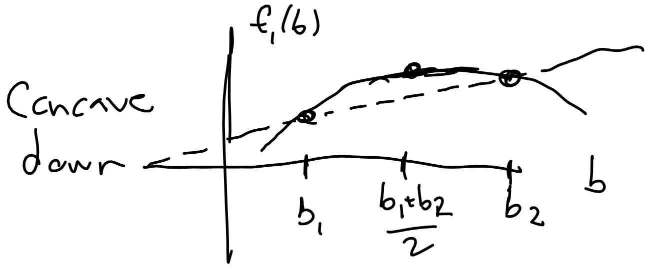
$$\frac{52-b}{52}(-1) + \frac{b}{52}\left(\qquad \right)$$

avg chance Alice wins $f(b) = \frac{1}{2}\left(\frac{51}{51} + \frac{51-b+1}{51}\right)$

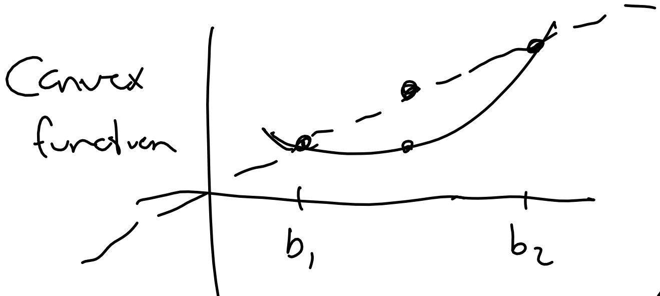
Betty calls:



$$\frac{b}{52}\Big(\underbrace{f(b)(2) + (1-f(b))(-2)}_{4f(b)-2}\Big)$$



$$f_1\left(\frac{b_1+b_2}{2}\right) \geq \frac{f(b_1)+f(b_2)}{2}$$



$$f\left(\frac{b_1+b_2}{2}\right) \leq \frac{f(b_1)+f(b_2)}{2}$$

$$\text{Alice bets } b \quad \begin{bmatrix} \vdots & \vdots \\ f_1(b) & f_2(b) \\ \vdots & \vdots \end{bmatrix}$$

$f_2(b) = \text{linear}(b)$

$f_1(b) =$ quadratic with $(-)(b^2)$ term

We want play

$$\begin{bmatrix} f_1(1) & f_2(1) \\ f_1(3) & f_2(3) \\ \\ f_1(5) & f_2(5) \end{bmatrix}$$

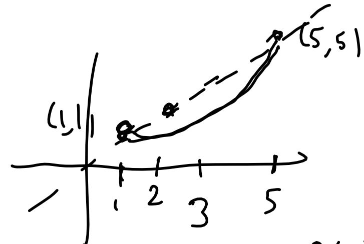
if

$$f_1(3) \le \frac{f_1(1) + f_1(5)}{2}$$

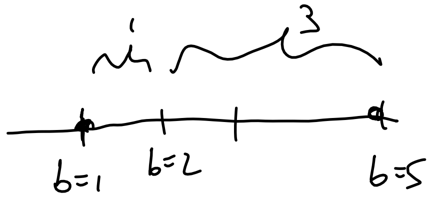
$$f_2(3) \le \frac{f_2(1) + f_2(5)}{2}$$

e.g.

$$\begin{array}{cc} \begin{bmatrix} 5 & 1 \\ . & 2 \\ 2.5 & 3 \\ . & 4 \\ 1 & 5 \end{bmatrix} & \begin{array}{l} b=1 \\ b=2 \\ . \\ . \\ b=5 \end{array} \\ f_1(b) \quad f_2(b) & \end{array}$$



$$f(3) \le \frac{f(1) + f(5)}{2}$$



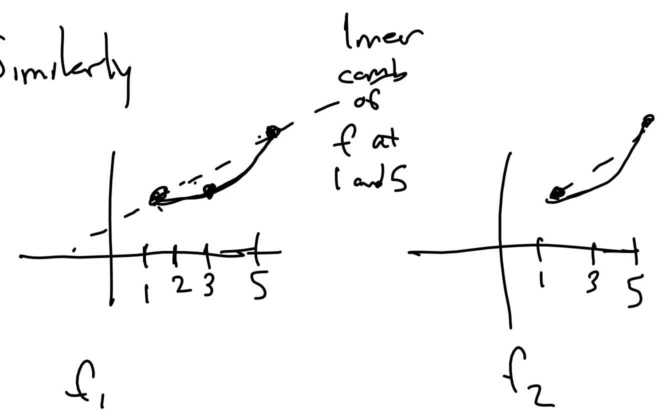
$$f_1(2) \le \frac{3}{4} f_1(1) + \frac{1}{4} f_1(5)$$

$$b^{th} \text{ row} \rightarrow \begin{bmatrix} f_1(1) & f_2(1) \\ f_1(b) & f_2(b) \\ \vdots & \vdots \\ f_1(5) & f_2(5) \end{bmatrix}$$

Concave up
$\Rightarrow$ play extremes

Concave down
play "middle"

Similarly



$$\begin{bmatrix} f_1(3) & f_2(3) \end{bmatrix} \le \frac{1}{2}\begin{bmatrix} f_1(1) & f_2(1) \end{bmatrix} + \frac{1}{2}\begin{bmatrix} f_1(5) & f_2(5) \end{bmatrix}$$